PHYSICS 8A Final Exam – Spring 2017 - C. Bordel

Lecture 2 - Thursday, May 11th, 3pm-6pm

- Student name:
- Student ID #:
- Discussion section #:
- Name of your GSI:
- Day/time of your DS:

<u>Physics Instructions</u>

In the absence of any information about the size of the objects, treat them as point masses. Assume that air resistance is negligible, and consider that the acceleration of gravity has magnitude g at the surface of the Earth.
Remember that you need to show your work in order to get full credit!

<u>Math Information Sheet</u>

- For circle of radius r:

  Circumference: $2\pi r$
  Area: πr^2

- Quadratic Equations
  The solution so the equation $0 = ax^2 + bx + c$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Derivatives

$$\frac{d(x^\alpha)}{dx} = \alpha x^{\alpha-1}$$

- Integrals

$$\int x^n dx = \frac{1}{n+1} x^{n+1}$$

Trigonometric Identities

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\sin\left(\theta \pm \frac{\pi}{2}\right) = \pm\cos\theta$$

$$\cos\left(\theta \pm \frac{\pi}{2}\right) = \mp\sin\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

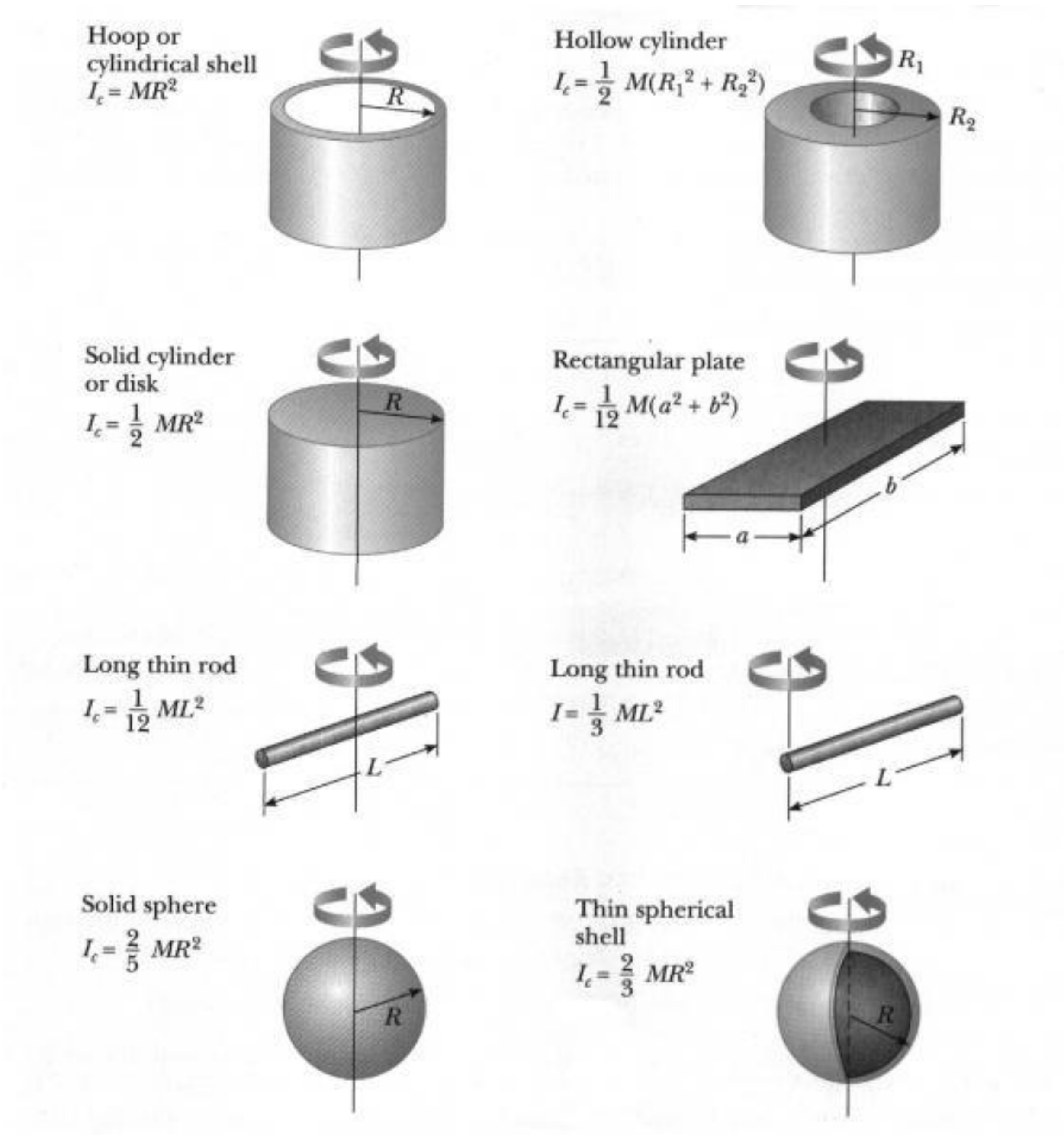
$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

<u>**Rotational inertias**</u>



<u>Problem 1</u> – Projectile game (25 pts)

A skeet (clay target) of mass M is fired at an angle θ from the horizontal with an initial speed v_0. When it reaches the maximum height h, it is hit from below by a pellet of mass m traveling vertically upward at a speed v (Fig. 1). Right after the collision, the pellet is embedded in the skeet.

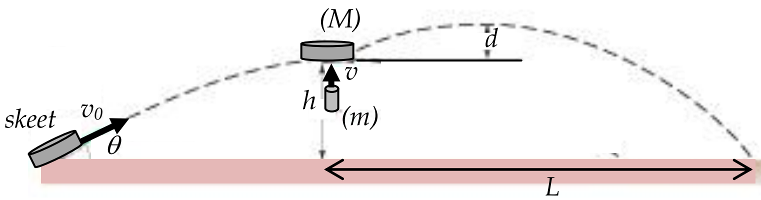


Figure 1

a- Determine the maximum height h reached by the skeet before the collision.

b- Determine the magnitude and direction of the velocity of the skeet-pellet system right after the collision.

c- By what distance d does the skeet-pellet system go higher than h due to the collision?

d- What is the horizontal distance L traveled by the skeet-pellet system after the collision?

<u>Problem 2</u> – Measuring Rotational Inertia (25 pts)

The apparatus shown in Figure 2 is used to measure the rotational inertia I of an object of non-traditional shape. To perform the experiment, the object is mounted to a vertical axle held in a frame with essentially frictionless bearings. A spool of radius b is also mounted to the axle, and an ideal string is wrapped around the spool. The combination of axle and spool has a non-negligible rotational inertia I_0 whose value is known. The string runs horizontally over a frictionless pulley and is tied to a block of mass m. The mass is suspended a height h above the floor and the rotating system is initially at rest. Then the block is released, and we measure the time t that it takes for the block to reach the floor.

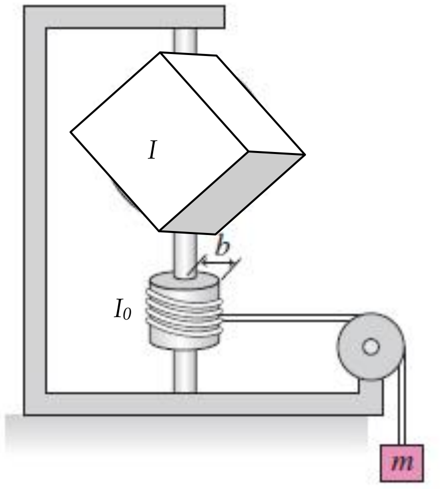


Figure 2

a- Using <u>kinematics</u>, determine the acceleration a of the block after it is released, in terms of h and t.

b- Determine the magnitude T of the tension force acting throughout the string in terms of m, g, h and t.

c- Determine the rotational inertia *I* of the object in terms of *m*, *g*, *h*, *t*, *b* and I_0.

d- If you were to mount this particular object differently, would you anticipate the rotational inertia to be the same or not?

Problem 3 – Cavendish experiment (25 pts)

The apparatus constructed by Cavendish is a torsion balance (Fig.3) made of a horizontal rod of mass m_r and length L suspended from a wire in the middle, with a small solid sphere of mass m and radius r attached to each end. Two larger and heavier solid balls of mass M and radius R are also located on a circle of radius $L/2$ from the torsion wire, and held in place with a separate suspension system (not shown in Fig.3). The two large balls are initially positioned along a line roughly perpendicular to the horizontal arm of the balance, initially in its equilibrium position ($\theta=0$). The gravitational attraction causes the arm to rotate, twisting the wire supporting the arm by an angle θ and thereby bringing the small spheres a distance d away from the large ones. You may consider that θ is close to 90 degrees, which results in a small angle and small distance between the large and small spheres. The wire has a torsional constant K and can be considered massless.

The experiment measures the faint gravitational attraction between the small balls and the larger ones, and provided the first accurate value of the gravitational constant G.

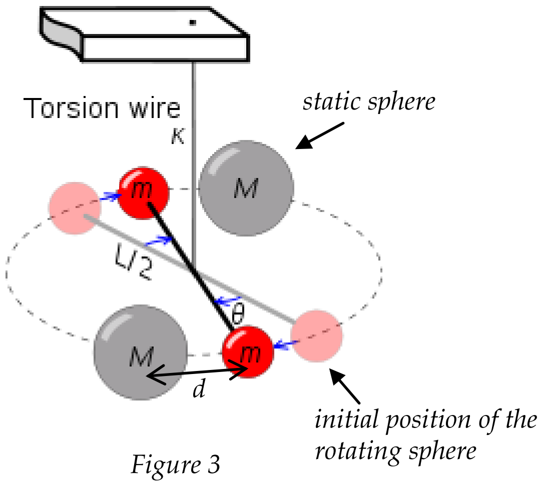


Figure 3

a- Assuming the system is in static equilibrium in translation, determine the magnitude of the net upward force acting against gravity on the system made of the wire, rod and 4 spheres.

b- Establish the equation satisfied by the system when it reaches static equilibrium in rotation.

c- Assuming that the distance d can be approximated by the length of the arc separating two closeby spheres, determine the gravitational constant G.
Hint: Use a proportionality equation based on the length of the circumference of a circle.

d- Explain why the gravitational force exerted by the Earth on the system does not play any role in the static equilibrium reached in rotation.

Problem 4 – Mass-spring systems and clamped rope (25 pts)

A simple harmonic oscillator is a system that can be described by the following differential equation: $\frac{d^2x}{dt^2} + \omega^2 x = 0$. In a traditional horizontal mass-spring system, the block of mass *m,* attached to an ideal spring of constant *k,* can slide without friction.

a- Establish the differential equation satisfied by the horizontal displacement x of the block, and determine the angular frequency ω of the oscillator.

In a less traditional mass-spring system, the object attached to the spring is a solid disk of mass M and radius R that can roll without slipping, as shown in Figure 4.

b- Establish the differential equation satisfied by the horizontal displacement x of the center of mass of the disk, and determine the angular frequency ω of the oscillator. *Hint: Write the statement of energy conservation for this system and differentiate it.*

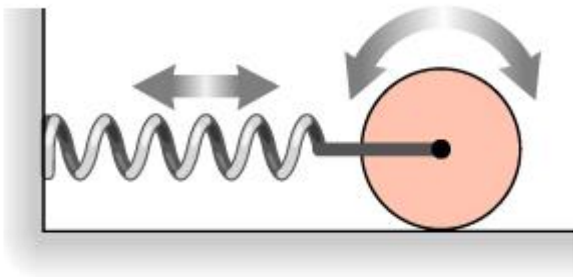


Figure 4

We consider a rope of length L, clamped at both ends, along which a sinusoidal wave travels in the *(+x)* direction. The reflected wave, which conserves the same amplitude and frequency, travels in the opposite direction after experiencing an inversion upon reflection. The interference between the two traveling waves results in a wave described by the following function of position and time: $y_{tot}(x,t) = 2A\sin(kx)\sin(\omega t)$, where A is the amplitude of the incoming (and reflected) wave, and k is the wave number.

c- From the mathematical expression of the resultant wave, explain why it qualifies as a standing wave.

d- Determine the condition that needs to be satisfied by the wavelength λ for the standing wave to exist on this rope clamped at both ends.

<u>Problem 5</u> – Fluids (25 pts)

Some mass m_{ice} of ice at temperature T_{ice} is dropped in a thermos containing some mass m_w of water at temperature T_w. The ice-water mixture can be considered as a closed system. Take L_f as the latent heat of fusion, c_w as the specific heat of water and c_{ice} as the specific heat of ice.

a- Assuming that all the ice melts and reaches a temperature that is above the melting temperature, determine the equilibrium temperature T_f of the system.

The thermos, which can be considered as a cubic box of side length L, is actually not perfectly insulating and allows some heat to be exchanged by conduction. Take k as the thermal conductivity of the thermos and d its thickness ($d \ll L$).

b- Assuming that the rate of heat flow has reached a steady state, where the water in the thermos is at temperature T_f and the outside of the thermos is at ambient temperature $T_a > T_f$, determine the amount of heat exchanged during the time Δt. Specify the direction of the heat flow.

Some water, of density ρ_w, is initially at rest in a tank open to atmosphere, until the cap is removed from the opening, a height h below the initial surface of the water (Fig. 5.1). You may consider that the size of the hole is very small compared to the diameter of the tank.

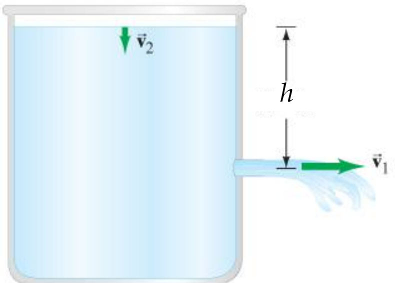


Figure 5.1

c- Determine the initial speed v_1 of the water that flows out of the tank, and explain if it increases or decreases as a function of time.

A beaker contains a thick layer of oil of density ρ_2 floating on water, which has density ρ_3. A cubic block of wood of density ρ_1 with side length L is gently lowered into the beaker, so as not to disturb the layers of liquid, until it floats peacefully between the layers, as shown in Figure 5.2.

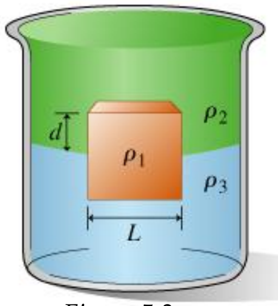


Figure 5.2

d- What is the distance d between the top of the wood cube (after it has come to rest) and the interface between the oil and water?

<u>Problem 6</u> – Thermodynamic Cycle (25 pts)

n moles of an ideal diatomic gas undergo the following processes. From point A, whose thermodynamic state is defined by a pressure P_A and volume V_A, the gas undergoes an isothermal expansion *(AB)*, followed by an isobaric compression *(BC)* at pressure P_C, and an adiabatic compression *(CA)*.

a- Based on the number of degrees of freedom of the gas, calculate the adiabatic exponent γ. *Hint: the internal energy and therefore the specific heat at constant volume depend on the number of degrees of freedom of the gas.*

b- Determine the unknown pressures, volumes and temperatures of points A, B and C, and plot the cycle on a PV diagram.

c- Determine the net work W_{net} done on the gas over a full cyclic process and comment on the sign.

d- Determine the efficiency of a heat engine operating under the cyclic process ($ABCA$). The efficiency is given by $e = \frac{|W_{net}|}{Q_{in}}$, where Q_{in} is the heat input over the entire cycle.