Physics 7b
Fall 2006
Midterm 1
R. Packard

Work all problems. Introduce and clearly define algebraic symbols to represent all physical quantities. Do not perform numerical work until you have a final algebraic answer within a box. Check the dimensions of your answer before inserting numbers. Work the easiest parts first, and the next hardest, etc. If you do not understand the question ask the proctor for assistance. All problems are weighted equally.

$k_B=1.38\text{x}10^{-23}$J/K, $N_A=6.02\text{x}10^{23}$, latent heat of fusion of water $3.3\text{x}10^5$J/kg, specific heat of ice, 2100J/kgC°, specific heat of liquid water 4186J/kgC°, $\sigma=5.6\text{x}10^{-8}\text{W/m}^2\text{K}^4$

When considering gas processes you may assume the gas follows the ideal gas law.

Name Solutions

SID__________

Sect. # or day and time __________

TA name (if known)__________

1._____

2._____

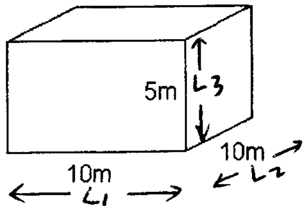
3._____

4._____

5._____

Total______

Problems 1,2, and 3 all are based on the same "house" described in problem 1.



1. Consider a house with a square "footprint" (i.e. base) 10m on each side and height 5m. Air is ~80% N_2 with a molecular weight M_W=28g/mole

a. (5pts) Estimate how many air molecules fill the interior space of the house when the inside temperature is 20°C. Tell what assumptions you made for the estimate.

b. (5pts) The walls of the house have a linear expansion coefficient $\alpha=1\times10^{-5}/°C$. If the temperature of the walls changes by 10°C, what is the fractional change in the interior volume?

c. (5pts) Assuming that the house is not air tight, what is the fractional change in the number density of molecules (i.e. number per unit volume) corresponding to this 10°C temperature rise.?

a) assume pressure is at 1 atm $\simeq 1.013\times10^5$ Pa

According to ideal gas law $PV = NKT$

$$N = \frac{PV}{KT} = \frac{1.013\times10^5 \times (5\times10\times10)}{1.38\times10^{-23}\times(273+20)} = \boxed{1.253\times10^{28}}$$

b) For linear expansion, $\Delta L = \alpha L_0 \Delta T$

$\Delta L_1 = 1\times10^{-5}\times10\times10 = 10^{-3}$ m

$\Delta L_2 = \Delta L_1 = 10^{-3}$ m

$\Delta L_3 = 1\times10^{-5}\times5\times10 = 5\times10^{-4}$ m

$\therefore$ final volume $= V_f = (10+10^{-3})\times(10+10^{-3})\times(5+5\times10^{-4})$ m^3

initial volume $= V_0 = 10\times10\times5$ m^3

(Here I assume the temperature rises 10°C, you can also assume the temperature drops 10°C.)

$$\frac{\Delta V}{V_0} = \frac{V_f - V_0}{V_0} \simeq \boxed{3\times10^{-4}}$$

OR: For volume expansion, $\Delta V = \beta V_0 \Delta T$

$\beta \simeq 3\alpha$

$\therefore \frac{\Delta V}{V_0} = \beta \Delta T = 3\times10^{-5} \times 10 = 3\times10^{-4}$

c) Since the house is not air tight. the pressure keeps constant. Also for this part, the temperature rises 10°C

Initially $PV_0 = N_0 k T_0 \Rightarrow \frac{N_0}{V_0} = \frac{P}{kT_0}$ (which is number density)

Finally $PV_f = N_f k T_f \Rightarrow \frac{N_f}{V_f} = \frac{P}{kT_f}$

$$\therefore \frac{\frac{N_f}{V_f} - \frac{N_0}{V_0}}{\frac{N_0}{V_0}} = \frac{\frac{P}{kT_f} - \frac{P}{kT_0}}{\frac{P}{kT_0}} = T_0 \left(\frac{1}{T_f} - \frac{1}{T_0} \right)$$

$$= 293 \left(\frac{1}{293+10} - \frac{1}{293} \right)$$

$$= -0.033$$

2a) In raising the temperature of a substance, the heat required will have the general form

$$Q = n C \Delta T \quad (\text{or } m C \Delta T).$$

The value of C depends on the process used to raise the temperature. There are two possibilities that will be accepted: constant volume (because limited by ability of house to expand) or constant pressure (because house might not be air tight, then pressure is determined by environment).

constant volume:

$$Q = \frac{N}{N_A} C_V \Delta T$$

$$C_V = \frac{D}{2} R$$

assume diatomic (air is mostly N_2) $D = 5$

$$Q = N \cdot \frac{D}{2} R / N_A \, \Delta T$$
$$(= \tfrac{D}{2} k)$$

$$= \frac{5}{2} \cdot 1.38 \times 10^{-23} \frac{J}{K} \cdot 15 K$$

$$= 5.175 \times 10^{-21} J$$

$$1 J = 1 W \cdot s = 10^{-3} kW \cdot s \cdot \frac{1 hr}{3600 s}$$
$$= 2.78 \times 10^{-7} kW \cdot hr$$

$$\boxed{Q = 1.44 \times 10^{-28} kWhr \cdot N}$$

constant pressure:

$$Q = N \frac{C_P}{N} \Delta T$$

$$C_P = \frac{D+2}{2} R$$

$$Q = \frac{7}{2} \cdot 1.38 \times 10^{-23} \frac{J}{K} \cdot 15 K$$
$$\boxed{= 2.01 \times 10^{-28} kWhr \cdot N}$$
$$(= 7.24 \times 10^{-22} J \cdot N)$$

b) Power is defined as $\frac{\text{energy}}{\text{time}}$

$$P = 5 kW = \frac{Q_{(a)}}{\Delta t}$$

$$\Delta t = \frac{Q_{(a)}}{P}$$

const vol.
$$\Delta t = \frac{1.44 \times 10^{-28} kWhr \cdot N}{5 kW}$$
$$= 2.88 \times 10^{-29} hr \cdot N$$
$$= 1.04 \times 10^{-25} s N$$

constant pressure: $\Delta t = 4.03 \times 10^{-29} hr \cdot N = 1.45 \times 10^{-25} s \cdot N$

First we need to know the rate heat flows out of the house in the summer. We use the rate in the winter to solve for the relevant combination of variables. $\frac{KA}{\ell}$

3. (15pts) This house requires 5kW of heat on a winter day when it is 13°C outside and 21°C inside. In the summer this house is cooled by an air conditioner (a refrigerator unit which cools the inside, pumping the heat to the outside) which operates at 75% of the Carnot coefficient of performance for a refrigerator. When it is 37° outside, how much does it cost to cool the house for one month if the inside temperature remains at 21°C inside and if the cost of electricity is \$0.14/kWhr. Assume the inside and outside temperatures remain constant.

Winter: $\boxed{H_w = \frac{KA}{\ell}(T_{out} - T_{in})} = 5\,kW = \frac{KA}{\ell}(21-13)$

$\boxed{\frac{KA}{\ell} = 0.625\,\frac{kW}{K}}$ Now solve for heat flow in summer.

Summer: $H_S = \frac{KA}{\ell}(T_{out} - T_{in}) = 0.625\,\frac{kW}{K}(37-21) = 10\,kW$

Now we find the total energy ~~[illegible]~~ removed in 1 month:

$\boxed{Q_L = H_s \cdot t_{1\,month}} = 10kW \cdot 1\,month \cdot \frac{30\,days}{1\,month} \cdot \frac{24\,hrs}{1\,day} = 7200\,kWhrs$

$e = \frac{3}{4}\, e_{\substack{Carnot \\ Refridgerator}} = \frac{3}{4}\left(\frac{T_H}{T_L} - 1\right)^{-1} = \frac{3}{4}\left(\frac{310}{294} - 1\right)^{-1}$

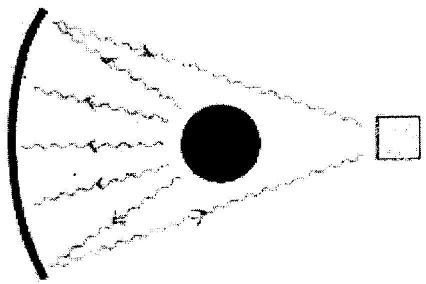
$e = 13.8$ $\quad e = \frac{Q_L}{W}$ $\quad W = \frac{Q_L}{e}$

$W = \frac{7200\,kWhrs}{13.8} = 522\,kWhrs$

$Cost = W \cdot Rate = 522\,kWhrs \cdot \frac{\$0.14}{kWhr} = \underline{\underline{\$73.10}}$

We can use the efficiency to find the work required to expell Q_L of heat. This work is what has to be paid for, because the electricity supplies the work.

4. (15pts) A metal ball of radius 1m is heated to a temperature of 800°C. The ball is placed near a curved mirror so that 20% of the emitted thermal radiation falls onto a small box containing 5kg of ice at -10°C. Assuming the ball's emissivity is ε=0.6 and the absorptivity of the box is 1, how long does it take to melt the ice and raise the water within to +20°C?



note:
We neglect the power radiated by the box because it is roughly $\left(\frac{273}{1073}\right)^4$ the size of the power radiated by the sphere. $\left(\frac{273}{1073}\right)^4 = .004$

First, we find the total power radiated by the sphere:

$P_{out} = \varepsilon_s A_s \sigma T_s^4$, the surface area of a sphere is $4\pi r^2$

Power is a change in energy, so $P = \frac{dQ}{dt}$, the ball does not change in time, so $P = \frac{\Delta Q}{\Delta t} = \frac{\Delta Q}{t}$

Next we find the power absorbed by the box

$P_{in} = \varepsilon_b \left(.2 P_{out}\right) = \frac{\Delta Q_{in}}{t}$ $\qquad$ $P_{in} = \varepsilon_b \varepsilon_s (.2) \sigma (4\pi r^2) T_s^4$

The contents of the box warm, melt, and then warm further, we can find the heat required for each of these steps

$\Delta Q_{ice\ warming} = m_{ice} C_{ice} \Delta T_{ice}$

$\Delta Q_{ice\ melting} = m_{ice} L_{ice}$

$\Delta Q_{water\ warming} = m_{water} C_{water} \Delta T_{water}$ (we note that $m_{ice} = m_{water}$, it is the same stuff)

so $\Delta Q_{tot} = m_{ice} C_{ice} \Delta T_{ice} + m_{ice} L_{ice} + m_{ice} C_{water} \Delta T_{water} = m\left(C_{ice} \Delta T_{ice} + L_{ice} + C_{water} \Delta T_{water}\right)$

All that is left is to solve for t

$P_{in} = \frac{\Delta Q_{in}}{t}$ so $t = \frac{\Delta Q_{in}}{P_{in}} = \boxed{\frac{m\left(C_{ice} \Delta T_{ice} + L_{fusion} + C_{water} \Delta T_{water}\right)}{(.2)\varepsilon_b \varepsilon_s \sigma (4\pi r^2) T_s^4}}$

$$t = \frac{5\,kg\left(2100 \frac{J}{kg\,°C} \cdot 10°C + 3.3\times10^5 \frac{J}{kg} + 4186 \frac{J}{kg\,°C} \cdot 20°C\right)}{.2 \cdot 1 \cdot .6 \left(5.6\times10^{-8} \frac{J}{s\,m^2\,K^4}\right)\left(4\pi\, 1\,m^2\right)\left((800+273)K\right)^4} = \frac{2.17\times10^6\,J}{1.12\times10^5\,\frac{J}{s}} = \boxed{19.4\text{ Seconds}}$$

5. (15pts) (Prob. 20-39 from homework) What is the total change in entropy when 2.5 kg of water at 0°C is frozen to ice at 0°C by being in contact with 450 kg of ice at –15°C?

As the water undergoes a phase transition, the heat it gives up is equal to the heat absorbed by the block of ice.

For the water: $T = 273\,K$

$$\Delta S_{water} = \int \frac{dQ}{T} = -\frac{Q}{T_{water}} = -\frac{m_{water} L_{fusion}}{T_{water}}$$

the sign is (-) because heat flows out.

so $\Delta S_{water} = \dfrac{-2.5\,kg\,(3.3\times10^5\,J/kg)}{273K} = -3.02\times10^3\,J/K$

For the ice: The heat absorbed raises the temperature slightly.

$$Q = mc\Delta T = m_{ice}c_{ice}(T_f - T_0)$$

$$m_{ice}c_{ice}T_f - m_{ice}c_{ice}T_0 = Q \qquad T_f = \frac{Q + m_{ice}c_{ice}T_0}{m_{ice}c_{ice}}$$

$$Q = m_{water}L_{fusion} = (2.5kg)(3.3\times10^5\,J/kg) = 8.25\times10^5\,J/kg$$

$$mc = 450\,kg\,(2100\,J/kg\,K) = 9.45\times10^5\,J/K$$

so $T_f = 258.87\,K \Rightarrow T_{avg} = \dfrac{258 + 258.87}{2}\,K = 258.4\,K$

Since the temperature change is so small, in finding the entropy change, it doesn't make much difference if we use 258 or 258.8K.

So, using 258.4 K (The avgerage temperature), $\Delta S_{ice} = \dfrac{m_{water} L_{fusion}}{T_{avg}} = 3.19\times10^3\,J/Kg$

Thus $\Delta S_{TOTAL} = \Delta S_{water} + \Delta S_{ice} = \boxed{170\,J/K}$