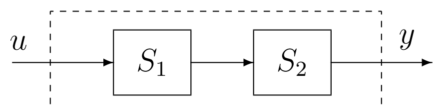
Be sure to check that all of your matrix manipulations have the correct dimensions, and that the concatenations have compatible dimensions (horizontal concatenations must have the same number of rows, vertical concatenation must have the same number of columns).

6. Consider the interconnection below. The transfer functions of systems S_1 and S_2 are

$$G_1(s) = \frac{3}{s+6}, \qquad G_2(s) = \frac{s+2}{s+1}$$

Determine the differential equation governing the relationship between u and y.

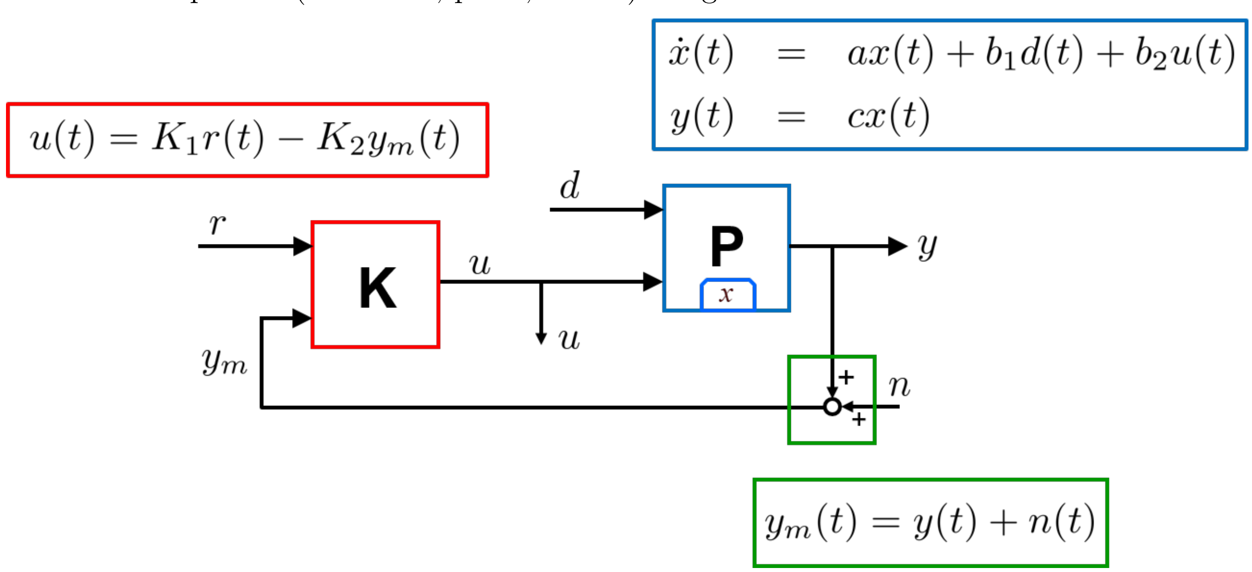


30 Recent exams

30.1 Fall 2017 Midterm 1

# 1	# 2	# 3	# 4	#5	#6	NAME
25	10	12	25	20	8	

1. A closed-loop feedback system is shown below. Signals are labeled and equations for each component (controller, plant, sensor) are given.



Keep the plant parameter a as a general value, but assume $b_1 = b_2 = c = 1$, for simplicity. K_1 and K_2 are gains (constants). Keep these as variables. Specific values will be designed in part (b) of the problem.

(a) For the closed-loop system above, fill in the 3×4 matrix which relates (x, r, d, n) to $(\dot{x}, y, u)$. Your expressions should involve variables (K_1, K_2, a).

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \\ u(t) \end{bmatrix} = \left[\begin{array}{c|c|c|c} \quad & \quad & \quad & \quad \\ \hline \quad & \quad & \quad & \quad \\ \hline \quad & \quad & \quad & \quad \end{array}\right] \begin{bmatrix} x(t) \\ r(t) \\ d(t) \\ n(t) \end{bmatrix}$$

(b) Now take $a = -1$ so that the plant, by itself, is stable, and hence has a time-constant of $\tau_P = 1$. Suppose one goal of feedback control is to achieve closed-loop stability, and **make the closed-loop system respond faster**, so that the closed-loop time constant is less than τ_P. Specifically, work in ratios, expressing this design requirement that the closed-loop time-constant, τ_{CL}, should be a fraction γ of the plant time constant, namely

$$\text{Design Requirement \#1 :} \qquad \tau_{CL} = \gamma \cdot \tau_P$$

where $0 < \gamma < 1$ is a given design target. The other design requirement is

$$\text{Design Requirement \#2 :} \qquad \text{SSG}_{r \to y} = 1$$

in words, the steady-state gain from $r \to y$ should equal 1. **Task:** As a function of γ, find expressions for K_1 and K_2 which simultaneously achieve the two Design Requirements.

(c) For the closed-loop system, what is the **instantaneous gain** from $r \to u$, as a function of the design parameter γ? Is this gain increasing or decreasing as γ decreases? Explain this relationship intuitively (ie., "if we require the system to respond more quickly, with perfect steady-state behavior from $r \to y$, the instantaneous effect that r must have on u....").

(d) For the closed-loop system, what is the **steady-state gain** from $r \to u$, as a function of the design parameter γ? How is this affected as γ decreases? Explain this relationship intuitively (ie., "if we require the system to respond more quickly, with perfect steady-state behavior from $r \to y$, the steady-state effect that r must have on u....").

(e) For the closed-loop system, what is the **steady-state gain** from $d \to y$, as a function of the design parameter γ? How is this affected as γ decreases?

2. **Basic System Properties:** The equations governing a 3-input, 3-output system are

$$\begin{bmatrix} \dot{x}(t) \\ y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 & 3 \\ 1 & 0 & 0 & -1 \\ 2 & 4 & -3 & 0 \\ 1 & -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$

(a) Is the system stable?

(b) What is the time-constant of the system?

(c) What is the steady-state gain from u_3 to y_2?

(d) What is the instantaneous-gain from u_2 to y_2?

(e) What is the frequency-response function $G(\omega)$ from u_1 to y_3?

(f) Suppose $x(0) = 10$ and

$$\lim_{t\to\infty} u_1(t) = 2, \quad \lim_{t\to\infty} u_2(t) = 1, \quad \lim_{t\to\infty} u_3(t) = 1.$$

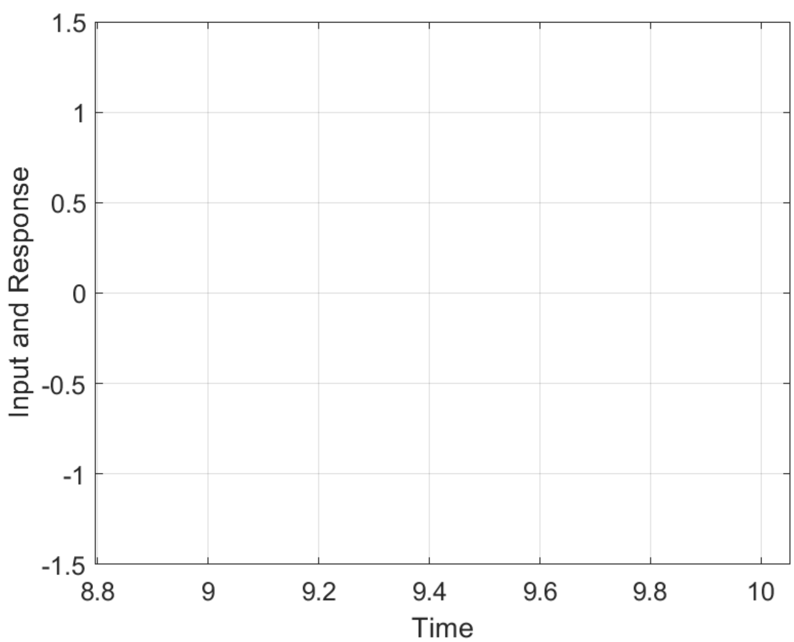
What is value of $\lim_{t\to\infty} y_3(t)$

3. (a) Define the complex number $G = \frac{19.5}{j5+12}$.
   i. Find the value of $|G|$

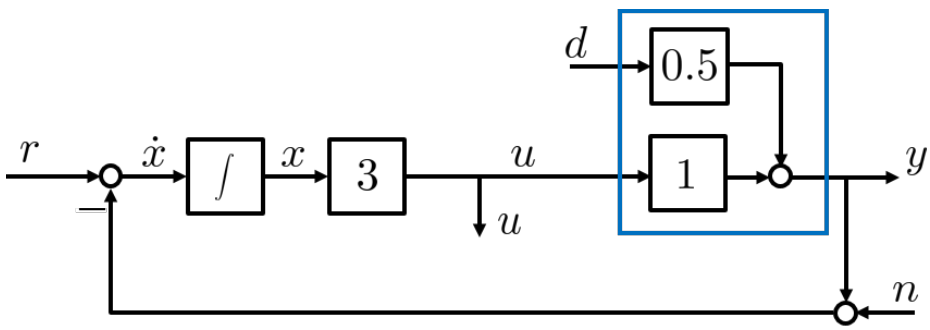
   ii. Find the value of $\angle G$

(b) Sketch the final output (in the axes) of the Matlab code. **Carefully focus on the period, amplitude and time-alignment of the input signal (dashed) and response signal (solid).**

```
w = 5;  TF = 10*2*pi/w; x0 = -2;
uH = @(z) sin(w*z);
fH = @(t,x) -12*x + 19.5*uH(t);
[tSol, xSol] = ode45(fH,[0 TF],x0);
plot(tSol, uH(tSol),'--', tSol, xSol);  % input=dashed; solution=solid
xlim((2*pi/w)*[7 8]);   % reset horz limits to exactly cover 1 period
```



4. A closed-loop feedback system is shown below. Signals are labeled. Note that the one marked summing junction has a $-$ sign, as typical of our negative feedback convention. Treat all <u>unmarked</u> summing junctions as $+$.



Fill in the 3×4 matrix which relates (x, r, d, n) to $(\dot{x}, y, u)$ as shown below

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \end{bmatrix} \begin{bmatrix} x(t) \\ r(t) \\ d(t) \\ n(t) \end{bmatrix}$$

(a) What is the time-constant of the closed-loop system?

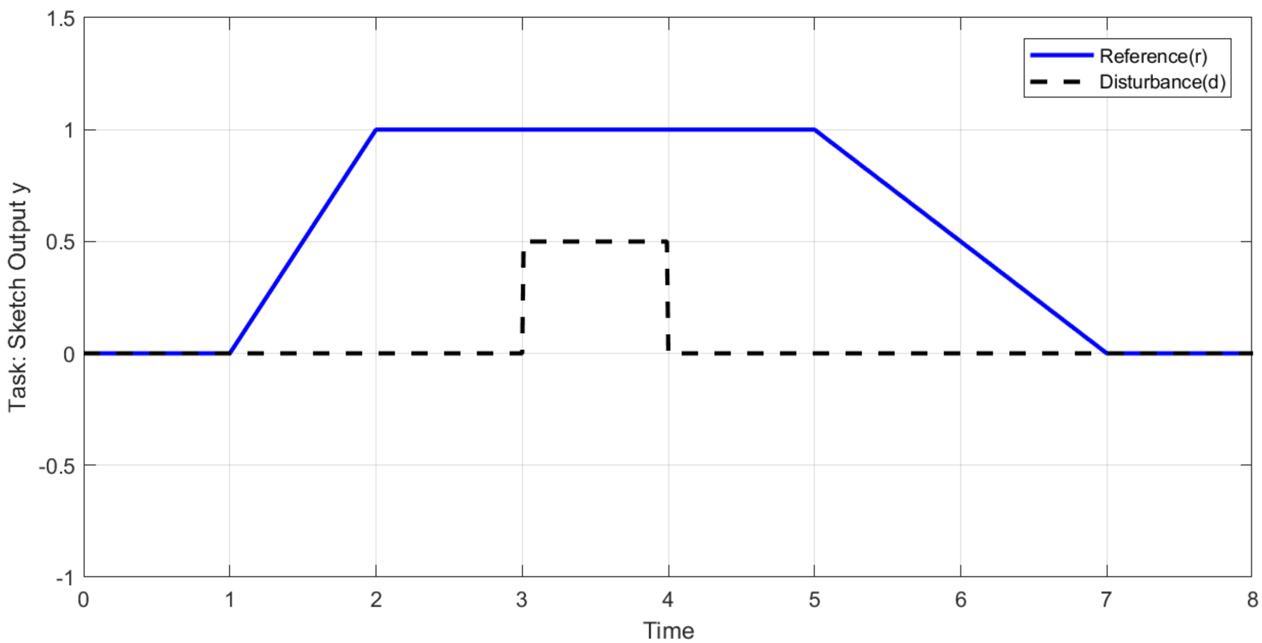
(b) What is the **steady-state gain** from $r \to y$?

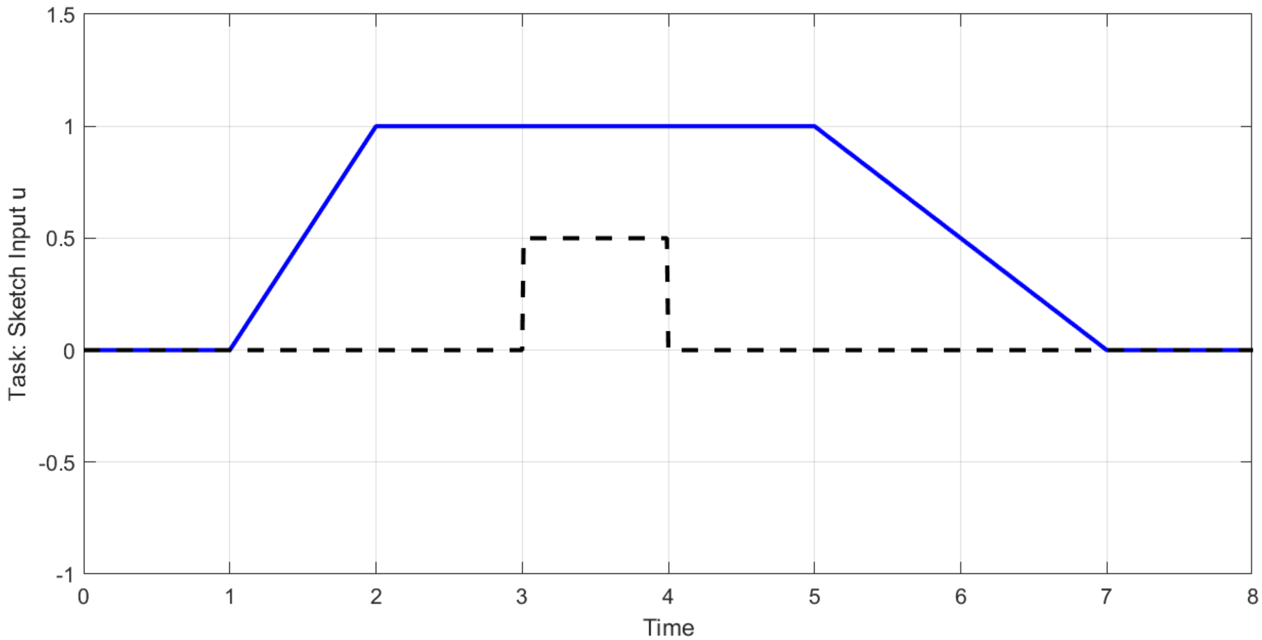
(c) What is the **steady-state gain** from $d \to y$?

(d) What is the **instantaneous gain** from $r \to y$?

(e) What is the **instantaneous gain** from $d \to y$?

(f) The two axes below show a specific reference input r (solid) and disturbance input d (dashed). *These are the same in both axes.* Assume $n(t) \equiv 0$ for all t. The closed-loop system starts from $x(0) = -0.2$, and is forced by this reference and disturbance input. Make careful sketches of $y(t)$ and $u(t)$ in the top and bottom axes, respectively (note that they are individually marked with task "Sketch Output y" and "Sketch Input u").




1.5
1
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
Time
Task: Sketch Input u

5. Consider the delay-differential equation

$$\dot{x}(t) = A_1 x(t) + A_2 x(t - T)$$

where A_1, A_2 and T are real-valued constants. $T \geq 0$ is called the "delay." Depending on the values, there are 3 cases:

- The system is unstable for $T = 0$; or
- The system is stable for all $T \geq 0$; or
- The system is stable for $T = 0$, but unstable for some positive value of T. In this case, we are interested in the smallest $T > 0$ for which instability occurs, and the frequency of the nondecaying oscillation that occurs at this critical value of delay.

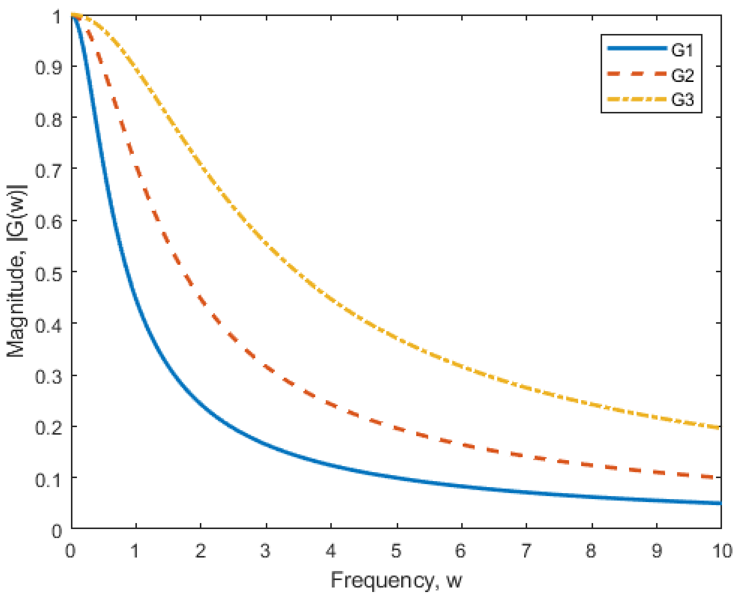
Fill in the table below. In each row, please mark/check one of the first three columns (from the three cases above). **If you check the 3rd column, then include numerical values in the 4th and 5th columns associated with the instability. Show work below.**

	unstable at $T = 0$	stable for all $T \geq 0$	stable at $T = 0$, but unstable at some finite $T > 0$	frequency at which instability occurs	smallest T at which instability occurs
$A_1 = -1$, $A_2 = -3$					
$A_1 = -4$, $A_2 = -2$					
$A_1 = 1$, $A_2 = -3$					
$A_1 = -2$, $A_2 = 3$					

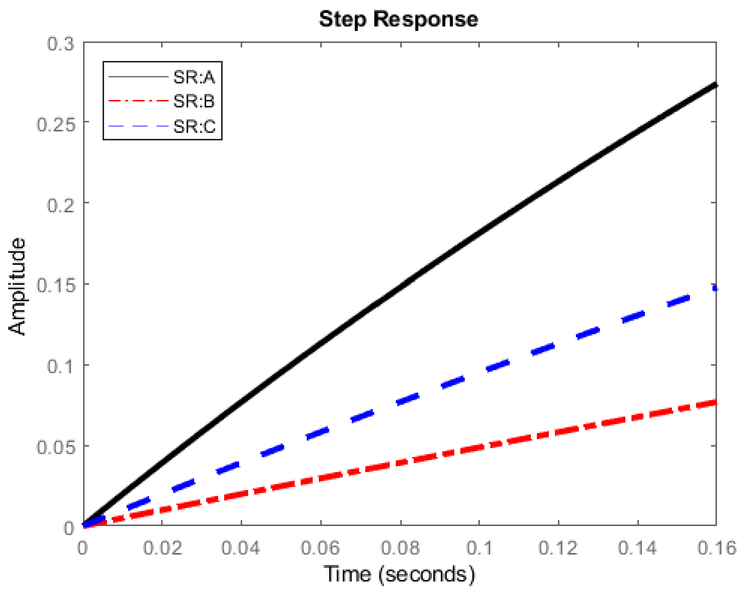
6. Three first-order systems, **Sys1**, **Sys2**, **Sys3**, all stable, have the familar form

$$\underbrace{\begin{array}{rcl} \dot{x}_1(t) & = & a_1 x_1(t) + b_1 u_1(t) \\ y_1(t) & = & c_1 x_1(t) + d_1 u_1(t) \end{array}}_{\textbf{Sys1}} \quad \underbrace{\begin{array}{rcl} \dot{x}_2(t) & = & a_2 x_2(t) + b_2 u_2(t) \\ y_2(t) & = & c_2 x_2(t) + d_2 u_2(t) \end{array}}_{\textbf{Sys2}} \quad \underbrace{\begin{array}{rcl} \dot{x}_3(t) & = & a_3 x_3(t) + b_3 u_3(t) \\ y_3(t) & = & c_3 x_3(t) + d_3 u_3(t) \end{array}}_{\textbf{Sys3}}$$

The Magnitude plot of the associated frequency-response functions $G_1(\omega), G_2(\omega)$ and $G_3(\omega)$ are shown below (note, G_1 is the frequency-response function of **Sys1**, etc).



The step-responses of the systems, labeled **SR:A**, **SR:B** and **SR:C** are shown below.



Match <u>each</u> step-response with the corresponding Frequency-response magnitude plot (eg., is **SR:A** the step response of **Sys1**, **Sys2** or **Sys3**?)