

EECS 20. Midterm 1. 2 October 1998

Please use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. **Write clearly and put a box around your answer**

Print your name below

Last Name \_\_\_\_\_ First \_\_\_\_\_

Problem 1

Problem 2

Problem 3

Problem 4

Problem 5

Total

1. **15 points**

(a) Find  $\theta$  so that

$$\operatorname{Re}[(1 + i) \exp i\theta] = -1.$$

(b) Define  $x : \mathit{Reals} \rightarrow \mathit{Reals}$

$$\forall t \in \mathit{Reals}, x(t) = \sin(\omega_0 t + 1/4\pi).$$

Find  $A \in \mathit{Comps}$  so that

$$\forall t \in \mathit{Reals}, x(t) = A \exp(i\omega_0 t) + A^* \exp(-i\omega_0 t),$$

where  $A^*$  is the complex conjugate of  $A$ .

2. **15 points**

Draw the following sets

(a)  $\{(x, y) \in \mathit{Reals}^2 \mid xy = 1\}$ .

(b)  $\{(x, y) \in \mathit{Reals}^2 \mid y - x^2 \geq 0\}$ .

(c)  $\{z \in \mathit{Comps} \mid z^5 = 1 + 0i\}$ .

3. 25 points

(a) Evaluate the truth values of

$$S = [P \wedge (\neg Q)] \vee R$$

for the following values of  $P, Q, R$ .

$P$	$Q$	$R$	$S$
True	False	False	
False	True	False	
True	False	True	

(b) The following sequence of statements is a complete context.

Let

$$x = 5, y = 6 \tag{1}$$

Then,

$$x \neq y \tag{2}$$

Now let

$$Z = \{z \in Reals \mid z \geq x + y\} \tag{3}$$

Then

$$x \in Z \tag{4}$$

Let

$$w = \text{smallest non-negative number in } Z \tag{5}$$

Answer the following:

- i. Are the two expressions in (1) both assignments or assertions?
- ii. Is the expression (2) an assertion or a predicate?
- iii. Is the equality in (3) an assignment or an assertion?
- iv. Is the expression “ $z \geq x + y$ ” in (3) an assertion or a predicate?
- v. Is (4) an assertion or a predicate?
- vi. Is (5) an assignment or an assertion?

**4. 20 points**

A signal is mathematically described as a function. We have studied signals described as functions of time and space and signals described as data and event sequences. For example, a mathematical model of a soundwave is a function  $Sound : Time \rightarrow Pressure$ . Propose mathematical models for the signals corresponding to the following intuitive descriptions. Give a very brief justification for your proposed models.

- (a) A gray-scale video with 256 gray-scale values.
- (b) The position of a bird in flight.
- (c) The sequence of buttons you press with your TV remote control.

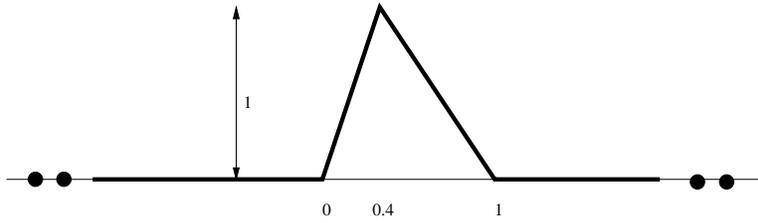


Figure 1: The graph of  $x$

5. **25 points** The function  $x : \mathbb{R} \rightarrow \mathbb{R}$  is given by its graph shown in Figure 1. Note that  $\forall t \notin [0, 1], x(t) = 0$ , and  $x(0.4) = 1$ . Define  $y$  by

$$\forall t \in \mathbb{R}, y(t) = \sum_{k=-\infty}^{\infty} x(t - kp)$$

where  $p \in \mathbb{R}$ .

- (a) Prove that  $y$  is periodic with period  $p$ , i.e.

$$\forall t \in \mathbb{R}, y(t) = y(t + p).$$

- (b) Plot  $y$  for  $p = 1$ .  
(c) Plot  $y$  for  $p = 2$ .  
(d) Plot  $y$  for  $p = 0.5$ .  
(e) Suppose the function  $z$  is obtained by advancing  $x$  by 0.4, i.e.

$$\forall t, z(t) = x(t + 0.4).$$

Define  $w$  by

$$\forall t \in \mathbb{R}, w(t) = \sum_{k=-\infty}^{\infty} z(t - kp)$$

What is the relation between  $w$  and  $y$ . Use this relation to plot  $w$  for  $p = 1$ .