Department of Physics
University of California, Berkeley

Mid-term Examination 1
Physics 7B, Section 1

6:00 pm - 8:00 pm, 4 October 2006

Name: Solutions

SID No:

Discussion Section:

Name of TA:

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	

Score:

1. [27 points] Short Questions

(a) [5 points] Circle T or F for True or False

(T) F (i) When a system goes from equilibrium state 1 to equilibrium state 2, the change in the internal energy is the same for all processes. State Function

(T) F (ii) A macroscopic description is given in terms of quantities that are an average over the microscopic details and are called state variables. Pressure and Temperature are state variables.

T (F) (iii) The heat capacity of a body is the amount of heat that it can store at a given temperature. heat capacity → how much T changes when Q added

T (F) (iv) Even though two bodies are in thermal equilibrium with a third, that does not mean that they are in thermal equilibrium with each other. 0th Law

(T) F (v) The mean free path for air molecules at standard temperature and pressure (STP) is about 10^{-7} m. see example in book

(T) F (vi) The internal energy of a given amount of an ideal gas at equilibrium depends only on its absolute temperature.

(T) F (vii) According to the principle of equipartition, energy is shared equally among the active degrees of freedom in an amount of $\frac{1}{2}kT$ in each on average.

T (F) (viii) If the absolute temperature of a body triples, the rate at which it radiates energy increases by a factor of 9. $P \sim T^4$

(T) F (ix) For a solid the mean distance between atoms is 2×10^{-10} m.

(T) F (x) Diffusion is described in the kinetic theory of gases as a result of the random motion of molecules.

(b) [5 points] Circle correct answer

(i) A concrete highway is built of slabs 12 m long at 20°C. How wide should the expansion cracks be between the slabs at 20°C to prevent buckling, if the temperature range is -30°C to +50°C.? For concrete $\alpha = 12 \times 10^{-6}/°$ C

(A) 4.3×10^{-5} m (B) 4.3×10^{-4} m
(C) 4.3×10^{-3} m (D) 4.3×10^{-2} m
(E) 4.3×10^{-1} m (F) 4.3 m

$\Delta X_{max} = 12\ m \cdot 12 \cdot 10^{-6}\ C^{-1} \cdot 30°C$
$= 4.3 \cdot 10^{-3}\ m$

(ii) An inventor claims he has produced a motor that takes heat from sea water, generates electricity, and rejects waste heat to warm buildings. He is seeking investors. You chose not to invest because this violates

(A) the First Law of Thermodynamics.
(B) the Second Law of Thermodynamics.
(C) the Third Law of Thermodynamics.
(D) the First and Second Law of Thermodynamics.
(E) the Second and Third Law of Thermodynamics.

(iii) Oceans as Global Thermometer: Various climate modelers predict that the Temperature of the Earth will rise between 1.5 and 5°C in the next 30 years. Water has a volume coefficient of expansion of $2.1 \times 10^{-4}°C^{-1}$. Assume the volume coefficient for earth is $0.1 \times 10^{-4}\ °C^{-1}$ The mean depth of the Ocean is about 12,000 ft (3700 m). How much will the level of the Ocean rise?

(A) no change at all
(B) 1 to 4 feet (0.3 to 1.3 m)
(C) 3 to 12 feet (1 to 4 m)
(D) 0.3 to 1.2 feet (0.1 0 to 0.4 m)
(E) 300 feet (100 m)

$\Delta V = V(\beta_1 - \beta_2)\Delta T$
$A\Delta x = Ax(2 \cdot 10^{-4}) \cdot 5$
$\Delta x = 10^{-3} x = 12\ ft.$

(iv) You are feeling feverish and take your temperature with a metric medical thermometer and find your temperature is 39°C. What is your temperature in Farhenheit and are you ill?

(A) 98.6° F and you are normal (B) 99° F and you have a very slight fever.
(C) 99.5° F and are a little ill. (D) 100° F and you are ill.
(E) 101° F and you are fairly sick. (F) 102° F and you are quite sick
(G) 103° F and you are very sick (H) 104° F and you are critically ill

(v) The fog blows in across the Bay and quickly (no time for heat to flow) up to the room of this exam an altitude of about 200 m. If the temperature on the Bay is 22°C and the pressure is $10^5 N/m^2$ and the density is roughly 1 kg/m³. What will the difference in its pressure and temperature in the room? ($\Delta T = T_{room} - T_{bay}$)

(A) Pressure is the same and temperature is same.
(B) Pressure difference about $2000 N/m^2$ lower and temperature 6°C higher
(C) Pressure difference about $2000 N/m^2$ lower and temperature 6°C lower
(D) Pressure difference about $2000 N/m^2$ lower and temperature 1.7°C lower
(E) Pressure difference about $2000 N/m^2$ lower and temperature 0.6°C lower
(F) Pressure difference about $200 N/m^2$ lower and temperature 0.6°C lower
(G) Pressure difference about $200 N/m^2$ lower and temperature 1.6°C lower

$P = P_0 - \rho g h$
adiabatic, so
$PV^{\gamma} = C$
$PT^{\gamma/1-\gamma} = C$

(c) [5 points] Circle correct answer

(i) What is the factor by which the number of accessible states changes when 1 kg of water temperature increases 1°C from 27°C?

(A) 10^{24}
(B) 10^{25}
(C) $10^{4.3849\times10^{23}}$
(D) $10^{4.3849\times10^{24}}$
(E) $10^{4.3849\times10^{25}}$
(F) $10^{4.3849\times10^{26}}$

$\Delta S = \frac{mC\Delta T}{T} = \frac{1\,kg \cdot 1\,kcal\,kg^{-1}\,C^{-1} \cdot 1°C \cdot 4200\,\frac{J}{kcal}}{300\,K} = \frac{4200\,J}{300\,K} = 14\,J/K$

$\Delta S = k \ln \frac{\Omega_f}{\Omega_i} \rightarrow \frac{\Omega_f}{\Omega_i} = e^{\frac{14\,J/k}{1.381\cdot10^{-23}}} = 10^{4.4\cdot10^{23}}$

(ii) What is the mean speed of air molecules at STP?

(A) 0.5 m/s
(B) 5 m/s.
(C) 50 m/s
(D) 500 m/s.
(E) 5000 m/s.

$v \sim \sqrt{\frac{3kT}{m}} \rightarrow 485\,m/s$

(iii) The critical temperature of a real gas is

(A) the temperature at and above which vapor of the substance cannot be liquefied, no matter how much pressure is applied

(B) the temperature of the PV isotherm curve goes horizontal and goes through the critical point.

(C) both (A) and (B).
(D) the division between gas and vapor
(E) All of the above.

(iv) The mean time between collisions of air molecules at standard temperature and pressure is roughly

(A) 10^{-25} sec
(B) 10^{-20} sec
(C) 10^{-15} sec.
(D) 10^{-10} sec.
(E) 10^{-5} sec

$v \sim 500\,m/s$, $l \sim 10^{-7}\,m \rightarrow \tau \sim 10^{-10}$

(v) What is the triple point?

(A) It is the critical point.
(B) The point at which liquid can no longer exist.
(C) The point at which vapor can no longer exist
(D) The point where solid, liquid, and vapor can coexist in equilibrium
(E) The point where solid, liquid, and gas can coexist in equilibrium
(F) The point where gas, vapor, and liquid can coexist in equilibrium

(d) [5 points] Circle correct answer

(i) The van der Walls equation of state is

(A) a simple correction to the ideal gas law taking into account the finite volume of the molecules

(B) a simple correction to the ideal gas law taking into account the small attractive forces of the molecules on each other

(C) $P\left(\frac{V}{n} - b\right) = RT$

(D) $\left(P + \frac{a}{(V/n)^2}\right)\left(\frac{V}{n} - b\right) = RT$

(E) $P_1 + P_2 = P_{total} = (n_1 + n_2)RT/V$.

(ii) What is the internal energy U of a monoatomic ideal gas?

(A) $U = \frac{1}{2}Nk_BT = \frac{1}{2}nRT$

(B) $U = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$

(C) $U = \frac{5}{2}Nk_BT = \frac{5}{2}nRT$

(D) $U = \frac{6}{2}Nk_BT = 3nRT$

(E) $U = \frac{7}{2}Nk_BT = \frac{7}{2}nRT$

(iii) Which of these is **not** a latent heat?

(A) heat of fusion.

(B) heat of vaporization.

(C) heat of phase change.

(D) heat of friction

(E) none of the above

(iv) What is **not** a statement of the First Law of Thermodyamics?

(A) Energy is conserved.

(B) $dU = dQ - dW$

(C) The change in a system's internal energy is the difference between the heat added to the system and the work done by the system.

(D) Heat is a transfer of energy due to a difference in temperature.

(v) Which of these is **not** true for free expansion of a gas. I.e. The gas is initially in only one side of the container and allowed to expand without interference or resistance.

(A) No work is done in free expansion.

(B) Free expansion is adiabatic.

(C) The internal energy of the gas remains constant.

(D) Free expansion is a good example of a non-equilibrium process..

(E) The entropy of the gas is unchanged in free expansion.

(f) [7 points] Circle correct answer: The Human Heat Engine

(i) [1 points] What is not a significant method by which the body rids itself of heat?

(A) Conduction - The thermal conductivity of human flesh is about 0.2 J/s/ m^2.
(B) Convection - blood flow
(C) Convection - air flow, e.g. fan
(D) Evaporation - perspire and transpire about 0.5 kg of water per day
(E) Radiation - thermal blackbody
(F) Cold drinks

(ii) [1 points] Suppose a human could live for two hours (120 minutes) unclothed in air at 45° F. How long could he live in water at 45° F? Water has 23 times the thermal conductivity that air has.

(A) 2760 minutes
(B) 276 minutes
(C) 120 minutes
(D) 52 minutes
(E) 5.2 minutes

less by a factor of 23...

(iii) Why do people become "flushed" when overheated?

(A) heat makes your skin turn redish
(B) perspiration makes the skin look splotchy and red
(C) People are embarrassed when hot and blush.
(D) The body's circulation system opens blood vessels near the skin's surface to improve the convection of heat by blood from the internal portion of the body to the skin.
(E) Hotter skin radiates in the red the way steel glows when it first gets hot.

(iv) How much heat power does the human body radiate? Assume a body temperature = 37° C and estimate surface area A = 1.5 m^2 and emissivity e = 0.70.

(A) 550 watts
(B) 440 watts
(C) 330 watts
(D) 220 watts
(E) 110 watts
(F) 55 watts
(G) 44 watts
(H) 33 watts
(I) 22 watts

$P = \varepsilon A \sigma T^4 = .7 \cdot 1.5 \cdot 5.67 \cdot 10^{-8} \cdot (310)^4$

$= 550$

(v) On a very cold day the net heat loss of a human is 219 Joules per second. How many food Calories of energy will be lost in one hour?

(A) 200,000 Calories
(B) 20,000 Calories
(C) 2000 Calories
(D) 200 Calories
(E) 20 Calories

219 W = .05 Cal s^{-1}

$.05 \cdot 3600 = 180$ Calories

(vi) Each person has chromosomes holding about 30,000 genes with about 5 billion base pairs. Each base pair holds 2 bits of information: A, T, C, or G. That's 10 billion bits, or 10 gigabytes.

Let's contrast this with the amount of mental information you learn and store in your brain over your lifetime. Psychological experiments have estimated [T.K.Landauer: "How much do people remember", Cognitive Sci. 10,4 (1986) 477-493; 12,2 (1988) 293-297] that you remember about 2 bits per second and end up holding somewhere between 0.5 and 3.4 gigabits of memory. Typically 2.5 gigbits.

What is the relative entropy increase during the lifetime to peak memory? How does that compare to the entropy increase per second from the regular human metabolism of about 70 watts or the stored entropy in a 50 kgm body?

$\Delta S = k \ln(2.5 \cdot 10^9)$

$= 21.6 k_B$

(A) 21.6 vs 70 or 200 J/K
(B) 21.6k_B vs 70 J/K or 200,000 J/K
(C) 2.5 × $10^9 k_B$ vs 70 J/K or 200,000 J/K
(D) 2.5 × 10^9 J/K vs 70 J/K or 200,000 J/K
(E) The entropy increases by a very large because of failure to remember most information and that exceeds the increase by heat loss and other stored entropy.

(vii) The human body is exceptionally efficient at extracting energy from food. Feces retain only about 5 per cent of the chemical energy originally present in the food consumed. Most of this energy is going into basic maintainance or metabolic work. Efficiencies are lower when we look at the energy consumed that goes into mechanical work. Cycling is one of the most efficient activities that humans can engage in. For a trained cyclist, efficiencies approach 20 per cent with mechanical work being generated at a rate of 370 W compared to a metabolic work of 1850 W. Cars are notoriously wasteful in comparison. Gasoline has an energy content of 477 MJ/kg and a density of 680 kg/m^3. If a car can travel 8500 km per m^3 of gasoline (8.5 km per liter or 15 mpg) the car will use 40 times more energy over the same distance. Muscle contractions could be up to 50 per cent efficient. What is the minimum effective temperature that food energy is being converted into heat energy, if the human body is a heat engine?

(A) 1000 K
(B) 500 K
(C) 375 K
(D) 310 K
(E) 240 K

use $e \leq e_{carnot}$

$.2 \leq 1 - T_L/T_h$

$\frac{T_L}{T_h} \leq .8$

$T_h \geq \frac{T_L}{.8} \approx \frac{310}{.8} \approx 375$ K

#2 Solution

2. [20 points] An air bubble at the bottom of a lake 37.0-m deep has a volume of 1.00 cm^3 . If the temperature at the bottom is 5.5.°C and at the top 21.0°C.

Give formulae and then evaluate.

(a) [15 points] what is the volume of the bubble just before it reaches the surface?

N constant $\Rightarrow \dfrac{P_i V_i}{T_i} = \dfrac{P_f V_f}{T_f}$

$V_f = \dfrac{T_f}{P_f} \dfrac{P_i V_i}{T_i}$, $P_i = P_f + \rho_w g h \Rightarrow \boxed{V_f = \dfrac{T_f}{P_f} \dfrac{V_i}{T_i} (P_f + \rho_w g h)}$

$T_i = 5.5 + 273.2 = 278.7\,K$, $V_i = 10^{-6}\,m^3$,

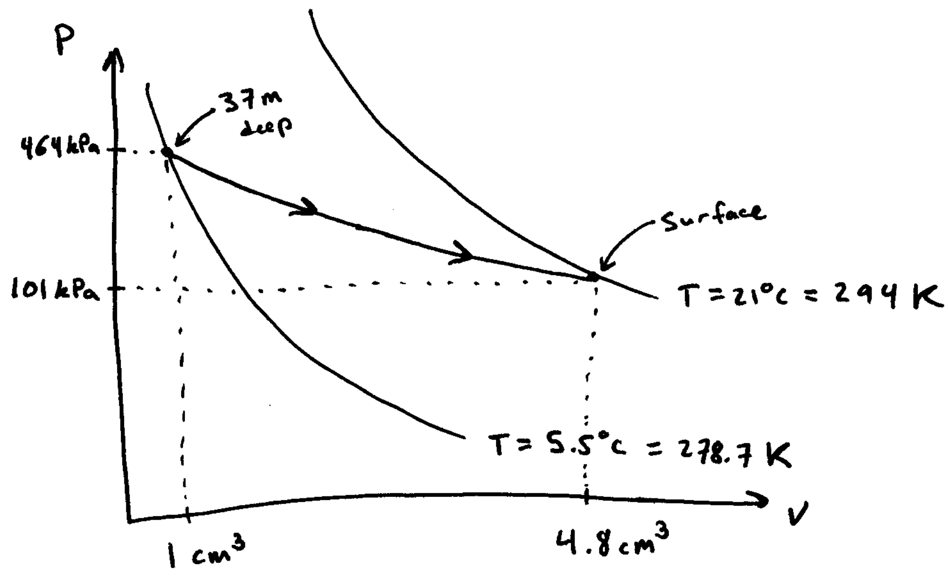
$T_f = 21 + 273 = 294.2K$, $P_f = 101300\,N/m^2$,

$\rho_w g h = 1000\,kg/m^3 \cdot 9.8\,N/kg \cdot 37\,m = 362600\,N/m^2$

$V_f = \dfrac{294.2 \cdot 10^{-6}}{101300 \; 278.7} \cdot (101300 + 362600) = 4.83 \cdot 10^{-6}\,m^3 = \boxed{4.83\,cm^3}$

463900

(b) [5 points] Draw and label a P-V diagram showing the trajectory of the bubble from the bottom of the lake to the surface. Show two isothermals with the temperature at the bottom and top of the lake for reference along with marking the bottom and top volumes and pressures.



(3) (a) How much work is done by the gas?

Gas expands from $V_1 = 3.50\,m^3$ to $V_2 = 7.00\,m^3$ isothermally.

$W = \int P\,dV \qquad P = \frac{NkT}{V} = \frac{nRT}{V}$

$W = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln\left(\frac{V_2}{V_1}\right) = (2.00\,mol)(8.31\,J/mol \cdot K)(300\,K)\ln(2)$

$W = 3456\,J$

(b) How much heat is added to the gas?

isothermal, so $\Delta T = 0$ and $\Delta U = 0$. Then, $Q = W$.

$Q = 3456\,J$

(c) what is the change in internal energy.

isothermal expansion, so $\Delta U = 0$

(d) The gas is compressed from $V_2 = 7.00\,m^3$ to $V_1 = 3.50\,m^3$ at constant pressure. How much work is done by the gas?

$W = \int P\,dV = P_2 \Delta V$ since P is constant.

$$P_2 = \frac{nRT_2}{V_2} = \frac{(2.00\,mol)(8.31\,J/mol \cdot K)(300\,K)}{7.00\,m^3} = 712\,Pa$$

$W = (712\,Pa)(-3.50\,m^3) = -2492\,J$

(e) Now the temp. is raised to $T_1 = 300\,K$ at constant volume $V_1 = 3.50$.

How much work is done? $W = 0$ because volume is constant.

what is the change in internal energy?

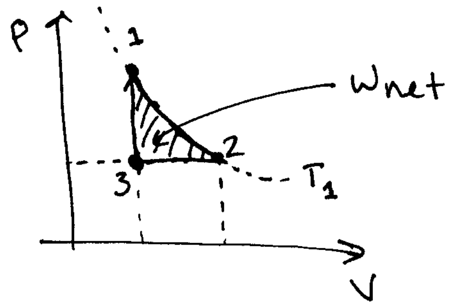
$\Delta U = \frac{5}{2} nR\Delta T = \frac{5}{2} nR(T_1 - T_3)$

$$T_3 = \frac{P_2 V_1}{nR} = \frac{(712\,Pa)(3.50\,m^3)}{(2.00\,mol)(8.31\,J/mol \cdot K)} = 150\,K$$

$\Delta U = \frac{5}{2}(2.00\,mol)(8.31\,J/mol \cdot K)(150\,K) = 6232.5\,J$

③ (f) For the full ~~cycle~~ cycle, what is the net work done?

$W_{net} = W_{1\to2} + W_{2\to3} = 3456\,J - 2492\,J = 964\,J$



(g) what is the efficiency of this process?

$e = \frac{W_{net}}{|Q_{in}|}$ $Q_{in} = Q_{1\to2} + Q_{3\to1} = 3456\,J + 6232.5\,J = 9688.5$

$e = \frac{964\,J}{9688.5\,J} = 0.10 = 10\%$

Compare to Carnot efficiency.

$e_c = 1 - \frac{T_L}{T_H} = 1 - \frac{150\,K}{300\,K} = 0.50 = 50\%$

Problem 4 solution 20 pts

a) 1 pt
max efficiency is Carnot

$$e_c = 1 - \frac{T_L}{T_H} = 1 - \frac{313\text{ K}}{573\text{ K}} = 0.454$$

b) 2 pts.

ideal $\Rightarrow |\Delta S_L| = |\Delta S_H|$

$$\frac{Q_L}{T_L} = \frac{Q_H}{T_H} \Rightarrow \frac{Q_L}{Q_H} = \frac{T_L}{T_H} = 0.546$$

in 1 sec, $Q_L = 1.2\times10^9$ J

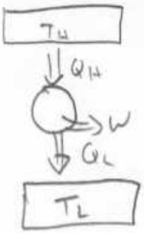
$\Rightarrow Q_H = 2.2\times10^9$ J $\Rightarrow$ $\boxed{\frac{dQ_H}{dt} = 2.2\times10^9\text{ W}}$

Entropy comes from ΔS_L (low temp reservoir) $= +\frac{Q_L}{T_L} = +3.8$ MJ/K
ΔS_H (high temp reservoir) $= -\frac{Q_H}{T_L} = -3.8$ MJ/K
] cancel b/c ideal powerplant

$$\boxed{\frac{dS_{tot}}{dt} = 0}$$

c) 3 pts.

$$e = \frac{W}{Q_H} = 1/3 \Rightarrow \frac{Q_L}{Q_H} = 1 - e = 2/3$$



$W = 10^9$ J in 1 sec

$\Rightarrow Q_H = 3\times10^9$ J

$Q_L = 2\times10^9$ J $\Rightarrow$ $\boxed{\frac{dQ_L}{dt} = 2\times10^9\text{ W}}$

$$\Delta S_H = \frac{-Q_H}{T_H} = \frac{-3\times10^9\text{ J}}{573\text{ K}} = -5.2\times10^6\text{ J/K}$$

$$\Delta S_L = \frac{+Q_L}{T_L} = \frac{+2\times10^9\text{ J}}{313\text{ K}} = +6.4\times10^6\text{ J/K}$$

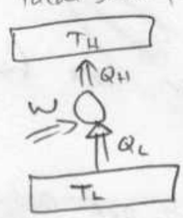
} $\Delta S_{tot} = 1.2\times10^6$ J/K

$$\boxed{\frac{dS_{tot}}{dt} = 1.2\times10^6\text{ W/K}}$$

d) 4 pts.

Bathroom $T_H = 295$ K | outside $T_L = 290$ K | $\Delta T = 5$ K

ideal heat pump $\Rightarrow |\Delta S_L| = |\Delta S_H|$



$$\frac{Q_L}{T_L} = \frac{Q_H}{T_H}$$

$Q_H = 1$ kJ in 1 sec

$Q_L = 0.983$ kJ

$\Rightarrow W = 0.0169$ kJ $= 16.9$ J $\Rightarrow$ $\boxed{\frac{dW}{dt} = 16.9\text{ W}}$

power plant
$T_L = 313$ K
$T_H = 573$ K

$e = 0.33 = \frac{W}{Q_H}$

$W = 16.9$ J in 1 sec
$Q_H = 51.2$ J
$Q_L = 34.3$ J

$\Rightarrow$ $\boxed{\frac{dQ_H}{dt} = 51.2\text{ W}}$

heat pump ideal $\Rightarrow \Delta S_{HP} = 0$ powerplant $\Delta S_{PP} = -\frac{Q_H}{T_H} + \frac{Q_L}{T_L} = 0.020$ J/K

$$\Rightarrow \boxed{dS/dt = 0.020\text{ W/K}}$$

e) 3 pts.

resistor creates heat from work

$W = Q_H = 1\ kJ$ in 1 sec

This is $\frac{1000}{16.9} = 59.2$ times as much energy as before in (d)

$\Rightarrow \boxed{\frac{dQ_H}{dt} = 51.2 \times 59.2\ W = 3030\ W}$

$\frac{dS_{PP}}{dt} = 0.020 \times 59.2 = 1.2\ W/K$

resistor <u>creates</u> entropy $\Delta S_R = \frac{Q_H}{T_{bathroom}} = \frac{1000\ J}{295\ K} = 3.4\ J/K$ in 1 sec

$\Rightarrow \boxed{\frac{dS_{tot}}{dt} = 4.6\ W/K}$

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f) 3 pts.

fuel $T = 573K$ —Q→ $\frac{2}{3}Q$ into bathroom at 295 K

→ $\frac{1}{3}Q$ outside at 290 K

$\frac{2}{3}Q = 1\ kJ$ every sec

$\Rightarrow Q = 1.5\ kJ$

$\boxed{\frac{dQ}{dt} = 1.5\ kW}$

Fuel creates heat and then expels heat so no <u>net</u> heat going into/out of 573K. $\Rightarrow \Delta S_{fuel} = 0$

Bathroom: $\Delta S = \frac{1\ kJ}{295\ K} = 3.4\ J/K$

outside: $\Delta S = \frac{0.5\ kJ}{290} = 1.7\ J/K$

$\Delta S_{tot} = 5.1\ J/K$

$\Rightarrow \boxed{\frac{dS}{dt} = 5.1\ W/K}$

Note: It is also <u>reasonable</u> to take $\Delta S_{fuel} = \frac{-1.5\ kJ}{573\ K} = -2.6\ J/K$ since heat is leaving the burning mix and we may avoid asking how it got there.

Then one finds $\boxed{\frac{dS}{dt} = 5.1 - 2.6 = 2.5\ W/K}$

Both are acceptable.

(g) [~~3~~ 4 points] Draw and label a PV diagram for the actual power plant . Show the beginning temperature and pressure and ending temperature and pressure as circled points on the PV diagram. To be specific consider one mole of steam. Sketch the following curves starting from the initial temperature and pressure: (i) isothermal expansion, (ii) adiabatic, (iii) estimated actual path.

actual path $\qquad n = 1\,\text{mol}$

$P_1 = 70\text{ atm} = 70.9 \times 10^5\text{ Pa} \qquad T_1 = 573\text{ K} \qquad \Rightarrow V_1 = 6.72 \times 10^{-4}\text{ m}^3$

$P_2 = 1\text{ atm} = 1.013 \times 10^5\text{ Pa} \qquad T_2 = 313\text{ K} \qquad V_2 = 2.57 \times 10^{-2}\text{ m}^3$

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isothermal path to same final volume

$P_1 = 70\text{ atm} \qquad T_1 = 573\text{ K} \qquad V_1 = 6.72 \times 10^{-4}\text{ m}^3$

$P_2 = 1.8\text{ atm} \Leftarrow T_2 = 573\text{ K} \qquad V_2 = 2.57 \times 10^{-2}\text{ m}^3$

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adiabatic path to same final volume

$P_1 = 70\text{ atm} \qquad T_1 = 573\text{ K} \qquad V_1 = 6.72 \times 10^{-4}\text{ m}^3$

$P_2 = 0.54\text{ atm} \Rightarrow T_2 = 169\text{ K} \qquad V_2 = 2.57 \times 10^{-2}\text{ m}^3$

$\gamma = 1 + \frac{2}{d} = 1 + \frac{2}{6} = 4/3$

$d = 6$ for steam (H_2O = H–O–H)

3 translation
3 rotation
assume no vibrations

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

solve for $P_2 = 0.54\text{ atm}$

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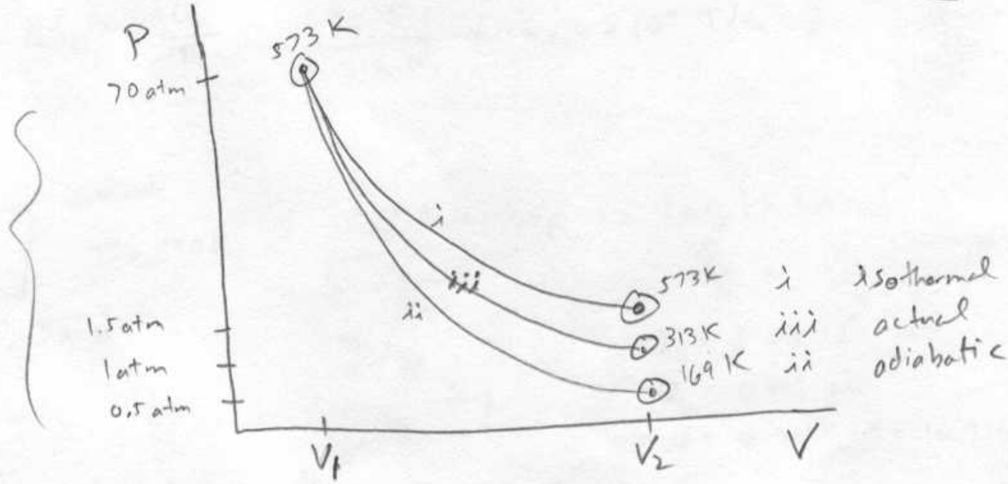
not really to scale



5. [20 points] A refrigerator absorbs heat from the freezer compartment at a temperature of -17°C and exhausts it into the room at 25°C with an actual coefficient of performance of CP = 5.

(a) [5 points] How much work must be done by the refrigerator to change 0.50 kg of water put in at 25°C into ice at -17°C. What would be the coefficient of performance if it were a "Carnot" (ideal) refrigerator? How much work would be required then?

$$Q_L = C_w M_w |0°C - 25°C| + M_w L_F + C_{ice} \cdot M_w |-17°C - 0°C| = 2.37 \times 10^5 \text{ J}$$

$$W = \frac{Q_L}{CP} = \boxed{4.74 \times 10^4 \text{ J}}$$

$$CP_{carnot} = \frac{Q_L}{W} = \frac{Q_H - W}{W} = \frac{1}{1-\varepsilon} - 1 = \frac{T_L}{T_H - T_L} = \boxed{6.095}$$

$$W_{carnot} = \frac{Q_L}{CP_{carnot}} = \boxed{3.89 \times 10^4 \text{ J}}$$

(b) [5 points] If the compressor output is 200 watts, what minimum time is needed to take 0.50 kg of 25°C water and freeze it at 0°C?

$$Q_L = C_w M_w |0°C - 25°C| + M_w L_F = 2.19 \times 10^5 \text{ J}$$

For CP = 5, $W = \frac{Q_L}{CP} = 4.39 \times 10^4 \text{ J}$

$$t = \frac{W}{P} = \boxed{220 \text{ sec}}$$

For CP = 6.095

$$W = \frac{Q_L}{CP} = 3.59 \times 10^4 \text{ J}$$

$$t = \frac{W}{P} = \boxed{180 \text{ sec.}}$$

(c) [5 points] Each time the door to the freezer is opened five moles of air (diatomic gas) at 25°C enters the refrigerator displacing the same amount of cold air (0°) and must be cooled to 0°C. How much heat must be removed? How much work is done by the ideal refrigerator each time the freezer is opened and then closed? If typically the time between opening the refrigerator door is an hour, what is the power used?

$$Q_L = nC_p\Delta T = 5\,\text{mol} \times \frac{7}{2} R \times 25^\circ C = \boxed{3.63 \times 10^3\,J}$$

$$CP_{carnot} = \frac{T_L}{T_H - T_L} = \frac{273}{25} = 10.92$$

$$W = \frac{Q_L}{CP_{carnot}} = \boxed{332\,J}$$

$$P = \frac{W}{t} = \frac{332\,J}{3600\,sec} = \boxed{0.092\text{ Watts}}$$

(d) [5 points] The freezer compartment is surrounded by foam insulation with $k_{foam} = 0.025$ J/(s m °C) which has a area of 6 m^2 and thickness of 4 cm. What is the heat load in watts from the 25°C to the 0°C refrigerator? If the motor for the cooling unit should run no more than 15 per cent of the time what is the minimum power requirements for the motor?

$$\frac{dQ}{dt} = \frac{kA\Delta T}{d} = \frac{0.025 \times 6 \times 25^\circ C}{0.04\,m} = \boxed{93.75\,J/s}$$

To limit run time 15% of all.

$$P = \left(\frac{93.75\,J}{0.15\,sec}\right) / CP = \sout{625\,W} / CP = \boxed{57.23\text{ Watts}}$$