

## FINAL EXAM – Section 3

Instructor: Prof. A. LANZARA

**TOTAL POINTS: 140**

**TOTAL PROBLEMS: 7**

*Read the whole exam before starting to solve problems. Start with the ones you are more familiar with to secure points.*

*Show all work, and take particular care to explain what you are doing. Show a logical progression of steps from equations on the equation sheet to your final answer. Partial credit is given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make. If you get stuck, skip to the next problem and return to the difficult section later in the exam period. All answers should be in terms of variables.*

**GOOD LUCK!**

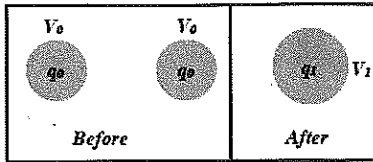
### **PROBLEM 1 (20 pts)**

Two samples of  $n$  moles of an ideal gas are initially at the same temperature and pressure,  $T_0$  and  $P_0$ . Each of the gases has mass  $M$ , specific heat at constant volume  $C_v$  and latent heat of vaporization  $L$ . They are each compressed reversibly from a volume  $V$  to a volume  $V/2$ , one isothermally, the other adiabatically.

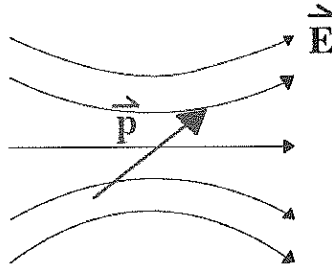
- a) (1pts) Draw the two processes in the same PV diagram.
- b) (2pts) From the PV diagram explain why you would expect for one of the two processes to have higher final pressure.
- c) (2pts) In which sample you expect the final temperature to be greater?
- d) (4pt) Determine the change of internal energy,  $\Delta U$ , for the two processes.
- e) (4pts) Determine the change in entropy,  $\Delta S$ , of the gas for each process by integration.
- f) (4pts) What is the entropy change of the environment for each process?
- g) (3pts) The gas that has undergone the isothermic compression is now cooled at constant pressure up to its liquefying temperature, till all the gas melts into liquid. The resulting liquid is at the same temperature as the gas. What is the change of internal energy?

**PROBLEM 2 (20 pts)**

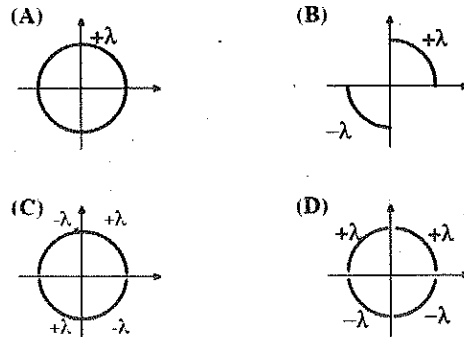
I) (5pts) Two isolated, identical spherical drops of mercury (a good electrical conductor) each carry a net charge of  $+q_0$  and electric potential  $V_0$  at the surface with respect to infinity. The two drops are pushed together to form a single drop of mercury. What is the total charge  $q_1$  and the electric potential  $V_1$  at the surface of the combined drop? Explain your answer.



II) (5pts) An electric dipole  $\vec{p}$  is placed in an external electric field  $\vec{E}$  as shown in the figure below. What is the magnitude and direction of the net torque and net force that the electric dipole will experience? Explain your answer.



III) (5pts) Which charge distribution shown below will yield a zero electric field and a non-zero potential at the origin of the coordinate system, if the potential is defined to be zero infinitely far away from the origin? (Note all semi- and quarter-circles have same radius and are all centered about the origin of the coordinate system). Show your thinking.

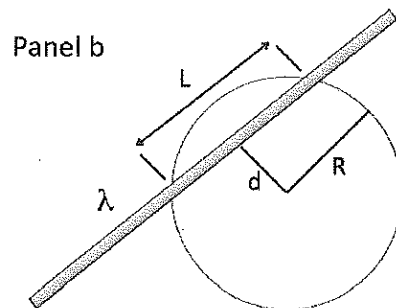
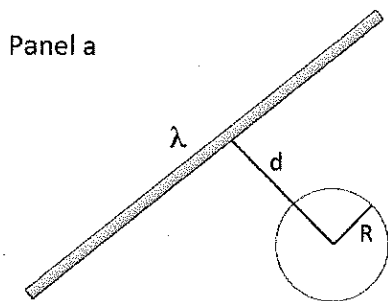


IV) (5pts) A large insulated hollow conductor carries a positive charge ( $q$ ). A small metal ball carrying a negative charge of the same magnitude is lowered by a thread through a small opening in the top of the conductor, allowed to touch the inner surface and then withdrawn. What then is the charge on the conductor and the ball?

**PROBLEM 3 (Tot 20pts)**

An infinite long line charge with uniform charge density per unit length  $\lambda$ , lies a distance  $d$  from point O.

- (8pts) Determine the total electric flux through a surface of a sphere of radius  $R < d$  (panel a) centered at O.
- (12pts) We now increase the radius of the sphere such that  $R > d$ , and a length  $L$  of the wire is now inside the sphere (see panel b). Find the total flux of the electric field through the sphere in this case.



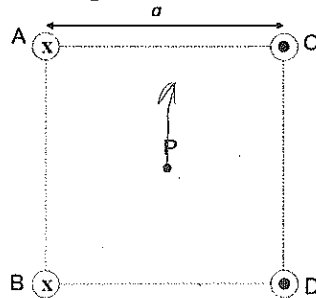
**PROBLEM 4 (Points 25)**

Four parallel wires of infinite length, each carrying a current  $I$  (see figure below) are located along the vertex of a square. The side of the square is  $a$ . The current direction is into the page in A and B and out of the page in C and D. The ends of the wires all lie in the same plane.

- (8pts) Calculate the magnitude and direction of the magnetic field  $B$  at point P, which is located at the center of the square.
- (6pts) If an electron were traveling in the  $+y$ -direction with velocity  $v_0$ , what would be the magnitude and direction of the magnetic force acting on it as it passed through P?

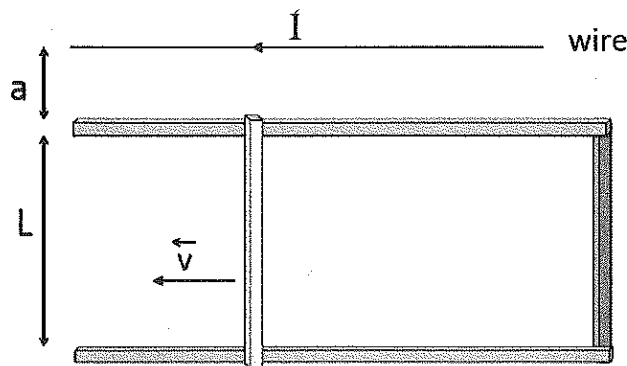
We now add a fifth wire of length  $L$ , parallel to the others. The wire is located at point P. The current  $I$  in this wire is out of the page.

- (6pts) Find the force acting on this wire and describe how it will move.



**PROBLEM 5 (Points 20)**

In the figure below a rod of length  $L$  and resistance  $R$  is moving at constant speed  $v$  along horizontal conducting rails. Neglect the resistance of the rails. In this case the magnetic field in which the rod moves is not uniform but is provided by a current  $I$  in a long parallel wire.



- (3pts) Draw the direction of the magnetic field inside and outside the loop and above and below the wire.
- (6pts) Calculate the emf induced in the rod.
- (5pts) What is the current in the conducting loop?
- (6pts) What force must be applied to the rod by an external agent to maintain its motion at a constant velocity?

**PROBLEM 6 (20pts)**

A hollow conducting pipe of inner radius  $a$  and outer radius  $b$  carries a current  $I$  parallel to its axis and distributed uniformly through the pipe material.

Find expressions for the magnetic field for

- a) (7pts)  $r < a$
- b) (7pts)  $a < r < b$
- c) (6pts)  $r > b$ , where  $r$  is the radial distance from the pipe axis.

**PROBLEM 7 (20pts)**

One spring is connected to an object with mass  $M$ , the arrangement being free to oscillate on a horizontal frictionless surface as in the figure below (see figure below). Following the discussion from lecture:

- a) (6pts) Sketch the circuit analog of this mechanical oscillating system.
- b) (6pts) If  $U$  is the potential energy of the compressed or extended spring and  $K$  is the kinetic energy of the moving block, write down the kinetic and potential energy of its circuit analog.
- c) (8pts) Describe the magnitudes of the two energies, in terms of the initial energy  $U_0$ , as a function of time  $t=0$ ,  $t=T/4$  and  $t=T/2$ . Draw a diagram of the energies as a function of time.

