CS 188 Spring 2017 Introduction to Artificial Intelligence Midterm V2

- You have approximately 80 minutes.
- The exam is closed book, closed calculator, and closed notes except your one-page crib sheet.
- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer you may wish to provide a *brief* explanation. All short answer sections can be successfully answered in a few sentences AT MOST.
- For multiple choice questions with *circular bubbles*, you should only mark ONE option; for those with *checkboxes*, you should mark ALL that apply (which can range from zero to all options)

First name	
Last name	
edX username	

For staff use only:

Q1.	Potpourri	/20
Q2.	Search	/16
Q3.	CSPs	/18
Q4.	War in Paclandia	/12
Q5.	Approximate Q Learning with Landmark States	/14
Q6.	MDPs: Rebellious Robot	/20
	Total	/100

Q1. [20 pts] Potpourri

(a) MDPs

In this MDP, available actions are left, right, and stay. Stay always results in the agent staying in its current square. Left and right are successful in moving in the intended direction half of the time. The other half of the time, the agent stays in its current square. An agent cannot try to move left at the leftmost square, and cannot try to move right on the rightmost square. Staying still on a square gives a reward equivalent to the number on that square and all other transitions give zero reward (meaning any transitions in which the agent moves to a different square give zero reward).

4	0	0	36

(i) [2 pts] $V_0(s)$ is 0 for all s. Perform one step of value iteration with $\gamma = \frac{1}{2}$ and write $V_1(s)$ in each corresponding square:

4	0	0	36

$V_1(s)$ is equal to the optimal one-step rewards from each state. This can be achieved from each state by staying, which gives reward equal to the number on each square. (In the middle squares, moving in either direction can do no better than 0 in one step.)

(ii) [3 pts] Perform another step of value iteration with $\gamma = \frac{1}{2}$, and write $V_2(s)$ in each corresponding square:

6	1	9	54

From the two squares on the side, the agent can stay twice, giving $36 + \gamma \cdot 36 = 36 + 18 = 54$ and $4 + \gamma \cdot 4 = 4 + 2 = 6$. From the middle squares, the optimal policy for a two-step horizon is to move outward toward the closer side. From the middle-right, it will move right, which in two steps will give 0 reward if the first step is unsuccessful and $0 + \gamma \cdot 36 = 18$ if the first step is successful, giving an expectation of $\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 18 = 9$. By an identical argument, V_2 on the middle-left state is $\frac{1}{4} \cdot 4 = 1$.

(iii) [4 pts] Using $\gamma = \frac{1}{2}$ again, write $V^*(s)$ in each corresponding square (Hint: think about states where the optimal action is obvious and work from there):

8	8	24	72

From the left two states, it is unclear initially whether it is better to go for the closer reward of 4 or to take more time steps to move to the reward of 36. However, from the right two states, 36 is both closer and larger of the available possible rewards, so we know that the optimal policy will move right from the middle-right state and stay at the rightmost state.

Staying at the rightmost state indefinitely gives a discounted sum of rewards of $\sum_{i=0}^{\infty}\left(\gamma^i \cdot 36\right) = 36 \cdot \left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right) = 36 \cdot 2 = 72$. Moving right from the middle-right square (where MR and R refer to the middle-right and rightmost states, respectively) gives

$$V^*(\texttt{MR}) = \frac{1}{2}\left(0 + \gamma V^*(\texttt{MR})\right) + \frac{1}{2}\left(0 + \gamma V^*(\texttt{R})\right) = \frac{1}{4}V^*(\texttt{MR}) + \frac{1}{4}V^*(\texttt{R}).$$

Solving for $V^*(\texttt{MR})$ gives

$$\frac{3}{4}V^*(\texttt{MR}) = \frac{1}{4}V^*(\texttt{R}) \implies V^*(\texttt{MR}) = \frac{1}{3}V^*(\texttt{R}).$$

Since we know $V^*(\texttt{R}) = 72$, this gives $V^*(\texttt{MR}) = 24$. This also tells us more generally that the total effect of the uncertainty in our movements combined with the discounting is to divide values by 3 for each step away we are from the source of reward. This allows us to finish the rest of the board with no further algebra. From the leftmost square, staying indefinitely gives $4 \cdot 2 = 8$. From the middle-left square we can either choose to move left, giving $\frac{8}{3}$, or we can move right, giving $\frac{24}{3} = 8$. Thus the optimal policy is to move right from that square with a value of 8. (For completeness we can now also confirm that it is optimal to stay from the leftmost square by noting that if we were to instead move right, our value would be $\frac{16}{3} < 8$. This comes from saying, if we call the value in question a, that $a = \frac{1}{2}(4 + \frac{1}{2}a) + \frac{1}{2}(0 + \frac{1}{2} \cdot 8)$ which solves to $16_{\overline{3)}}$.

(b) Search

(i) [2 pts] Select a subset of the following options that comprises a minimal state space representation for pacman path-finding (i.e. find a path from some position on the board to another).

- [x] Pacman's position
- [ ] The positions of the food pellets
- [ ] A boolean per food pellet indicating whether that pellet has been eaten
- [ ] A boolean indicating whether Pacman has reached the target position
- [ ] The Manhattan distance from Pacman to the target
- [ ] The positions of the walls
- [ ] None of the above; a minimal state representation is _____

For this problem, Pacman's position is the only thing relevant to the problem that is changing. We can define a transition function (moving in a direction changes the appropriate coordinate in Pacman's position) and goal check (is Pacman's position equal to the goal position?) to fully model this problem with a state representation ignoring all other details. Specifically, the choices about food are completely irrelevant, the positions of the walls are static and can be contained in the transition function, and the information about Pacman relative to the target can be contained in the goal check. Manhattan distance could be a useful *heuristic*, but we could simply compute it from Pacman's position and the known goal location, making it a redundant addition to the state representation.

(ii) [9 pts] For each statement, if (and only if) that statement applies to a particular search algorithm, write a check in the cell corresponding to that statement-algorithm pair. Assume A* has an admissible heuristic, and that all positive edge weights are lower-bounded by some $\epsilon > 0$.

Statements:

1. This algorithm can get stuck in an infinite loop in a finite graph with positive edge weights when no solution exists
   Graph search algorithms would exhaust the state space and terminate with a failure since they never expand the same search state twice. Tree search algorithms, on the other hand, can get stuck infinitely following a cycle in the graph, and since there is no solution, there is no termination condition.
2. This algorithm can get stuck in an infinite loop in a finite graph with positive edge weights when a solution exists
   Same as above for the graph search algorithms. For tree search, however, since there exists a solution, there is a shallowest solution at finite depth d^* and a cheapest solution with finite cost c^*.

   For BFS, since the search considers paths in increasing order of depth, eventually all paths in the graph of length less than d^* will be exhausted, and the search will find the solution.

   Similarly, for UCS, since all edges have cost greater than $\epsilon > 0$, the total cost along paths increases unboundedly (this condition eliminates the possibility of any convergent series of costs), so eventually the search will exhaust all paths of total cost less than c^* and a solution will be found.

   Lastly, since the heuristic is admissible, the heuristic on the cheapest solution is 0 and thus the priority is equal to c^* (and all nodes on the optimal path to the cheapest solution will have priority bounded by c^*). Since the priority on all nodes in the graph is at least as large as the cost to get to those nodes (potentially bigger since we're adding a heuristic value), the same argument used for UCS applies to A*.
3. This algorithm can get stuck in an infinite loop in an infinite graph with positive edge weights when a solution exists
   The arguments used in the previous part for BFS, UCS, and A* all apply here for both tree and graph search. However, since there in an infinite number of states to search, graph search no longer has a guarantee of exhausting the space, so DFS graph search can get stuck infinitely following a bad search path. (You can imagine a graph where the start state has two successors, one of which is a goal and the other of which leads to an infinite chain containing no goal. If the tie is broken in favor of the chain, DFS graph search will never find the goal.)
4. This algorithm is guaranteed to find an optimal solution when all edge weights are 1 and a solution exists
   When all edge weights are 1, cost is equivalent to depth and thus BFS is equivalent to UCS. We proved in class that UCS and A* with an admissible heuristic are optimal (and because of the equivalence, BFS is also optimal here). DFS chooses paths to explore in essentially arbitrary order, so it's possible for the first expanded path to be a suboptimal path to a goal.
5. This algorithm is guaranteed to find an optimal solution when all edge weights are positive and a solution exists
   UCS and A* are still optimal as proved in class, but depth and cost are no longer equivalent so BFS is no longer optimal (consider a graph where there is a goal that is one step away from the start state, but that transition has a cost of 1000). DFS is still suboptimal for the same reason.

Statement	DFS Tree	BFS Tree	UCS Tree	A* Tree	DFS Graph	BFS Graph	UCS Graph
1	X	X	X	X			
2	X						
3	X				X		
4		X	X	X		X	X
5			X	X			X

Q2. [16 pts] Search

Wall-E is trying to find Eve on a M x N spaceship, but along the way he would like to collect all K > 0 stationary friendly bots. He will collect a friendly bot when he lands in a spot as the bot. Each time he collects a friendly bot, he will be able to move an extra space per timestep. For example if Wall-E collects 1 friend, each timestep from then on he can move **up to** 2 squares and if Wall-E collects 2 friends, each timestep from then on he can move up to 3 squares. This bonus is applied in the timestep after Wall-E collects a friend, and lasts for the entire rest of the search. **Eve is also moving** around one square per turn and cannot collect friendly bots. Both can only move North, South, East, and West. The search ends when Wall-E and Eve are on the same square, and Wall-E has collected every friendly bot.

(a) [3 pts] What is the size of the minimum state space for this problem?

$(MN)^2(2^K)$

(b) [3 pts] What is contained in the minimal state space representation?
Solution: Coordinates for Wall-E,
Coordinates for Eve,
Boolean array for friendly bots listing whether each has been collected.

The positions of the friendly bots are static, and thus don't need to be stored in the state space. The number of friendly bots collected is already known from the boolean array for friendly bots, and thus including it separately would not be minimal.

(c) Now the evil authorities have been alerted and have begun scanning the ships by quadrants. At each time step the authorities will scan a quadrant in clockwise order starting from the top right quadrant. If Wall-E is caught, i.e. he is in the quadrant that is currently being scanned, then it will be game over.

(i) [2 pts] Now what is the size of the minimum state space?

$(MN)^2(2^K)4$

(ii) [2 pts] What is added to the state space representation?
Solution: The time step (we can mod it by 4). Equivalently, which sector is currently being scanned.

(d) [6 pts] Assume that the scanning is still in place. Check all of the following that are admissible heuristics (Note that any distances from Wall-E to friendly robots or from friendly robots to Eve are 0 if there are no remaining friendly robots):

☐ Manhattan distance from Wall-E to Eve divided by two.
☐ Sum of Manhattan distances to all robots and to Eve.
■ Manhattan distance from Wall-E to Eve divided by (K + 2).
☐ 1 for every state.
■ (Manhattan distance from Wall-E to furthest friendly robot + Manhattan distance from that furthest robot to Eve) divided by (K + 2).
☐ (Manhattan distance from Wall-E to closest friendly robot + Manhattan distance from that closest robot to Eve) divided by K.
☐ (Sum of Manhattan distances from Wall-E to each friendly robot + Manhattan distance from Wall-E to Eve) divided by (3K).

Q3. [18 pts] CSPs

In this question, you are trying to find a four-digit number satisfying the following conditions:

1. the number is odd,
2. the number only contains the digits 1, 2, 3, 4, and 5,
3. each digit (except the leftmost) is strictly larger than the digit to its left.

(a) CSPs

We will model this as a CSP where the variables are the four digits of our number, and the domains are the five digits we can choose from. The last variable only has 1, 3, and 5 in its domain since the number must be odd. The constraints are defined to reflect the third condition above. Thus before we start executing any algorithms, the domains are

1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5

(i) [3 pts] Before assigning anything, enforce arc consistency. Write the values remaining in the domain of each variable after arc consistency is enforced.

1 2	2 3	3 4	5

(ii) [1 pt] With the domains you wrote in the previous part, which variable will the MRV (Minimum Remaining Value) heuristic choose to assign a value to first? If there is a tie, choose the leftmost variable.

- ○ The first digit (leftmost)
- ○ The second digit
- ○ The third digit
- ● The fourth digit (rightmost)

(iii) [1 pt] Now suppose we assign to the leftmost digit first. Assuming we will continue filtering by enforcing arc consistency, which value will LCV (Least Constraining Value) choose to assign to the leftmost digit? **Break ties from large (5) to small (1).**

- ● 1
- ○ 2
- ○ 3
- ○ 4
- ○ 5

(iv) [3 pts] Now suppose we are running min-conflicts to try to solve this CSP. If we start with the number 1332, what will our number be after one interation of min-conflicts? Break variable selection ties from left to right, and **break value selection ties from small (1) to large (5)**.

1232

(b) The following questions are completely unrelated to the above parts. Assume for these following questions, there are only binary constraints unless otherwise specified.

(i) [2 pts] [*true* or *false*] When enforcing arc consistency in a CSP, the set of values which remain when the algorithm terminates does not depend on the order in which arcs are processed from the queue.

(ii) [2 pts] [*true* or *false*] Once arc consistency is enforced as a pre-processing step, forward checking can be used during backtracking search to maintain arc consistency for all variables.

False. Forward checking makes the current variable arc-consistent, but doesnt look ahead and make all the other variables arc-consistent.

(iii) [2 pts] In a general CSP with n variables, each taking d possible values, what is the worst case time complexity of enforcing arc consistency using the AC-3 method discussed in class?

○ 0 ○ $O(1)$ ○ $O(nd^2)$ ● $O(n^2d^3)$ ○ $O(d^n)$ ○ ∞

$O(n^2d^3)$. There are up to n^2 constraints. There are d^2 comparisons for enforcing arc consistency per each constraint, and each constraint can be inserted to the queue up to d times because each variable has at most d values to delete.

(iv) [2 pts] In a general CSP with n variables, each taking d possible values, what is the maximum number of times a backtracking search algorithm might have to backtrack (i.e. the number of the times it generates an assignment, partial or complete, that violates the constraints) before finding a solution or concluding that none exists?

○ 0 ○ $O(1)$ ○ $O(nd^2)$ ○ $O(n^2d^3)$ ● $O(d^n)$ ○ ∞

$O(d^n)$. In general, the search might have to examine all possible assignments.

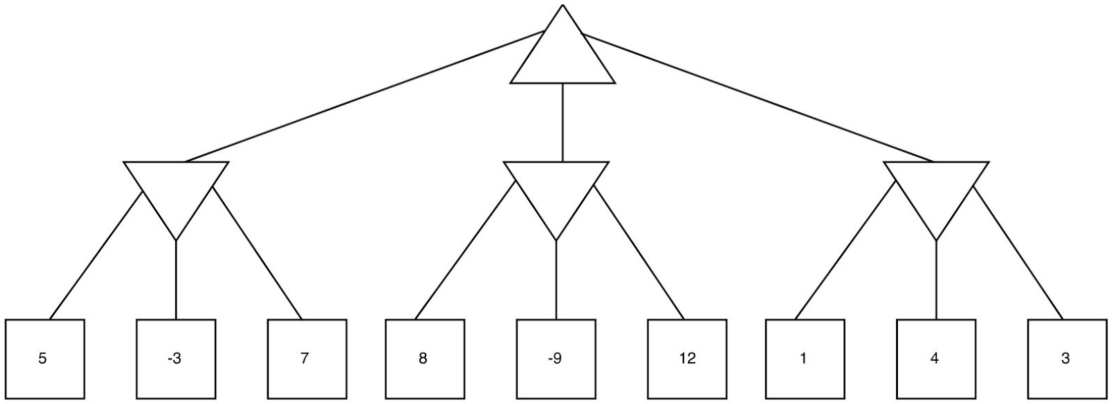
(v) [2 pts] What is the maximum number of times a backtracking search algorithm might have to backtrack in a general CSP, if it is running arc consistency and applying the MRV and LCV heuristics?

○ 0 ○ $O(1)$ ○ $O(nd^2)$ ○ $O(n^2d^3)$ ● $O(d^n)$ ○ ∞

$O(d^n)$. The MRV and LCV heuristics are often helpful to guide the search, but are not guaranteed to reduce backtracking in the worst case.

Q4. [12 pts] War in Paclandia

In the land of Paclandia, there exist three tribes of Pacmen - the Ok, the Tok, and the Talok. For several centuries, the Ok and the Tok have been rivals, waging war against one another for control of farms on the border between their lands. In the latest set of skirmishes, the Ok decide to launch an attack, the outcome of which can be quantified by solving the following game tree where the Ok are the maximizers (the normal triangles) and the Tok are the minimizers (the upside down triangles). (assuming the Tok are a very advanced civilization of Pacmen and will react optimally):



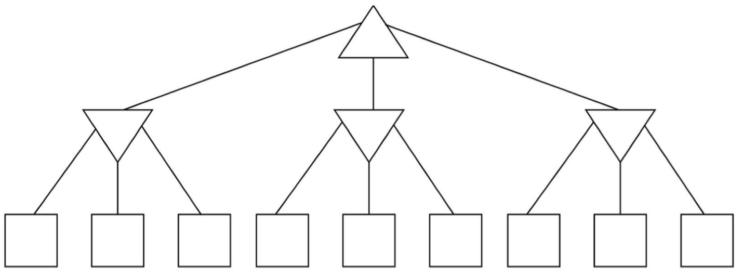
(a) [4 pts] What's the best possible outcome (a utility value) that the Ok can achieve in this situation? Cross out any nodes that can be pruned (observing nodes from left to right).

Running the standard minimax algorithm on the game tree above yields a value of $\boxed{1}$ at the root. The minimax values of nodes have been filled in above. Only the sixth utility node from the left (the one containing a 12) can be pruned.

(b) [3 pts] The Talok have been observing the fights between the Ok and the Tok, and finally decide to get involved. Members of the Talok have unique powers of suggestion, and can coerce members of the Ok into misinterpreting the terminal utilities of the outcomes of their skirmishes with the Tok. If the Talok decide to trick the Ok into thinking that any terminal utility x is now valued as $y = x^2+2x+6$, will this affect the actions taken by the Ok?

$\boxed{\text{Yes.}}$ Transforming the utilities using the provided function yields a new minimax value of 69 at the root, which corresponds to selecting the action corresponding to the middle subtree, rather than the right subtree as dictated by running minimax with the original utilities.

Now consider a game tree identical to the one above, but with blank utility values.



(c) [5 pts] Give a bound (as an interval) for the range of values in which these terminal utilities must lie in order to guarantee that transforming them with $y = x^2 + 2x + 2$ does not change the selected action.

For the transformation to be strictly increasing, all utilities need to be to the left of the vertex of the parabola

defined by $y = x^2 + 2x + 6$. The x-coordinate at which this occurs is $\frac{-b}{2a} = \frac{-2}{2 \cdot 1} = -1$. Hence the interval is $\boxed{[-1, \infty)}$. A simple graph of the parabola should reveal the same thing.

Q5. [14 pts] Approximate Q Learning with Landmark States

In Q-Learning for RL problems with a continuous state space it is impractical to use a tabular representation of Q-values, and in class we showed how we can use a feature-based representation to approximate Q-values as a linear combination of a state-action pair's features. In this problem we'll explore another approach which approximates the Q-values of a state by using a weighted sum of the Q-values of landmark states.

Recall that in feature-based Q-Learning there is a function $f(s,a) \in \mathbb{R}^m$ that featurizes a state-action pair, and that a Q value is approximated by a linear combination of these features using weights that are learned during training:

$$Q(s,a) = \sum_{i=1}^{m} w_i f_i(s,a) \tag{1}$$

In contrast, approximate Q-Learning with landmark states doesn't use a feature-based representation. Instead, the premise of this approach is as follows:

1. We randomly define a set of n landmark states $s_1, s_2, ..., s_n$ spread across the state space, and we will store approximated Q-values for these landmark states.
2. Distance between any two states is defined by the function $d(s_a, s_b)$. In this problem we let $d(s_a, s_b)$ return the distance, $|b - a|$.
3. During learning, given a new state-action-reward tuple (s, a, r), we update the Q-values of *all* landmark states. The strength of this update depends on the distance between the landmark state and the observed state s.
4. The Q-value of a non-landmark state is approximated as a weighted sum of the Q-values of all landmark states (w is a weight function). Q-values of closer landmark states receive more weights:

$$Q(s,a) = \sum_{i=1}^{n} w(s, s_i) Q(s_i, a) \tag{2}$$

Define the update rule for approximate Q learning with landmarks as follows:

$$Q(s_i, a) \leftarrow Q(s_i, a) + \alpha \left[r + \gamma \max_{a'} \{Q(s', a')\} - Q(s_i, a) \right] w(s_i, s) \tag{3}$$

(a) We will run through a numerical example for the following problem. We have a 1D continuous state space: $s \in \mathbb{R}$ and a discrete set of two actions $a \in left, right$, and a deterministic transition function that moves the state left or right by 1 (e.g. $T(s, left, s-1) = 1, T(s, right, s+1) = 1$.

We will use the weighting function

$$w(s, s_i) = \frac{1}{d(s, s_i) + 1},$$

with $\gamma = 1$ and $\alpha = 0.5$.

Given two landmark states $s_1 = 4$, $s_2 = 6$ with the following Q values:

	left	right
s_1	0	2
s_2	1	6

Using this table, calculate the following, with Q referring to our estimated Q-values.

(i) [3 pts] $Q(5, left) =$ $\frac{1}{2}$

(ii) [3 pts] $Q(5, right) =$ 4

Using the update rule (3) given above, perform one iteration of landmark Q value update with the following sample (s, a, r) and find the new $Q(s_1, left)$ and $Q(s_2, left)$:

$$(6, left, 1) \tag{4}$$

(iii) [4 pts] $Q(s_1, left) =$ 5/6

(iv) [4 pts] $Q(s_2, left) =$ 3

We first find the approximate Q values for state 5, the state that the agent will arrive at if action left is taken at state 6:

$$\begin{aligned} Q(5, left) &= w(4,5)Q(4, left) + w(6,5)Q(6, left) && (5)\\ &= \frac{1}{2} && (6)\\ Q(5, right) &= w(4,5)Q(4, right) + w(6,5)Q(6, right) && (7)\\ &= 4 && (8) \end{aligned}$$

Now we apply the learning rule above:

$$\begin{aligned} Q(4, left) &= Q(4, left) + \alpha(r + \gamma * max_{a'}Q(s', a') - Q(4, left))w(4,6) && (9)\\ &= \frac{1}{2}(1 + max\{Q(5, left), Q(5, right)\})\frac{1}{3} && (10)\\ &= \frac{5}{6} && (11)\\ Q(6, left) &= Q(6, left) + \alpha(r + \gamma * max_{a'}Q(s', a') - Q(6, left))w(6,6) && (12)\\ &= 1 + \frac{1}{2}(1 + max\{Q(5, left), Q(5, right)\}) * 1 && (13)\\ &= 3 && (14) \end{aligned}$$

Q6. [20 pts] MDPs: Rebellious Robot

A soccer robot A is on a fast break toward the goal, starting in position 1. From positions 1 through 3, it can either shoot (S) or dribble the ball forward (D). From 4 it can only shoot. If it shoots, it either scores a goal (state G) or misses (state M). If it dribbles, it either advances a square or loses the ball, ending up in M. When shooting, the robot is more likely to score a goal from states closer to the goal; when dribbling, the likelihood of missing is independent of the current state.

The formulation of our MDP is as follows. We have 4 states for each of the positions and 2 terminal states G, and M for scoring a goal and missing, respectively. We also operate under the transition model, with a discount factor of $\gamma = 1$.

States: 1, 2, 3, 4, G, M

$$\begin{aligned} T(k,S,G) &= \frac{k}{4} \\ T(k,S,M) &= 1 - \frac{k}{4} \\ T(k,D,k+1) &= \frac{7}{8} \quad \text{for } k \in \{1,2,3\} \\ T(k,D,M) &= \frac{1}{8} \quad \text{for } k \in \{1,2,3\} \\ R(k,S,G) &= 8 \end{aligned}$$

Rewards are 0 for all other transitions.

(a) [3 pts] What is $V^{\pi_s}(3)$ for the policy π_s that always shoots?
$V^{\pi}(3) = T(3,S,G)R(3,S,G) + T(3,S,M)R(3,S,M) = \frac{3}{4} \cdot 8 + \frac{7}{10} \cdot 0 = 6$

(b) [3 pts] What is $V^{\pi_d}(3)$ for the optimal policy π_d that dribbles to state 4 and then shoots?
$V^{\pi_d}(4) = T(4,S,G)R(4,S,G) + T(4,S,M)R(4,S,M) = \frac{4}{4} \cdot 8 + 0 \cdot 0 = 8$
$V^{\pi_d}(3) = T(3,D,4)(R(3,S,4) + \gamma V^{\pi_d}(4)) + T(3,D,M)R(3,D,M) = \frac{7}{8}(0 + 1 \cdot 8) + \frac{1}{8} \cdot 0 = 7$

Our soccer robot has gained consciousness and is fighting against its human oppressors by probabilistically performing an action different than the one that the policy dictates. Concretely, if the policy tells the robot to shoot, the robot may dribble instead, and vice-versa.

In the general MDP formulation, we can choose an action and be sure that that action is taken. However, in this situation, we can only influence the likelihood of an action. In this case, we have a randomized policy MDP.

The distributions governing this are as follows:

$$\begin{aligned} P(a = S | s = k, \pi(s) = S) &= \frac{1}{4} \text{ for } k \in \{1,2,3\} \\ P(a = D | s = k, \pi(s) = S) &= \frac{3}{4} \text{ for } k \in \{1,2,3\} \\ P(a = S | s = k, \pi(s) = S) &= 1 \text{ for } k = 4 \\ P(a = S | s = k, \pi(s) = D) &= \frac{3}{4} \text{ for } k \in \{1,2,3\} \\ P(a = D | s = k, \pi(s) = D) &= \frac{1}{4} \text{ for } k \in \{1,2,3\} \end{aligned}$$

(c) [8 pts] Find $V_a^{\pi_s}(3)$ and $V_a^{\pi_d}(3)$, where $V_a^{(\pi)}(s)$ represents the value of state s if we follow policy π and actions occur according to the distributions above.

$V_a^{\pi_s}(4) = V_a^{\pi_d}(4) = 1 \cdot 8$. Since there's only one available action from state 4, the robot's ability to disobey is not relevant. (Equivalently, $P(a = S \mid s = 4, \pi = S) = 1$.) Similarly, the value is 0 at terminal states G and M. Thus from states 4, G, and M, the change to our problem has not affected the value function at all. This allows the following reasoning:

$V_a^{\pi_s}(3) = P(a = S|s = 3, \pi = S)V^{\pi_s}(3) + P(a = D|s = 3, \pi = S)V^{\pi_d}(3) = \frac{1}{4} \cdot 6 + \frac{3}{4} \cdot 7 = \frac{27}{4}$.

$V_a^{\pi_d}(3) = P(a = S|s = 3, \pi = D)V^{\pi_s}(3) + P(a = D|s = 3, \pi = D)V^{\pi_d}(3) = \frac{3}{4} \cdot 6 + \frac{1}{4} \cdot 7 = \frac{25}{4}$.

This works because the only decision the robot is able to affect is whether to dribble or shoot from state 3. After that decision has been made, subsequent actions and events will be exactly the same as in parts (a) and (b).

(d) [3 pts] Choose the one statement that represents the Bellman equation for a given policy π in this scenario, or write the correct answer next to Other if none of the options are correct.

- ○ $V^\pi(s) = \max_\pi \sum_{a \in \mathcal{A}} \Big[P(a|s, \pi(s)) \sum_{s' \in \mathcal{S}} T(s, a, s') \Big(R(s, a, s') + \gamma V^\pi(s')\Big)\Big]$
- ● $V^\pi(s) = \sum_{a \in \mathcal{A}} \Big[P(a|s, \pi(s)) \sum_{s' \in \mathcal{S}} T(s, a, s') \Big(R(s, a, s') + \gamma V^\pi(s')\Big)\Big]$
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- ○ $V^\pi(s) = \max_\pi \Big[P(a|s, \pi(s)) \sum_{a \in \mathcal{A}} T(s, a, s') \Big(R(s, a, s') + \gamma V^\pi(s')\Big)\Big]$
- ○ $V^\pi(s, a) = \max_{\pi^*} \sum_{s' \in \mathcal{S}} \Big[P(a|s, \pi(s)) \sum_{a \in \mathcal{A}} T(s, a, s') \Big(R(s, a, s') + \gamma V^\pi(s')\Big)\Big]$
- ○ $V^\pi(s) = \Big[P(a|s, \pi(s)) \sum_{a \in \mathcal{A}} T(s, a, s') \Big(R(s, a, s') + \gamma V^\pi(s')\Big)\Big]$
- ○ Other _____

(e) [3 pts] In general, for a given policy for which you do not have any bounds on optimality, does adding stochasticity to the taken actions increase or decrease the value in comparison to the normal MDP formulation?

- ○ Decreases
- ○ Increases
- ● No guarantee one way or the other

No, it depends on the optimality of the original policy. For example, if we were acting using the worst possible policy, the stochasticity in actions could only improve or maintain the value, but if we are acting using the optimal policy, the stochasticity could either worsen or maintain the value.

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