Statistics 134 - Instructor: Adam Lucas

MIDTERM

Friday, March 2, 2018

Print your name: _____

SID Number: _____

Exam Information and Instructions:

- You will have 45 minutes to take this exam. Closed book/notes/etc. No calculator or computer.
- We will be using Gradescope to grade this exam. Write any work you want graded on the front of each page, in the space below each question. Additionally, write your SID number in the top right corner on every page.
- Please use a dark pencil (mechanical or #2), and bring an eraser. *If you use a pen and make mistakes, you might run out of space to write in your answer.*
- Provide calculations or brief reasoning in every answer.
- Unless stated otherwise, you may leave answers as unsimplified numerical and algebraic expressions, and in terms of the Normal c.d.f. Φ. Finite sums are fine, but simplify any infinite sums.
- Do your own unaided work. Answer the questions on your own. The students around you have different exams.

I certify that all materials in the enclosed exam are my own original work.

Sign your name: _____

GOOD LUCK!

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Suppose you own n unique pairs of socks, each pair with a different pattern. In the first morning of the semester, you grab 2 socks chosen randomly without replacement from the pile of $2n$ socks, and wear them for the day. The next day, you grab a new random pair from the $2n-2$ remaining socks, and so on, until all socks are dirty.

1. (4 pts) Let N be the number of days on which you wear correctly matched socks, based on the setting provided above. Find $\mathbb{E}(N)$.

2. (6 pts) Continued from Q1: Find $Var(N)$.

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3. (5 pts) In a game of poker, you are dealt a five card hand from a standard deck. Given that you have the king of spades and the king of clubs, what is the chance that you have a full house? (A full house is ranks *aaabb*. Note that the two kings are of the same rank.)

4. (5 pts) A pearl diver goes from clam to clam, checking to see if a pearl is inside. Suppose 10% of clams contain a pearl inside. **Approximate** the chance that the diver has to check less than 175 clams in order to find his 15th pearl.

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5. (5 pts) Suppose X is a Poisson (μ) random variable, where $\mu = 0.5$. Calculate $\mathbb{E}(X!)$.

6. (5 pts) Suppose 3% of individuals of a population have a disease. There is a diagnostic test available, but it isn't perfect: healthy people are falsely diagnosed as having the disease 4% of the time, and people with the disease are falsely diagnosed as healthy 15% of the time. Given that a randomly selected individual has two positives and one negative result in three independent tests, what is the chance that the individual has the disease?

Discrete

name and range	$P(k) = P(X = k)$ for $k \in$ range	mean	variance
uniform on $\{a, a+1, \ldots, b\}$	$\dfrac{1}{b-a+1}$	$\dfrac{a+b}{2}$	$\dfrac{(b-a+1)^2-1}{12}$
Bernoulli (p) on $\{0,1\}$	$P(1) = p;\ P(0) = 1-p$	p	$p(1-p)$
binomial (n,p) on $\{0,1,\ldots,n\}$	$\binom{n}{k} p^k (1-p)^{n-k}$	np	$np(1-p)$
Poisson (μ) on $\{0,1,2,\ldots\}$	$\dfrac{e^{-\mu}\mu^k}{k!}$	μ	μ
hypergeometric (n,N,G) on $\{0,\ldots,n\}$	$\dfrac{\binom{G}{k}\binom{N-G}{n-k}}{\binom{N}{n}}$	$\dfrac{nG}{N}$	$n\left(\dfrac{G}{N}\right)\left(\dfrac{N-G}{N}\right)\left(\dfrac{N-n}{N-1}\right)$
geometric (p) on $\{1,2,3\ldots\}$	$(1-p)^{k-1}p$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$
geometric (p) on $\{0,1,2\ldots\}$	$(1-p)^k p$	$\dfrac{1-p}{p}$	$\dfrac{1-p}{p^2}$
negative binomial (r,p) on $\{0,1,2,\ldots\}$	$\binom{k+r-1}{r-1} p^r (1-p)^k$	$\dfrac{r(1-p)}{p}$	$\dfrac{r(1-p)}{p^2}$