CEC30/MEC85 Midterm Examination 1

February 21th, 1710–1800

NAME : ______________________________

SID : ______________________________

Problem 1: _____ /22 points

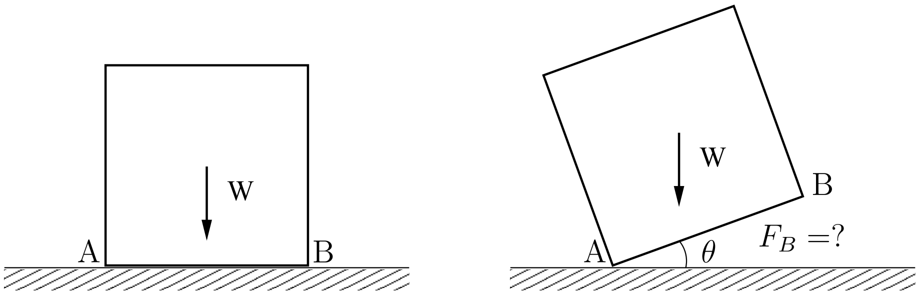
Problem 2: _____ /21 points

Problem 3: _____ /21 points

Notes:
1. Write your name and SID on the cover page.
2. Turn off your cell phone.
3. Record your answers only in the pages provided.
4. You may not ask questions during the exam.

Problem 1 (6+12+4 points)

A homogeneous square box of side a and weight W is initially at rest on a frictionless horizontal plane. A force acting at the bottom right corner B lifts the box to an angle θ with the horizontal plane, as shown in the figure.

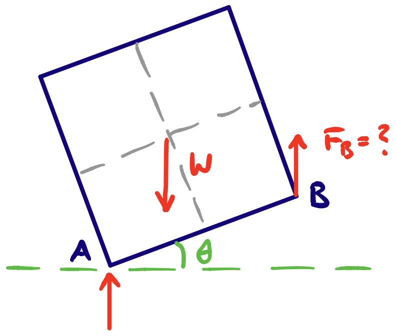


(a) Draw the free-body diagram of the box in the rotated configuration assuming that it is in equilibrium.

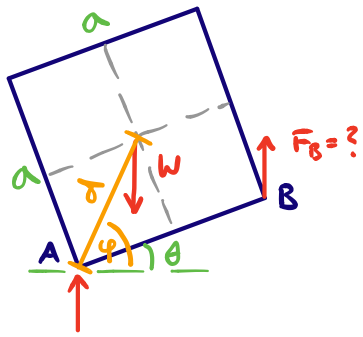
(b) Find the force F_B acting at B as a function of the angle θ.

(c) To check the accuracy of the formula derived in part (b), examine the value of F_B for $\theta = 0+$ (that is, just as the right side of the box is lifted from the plane) and $\theta = \pi/4$. Are they consistent with your intuition?

(a)



(b) Moment at A: $\sum M_A = 0$ (From $\sum F_x$: $\underline{F}_B = F_B \underline{j}$)



$$d = \frac{1}{\sqrt{2}} a$$

$$\varphi = \theta + 45^\circ$$

$$a \cos\theta \, F_B = d \cos\varphi \, W$$

$$F_B = \frac{1}{\sqrt{2}} \frac{\cos\varphi}{\cos\theta} W$$

$$= \frac{1}{\sqrt{2}} \frac{\cos(\theta + 45^\circ)}{\cos\theta} W$$

(c) for $\theta = 0+$ $\quad F_B = \frac{1}{\sqrt{2}} \frac{\cos(45^\circ)}{\cos(0^\circ)} W = \frac{1}{2} W$

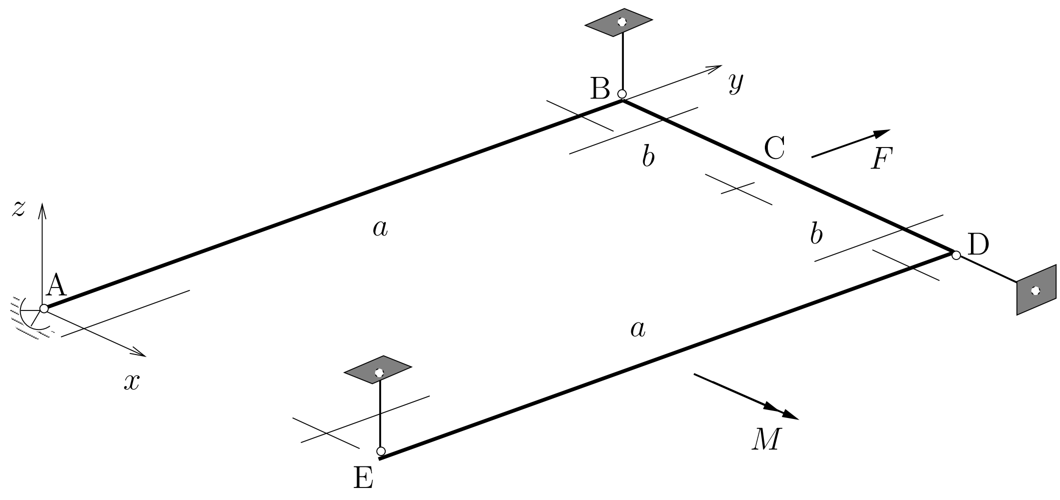
$\left(\sum F_y: \; A_y = W - F_B = \frac{1}{2} W \checkmark.\right)$

for $\theta = \frac{\pi}{4}$ $(= 45^\circ)$ $\quad F_B = \frac{1}{\sqrt{2}} \frac{\cos(90^\circ)}{\cos(45^\circ)} W = 0$

(W and A_y collinear $\checkmark$.)

Problem 2 (6+12+3 points)

A three-dimensional massless solid is kept in place by a ball-socket support at point A and three rigid links at points B, D, and E, and is subject to an external force F and an external moment M, as in the figure below.

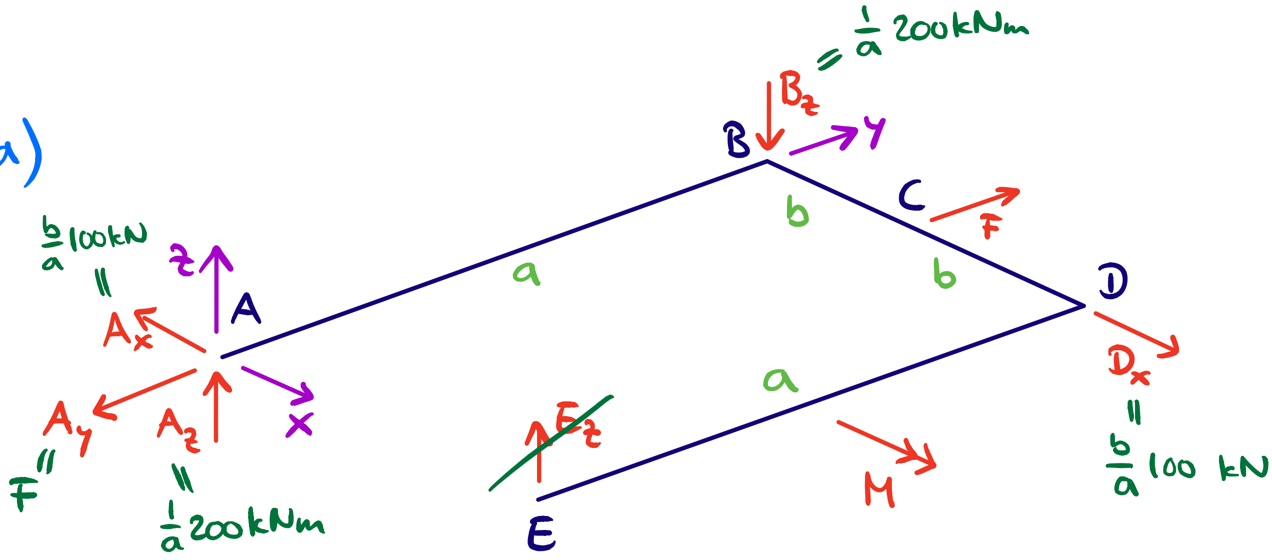


(a) Draw the free body diagram of the solid.

(b) Determine all the reactions.

(c) Given the external load, which of the three rigid links would be possible to replace with inextensible cables?

(a)



(b) ΣF_y: $\quad A_y = F \; (= 100\,\mathrm{kN})$

ΣM_y^A: $\quad 2b \cdot E_z = 0 \quad \Rightarrow E_z = 0$

ΣM_x^A: $\quad M = a\,B_z \quad \Rightarrow B_z = \frac{1}{a} M = \frac{1}{a} 200\,\mathrm{kNm}$

ΣF_z: $\quad A_z = B_z \quad \Rightarrow A_z = \frac{1}{a} 200\,\mathrm{kNm}$

ΣM_z^A: $\quad a \cdot D_x = b\,F \quad \Rightarrow D_x = \frac{b}{a} F = \frac{b}{a} 100\,\mathrm{kN}$

ΣF_x: $\quad A_x = D_x \quad \Rightarrow A_x = \frac{b}{a} F = \frac{b}{a} 100\,\mathrm{kN}$

(c) Which links are <u>not</u> in compression?

Only at E $(E_z = 0)$

Check: $\quad \underline{M}_A = (M - a B_z)\,\underline{i} + (-2b E_z)\,\underline{j} + (bF - a D_x)\,\underline{k}$

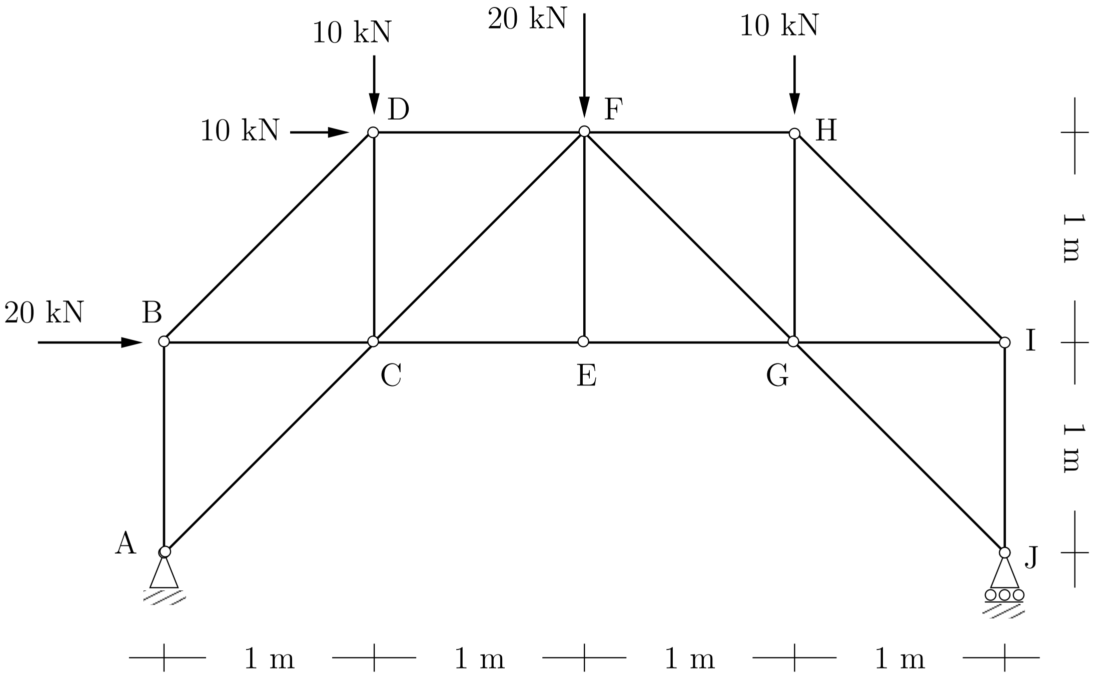
$\underline{R} = (D_x - A_x)\,\underline{i} + (F - A_y)\,\underline{j} + (E_z + A_z - B_z)\,\underline{k}$

$\Rightarrow B_z = \frac{1}{a} M \qquad E_z = 0 \qquad D_x = \frac{b}{a} F$

$\Rightarrow A_x = \frac{b}{a} F \qquad A_y = F \qquad A_z = \frac{1}{a} M$

Problem 3 (3+3+6+9 points)

Consider the simply-supported two-dimensional truss shown in the figure below.

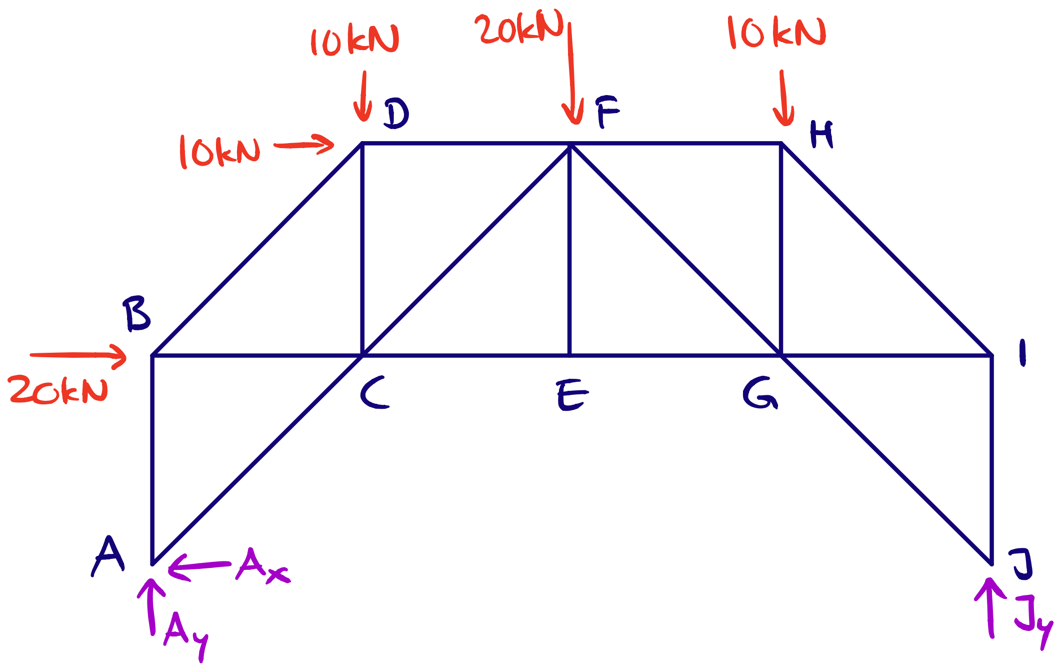


(a) Argue that the truss is statically determinate.

(b) Determine the external reactions at points A and B.

(c) Determine the forces of members IJ and GJ, and state explicitly if they are in tension or compression.

(d) Determine the forces of members CE, EF, and EG, and state explicitly if they are in tension or compression.



(a) $\left.\begin{array}{l} r=3 \\ n=17 \end{array}\right\}$ 20 unknowns $j=10 \rightarrow 20$ eqns

All consequtive triangles & proper external support ✓.

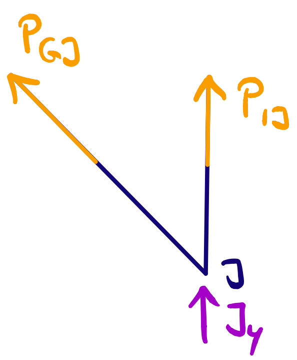
(b) ΣM_A: $\quad 4m \cdot J_y = \underbrace{2m \cdot 40kN}_{\text{exploit symmetry of vertical forces}} + 1m \cdot 20kN + 2m \cdot 10kN$

$$= 120 \text{ kNm} \quad \Rightarrow J_y = 30 \text{ kN}$$

ΣF_x: $\quad A_x = 20kN + 10kN = 30kN$

ΣF_y: $\quad A_y = 10kN + 20kN + 10kN - J_y = 10 \text{ kN}$

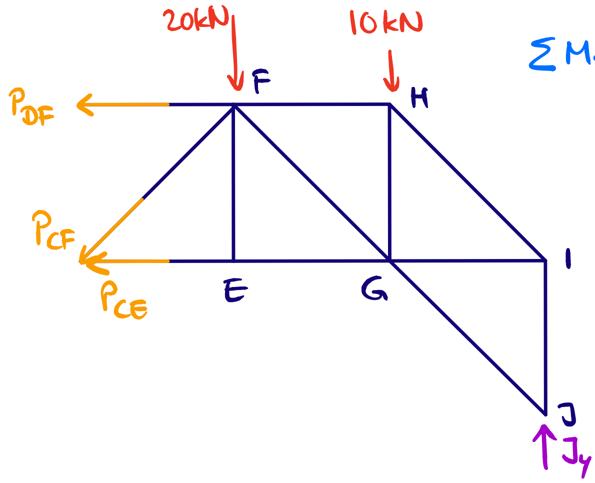
(c)



ΣF_x: $\quad P_{GJ} = 0$ (neither)

ΣF_y: $\quad P_{IJ} = -J_y = -30kN$
(compression)

(d)



$$\Sigma M_F: \quad 2m \cdot \underbrace{J_y}_{30kN} = 1m \cdot 10kN + 1m \cdot P_{CE}$$

$$\Rightarrow P_{CE} = 50kN$$

(tension)

Joint E:

$$\Sigma F_x: \quad P_{EG} = P_{CE} = 50 \text{ kN} \quad \text{(tension)}$$