Solutions to Midterm 1

February 20, 2018

Problem 2

Use the lens equation to find the image distance

$$\frac{1}{\frac{5f}{3}} + \frac{1}{d_{i_1}} = \frac{1}{f}$$
$$d_{i_1} = \frac{5f}{2}$$

This is a real image. Now for the second lens, object distance is $d_{o_2} = -\frac{f}{2}$ since it is to the right of the lens. Use the lens equation to find the image distance

$$-\frac{2}{f} + \frac{1}{d_{i_2}} = -\frac{1}{f}$$
$$d_{i_2} = f$$

This is also a real image. Important features of the ray diagram - show two different light rays that converge to appear to meet at d_{i_1}, diverge to meet at d_{i_2} in the end

Point distribution for each image

1. Using lens equation - 3 points
2. Correct object distance - 3 points
3. Correct focal length - 1 point
4. Finding correct image distance - 3 points
5. Correctly saying whether real or virtual image - 3 points

For the ray diagram - 1 point for each feature of each ray - total 4 points

Problem 3

From the geometry of the problem with r being the angle of refraction ,

$$y_1 = W\ \tan r$$

From Snell's law, we have

$$n \sin r = \sin\theta$$

Now

$$\begin{aligned}\tan r &= \frac{\sin r}{\cos r} \\ &= \frac{\frac{\sin\theta}{n}}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}} \\ &= \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}\end{aligned}$$

Thus, $y_1 = \frac{W \sin\theta}{\sqrt{n^2 - \sin^2\theta}}$.
Now, $y_2 = y - y_1$ where $y = W\ \tan\theta$. Thus, we have

$$y_2 = W \sin\theta \left(\frac{1}{\cos\theta} - \frac{1}{\sqrt{n^2 - \sin^2\theta}} \right)$$

In the limit of $n \to 1$, the denominator of the second term becomes $\cos\theta$ as well and thus, they cancel to give $y_2 = 0$. This is precisely what is expected since there should be no deviation in that limit.

Point distribution

1. Using trigonometry to relate y_1 to W - 4 points
2. Using Snell's law correctly - 4 points
3. Expressing y_1 in terms of n, W and θ - 3 points
4. Finding y_2 using trigonometry to get y - 4 points
5. Expressing y_2 in terms of n, W and θ - 2 points
6. Showing that y_2 goes to zero in the limit - 3 points

Solutions to Midterm Problems 1 and 4

Canxun Zhang

February 21, 2018

Problem 1

(a) The lens equation: (2pts)

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

The focal length: (1pt)

$$f = D$$

The object distance: (1pt)

$$d_o = D - \epsilon$$

The image distance: (2pts)

$$d_i = -\frac{D(D-\epsilon)}{\epsilon}$$

The image is virtual since $d_i < 0$. (2pts)
Total: 8pts.

(b) The lens equation: (2pts)

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

The focal length: (1pt)

$$f = D$$

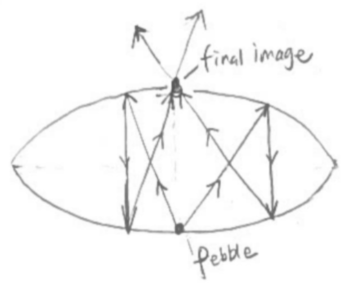
The object distance: (1pt)

$$d_o = D - \left(-\frac{D(D-\epsilon)}{\epsilon}\right) = \frac{D^2}{\epsilon}$$

The image distance: (2pts)

$$d_i = \frac{D^2}{D-\epsilon} \to D$$

The image is real since $d_i > 0$. (2pts)
Total: 8pts.



(c) (4pts) (by Wenxin Zhang)

Problem 4

(a) The path difference between two adjacent slits: (3pts)

$$\Delta l = d \sin\theta$$

The phase difference between two adjacent slits: (3pts)

$$\delta = kd\sin\theta = 2\alpha$$

The modulating factor: (4pts)

$$f = \frac{\sin(3\delta/2)}{\sin(\delta/2)} = \frac{\sin 3\alpha}{\sin\alpha}$$

Total: 10pts.

(b)

$$\frac{\sin 3\alpha}{\sin\alpha} = \frac{\sin 2\alpha\cos\alpha + \cos 2\alpha\sin\alpha}{\sin\alpha} = 2\cos^2\alpha + \cos 2\alpha = 2(1-\sin^2\alpha) + 1 - 2\sin^2\alpha = 3 - 4\sin^2\alpha$$

2 pts for every step; total: 8pts.

(c) Maximum for f^2 corresponds to minimum for $\sin^2\alpha$: (3pts)

$$\sin\alpha = 0$$

The two smallest positive values: (3pts)

$$\alpha = \frac{\pi}{2}, \pi$$

Minimum for f^2 corresponds to $f = 0$. (2pts)
Solve for $\sin\alpha$: (2pts)

$$\sin\alpha = \pm\frac{\sqrt{3}}{2}$$

The two smallest positive values: (2pts)

$$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$$

Total: 12pts.