University of California, Berkeley
Physics 110B Fall 2003 (*Strovink*)

FINAL EXAMINATION

Directions: Do all six problems, which have unequal weight. This is a closed-book closed-note exam except for Griffiths, Pedrotti, a copy of anything posted on the course web site, and anything in your own original handwriting (not Xeroxed). A table of Fourier transforms is included with the exam. Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (30 points)
A point charge e travelling on the z axis has position

$$\begin{aligned}\mathbf{r}(t) &= +\hat{\mathbf{z}}\beta ct \quad (t<0)\\ &= -\hat{\mathbf{z}}\beta ct \quad (t>0)\ ,\end{aligned}$$

where β is a positive constant that is not $\ll 1$. That is, the charge reverses direction instantaneously at $t=0$, while it is at the origin. The fields that the charge produces are viewed by an observer at $(x,0,0)$, where $x>0$.

(a.) (15 points)
What magnetic field $\vec{B}$ does the observer see at $t=0$?

(b.) (15 points)
At time t such that $ct=x$ (exactly!), what is the direction of the electric field $\vec{E}$ seen by the observer? (You need consider only the part of the total electric field which is dominant at exactly that time.) Justify your answer.

Problem 2. (30 points)
A plane wave of wavelength λ is normally incident on a thin film that is tinted various shades of gray. The ratio of incident to transmitted field *amplitude* is the aperture function $g(\vec{s})$, where $\vec{s}$ is a vector from the origin (taken on the downstream surface of the film) to another point on its downstream surface. This film has an aperture function

$$g(\vec{s}) = e^{-s^2/2d^2}\ .$$

Because this aperture function is cylindrically symmetric, the outgoing diffraction pattern is also cylindrically symmetric: it is a function of θ, the polar angle of the observer relative to the beam axis. You may make the approximations

$$\begin{aligned}\theta &\ll 1\\ d^2 &\ll 2\lambda D\ ,\end{aligned}$$

where D is the distance from the film to the observer.

Calculate the *irradiance* ratio

$$\mathcal{R}(\theta) = \frac{I(\theta)}{I(\theta=0)}\ .$$

Problem 3. (35 points)
The irradiance I_0 of a mystery light beam is attenuated by each of four devices, applied one at a time: (A) a grey filter passing half the incident irradiance; (B) an $\hat{x}$ polarizer; (C) a $\frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$ polarizer; and (D) a device consisting of a quarter-wave plate (QWP) with slow axis at $+45°$ to $\hat{x}$, followed by an $\hat{x}$ polarizer, followed by a QWP with slow axis at $-45°$ to $\hat{x}$. The attenuated irradiances observed are, respectively,

$$\begin{aligned}I_A &= \tfrac{1}{2}I_0\\ I_B &= \tfrac{1}{2}\big(1+\tfrac{1}{\sqrt{2}}\big)I_0\\ I_C &= \tfrac{1}{2}I_0\\ I_D &= \tfrac{1}{2}\big(1+\tfrac{1}{\sqrt{2}}\big)I_0\ .\end{aligned}$$

With devices (A) through (D) no longer in the picture, device (E) is inserted into the beam. It is the same as device (D) except that the $\hat{x}$ polarizer is rotated to become a $\hat{y}$ polarizer. What irradiance I_E is observed?

Problem 4. (35 points)
Electromagnetic waves of frequency ω are normally incident from vacuum upon a thin region of dilute material with constant ohmic conductivity σ. The space downstream of this region is also vacuum. Within the material, denote the complex wave vector $\tilde{k}$ by

$$\tilde{k} = k + i\kappa \ ,$$

where k and κ are real constants. (For this problem, κ is not supplied; you must calculate it.) The material has negligible magnetic properties, so you may approximate $\mu \approx \mu_0$. The effect of its bound electrons is also negligible, so you may approximate the *real part* of $\tilde{k}$ by

$$\mathrm{Re}\,\tilde{k} \equiv k \approx \frac{\omega}{c} \ .$$

(This is equivalent to taking $\epsilon \approx \epsilon_0$, where ϵ is the real part of the dielectric constant.) Finally, when it is multiplied by $\mathrm{Re}\,\tilde{k}$, the thickness d of the material is chosen so that

$$kd = \pi$$

exactly. Define

$$\beta \equiv \frac{\sigma}{\epsilon_0 \omega} \ .$$

In the limit $\beta \ll 1$, including terms of order β but neglecting those of higher order, calculate the (possibly complex) fraction r of the incident complex electric field amplitude that is reflected.

Problem 5. (35 points)
At a future linear positron-electron (e^+e^-) collider, (not necessarily resonant) production of top quark-antiquark pairs ($t\bar{t}$) is expected to occur via the reaction

$$e^+ + e^- \to t + \bar{t} \ .$$

Very soon thereafter, the top quarks decay according to

$$t \to W^+ + b$$
$$\bar{t} \to W^- + \bar{b} \ .$$

Here you may neglect all rest masses except for those of the t and W (particle and antiparticle masses are the same); for the purposes of this problem, you may approximate

$$m_t = 2\, m_W \ .$$

Under all circumstances, it would be highly improbable for one of the W's to be produced *completely at rest* in the laboratory. But is it kinematically *possible*? If so, what restriction, if any, is placed on the equal total energy E of the positron or electron beams?

Problem 6. (35 points)
Lorentz frame $\mathcal{S}_{BA}$ is related to the laboratory frame $\mathcal{S}$ by a sequence of two Lorentz transformations: (1) A boost (A) by rapidity η along $\hat{x}$; (2) a boost (B) by the same η along $\hat{y}$. Lorentz frame $\mathcal{S}_{AB}$ is related to the lab frame by applying the same two boosts, but in the opposite order. Here, as usual, $\eta = \tanh^{-1}\beta$, where βc is the relative speed characterizing each transformation.

In the limit $\beta \to 0$, retaining terms of order β^2, show that frames $\mathcal{S}_{BA}$ and $\mathcal{S}_{AB}$ are the same, except that one is *rotated* with respect to the other. Solve for the angle θ by which frame $\mathcal{S}_{AB}$ is rotated with respect to frame $\mathcal{S}_{BA}$, and specify the axis about which this rotation occurs.