Math 1A (Fall 2017) Midterm I (Thursday September 14, 3:40-5:00)

Name: SID:

Please write clearly and legibly. Justify your answers except Problem 1. Partial credits may be given to Problems 2, 3, 4, 5, and 6. When submitting the exam, include scratch paper if you put solutions there or if you think you may get partial credit from scratch work. In that case write your name on scratch paper to avoid a mix-up.

1. Mark each of the following True (T) or False (F). No justification is necessary. (For each sub-problem, correct = 4 pts, no response = 2 pts, wrong = 0 pts.)

(1) ( ) If f is an odd function and g is an even function then the composite function $f \circ g$ is odd.

(2) ( ) If f is a one-to-one function defined on $\mathbb{R}$ then f^{-1} is also a one-to-one function.

(3) ( ) If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} f(x)g(x)$ is either ∞ or 0.

(4) ( ) The line $x = 1$ is a vertical asymptote of $y = \dfrac{1}{x-1}$ because $\lim_{x\to 1^-} \dfrac{1}{x-1} = -\infty$.

(5) ( ) If f is defined on $(0, \infty)$ and $f(\frac{1}{n}) = 0$ for $n = 1, 2, 3, \ldots$ then $\lim_{x\to 0^+} f(x) = 0$.

2. (15 pts) Answer the following.

(1) (5 pts) Compute $\sin(\tan^{-1}(2))$. (Express answer as a number without trigonometric functions.)

(2) (10 pts) Let $f(x) = \ln(x-1)$ and $g(x) = \dfrac{3x+1}{x}$. What is the domain of the composite function $g \circ f$?

3. (15 pts) Let $f(x) = \begin{cases} |x|/x^3, & x < 0, \\ \sin(1/x), & x > 0. \end{cases}$ Find $\lim_{x \to 0^-} x^2 f(x)$ and $\lim_{x \to 0^+} x^2 f(x)$. For either limit, if the limit does not exist, explain why.

4. (15 pts) Explain how the graph of the function $\sqrt{2-x}+3$ is obtained from the graph $y=\sqrt{x}$ by a sequence of basic transformations (shifting, expanding, shrinking, or reflecting the graph). Make sure to indicate in what order the transformations are performed. Using this, sketch the graph of $\sqrt{2-x}+3$ in the xy-plane.

5. (15 pts) Consider the function $f(x) = \ln(2 + e^x)$ on $\mathbb{R}$, which is one-to-one. Find the formula for $f^{-1}(x)$.

6. (20 pts) Compute the following limits if they exist. For either limit, if the limit does not exist, explain why.

(1) $\lim_{x \to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$

(2) $\lim_{t \to 0} \left(\frac{\sqrt{1+t} - \sqrt{1-t}}{t} \right)$