Math 53 First Midterm Exam, Prof. Srivastava
February 22, 2018, 5:10pm–6:30pm, 155 Dwinelle Hall.

Name: ______________________________

SID: ______________________________

GSI: ______________________________

Name of the student to your left: ______________________________
Name of the student to your right: ______________________________

Instructions: Write all answers clearly in the provided space. This exam includes some space for scratch work at the bottom of pages 2 and 4 which will not be graded. Do not under any circumstances unstaple the exam. Write your name and SID on every page. Show your work — numerical answers without justification will be considered suspicious and will not be given full credit. Calculators, phones, cheat sheets, textbooks, and your own scratch paper are not allowed.
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Sign here: ______________________________

Question	Points
1	10
2	10
3	13
4	10
5	21
6	10
7	10
8	16
Total:	100

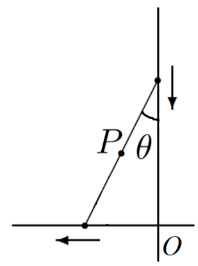
Do not turn over this page until your instructor tells you to do so.

1. (10 points) Find a parametric equation of the line defined by the intersection of the planes:

$$x - y + 3z = 5$$
$$3x + y - z = 3.$$

[Scratch Space Below]

2. The top extremity of a ladder of unit length rests against a vertical wall, while the bottom is being pulled away, as shown below.



(a) (8 points) Find a parameterized curve $\mathbf{r}(\theta)$ tracing the trajectory of the midpoint P of the ladder as it goes from fully vertical to horizontal, using as parameter the angle θ between the ladder and the vertical wall, and treating the point at which the wall and the ground meet as the origin.

(b) (2 points) Is the speed of P (as a function of θ) increasing, decreasing, or constant as θ varies from 0 to $\pi/2$?

3. Consider the function $f(x, y) = xy$.

   (a) (5 points) Roughly sketch the level curves $f(x, y) = 1$, $f(x, y) = 2$, and $f(x, y) = -1$, labeling which is which.

   (b) (5 points) Find a point P on the curve $f(x, y) = 1$ at which the directional derivative along the direction $\mathbf{u} = \frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}$ is equal to zero.

   (c) (3 points) Sketch the direction of the vector ∇f at the point P that you found above, originating from P in the drawing in part (a).

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4. (10 points) The parameterized curves $\mathbf{a}(t) = \langle 3t, t^2 - 2, e^t + 1 \rangle$ and $\mathbf{b}(s) = \langle s^2 - s, -2s, s^3 + s^2 \rangle$ intersect at the point $P = (0, -2, 2)$. Find the cosine of the (acute) angle of their intersection at P.

[Scratch Space Below]

5. Consider the function $f(x, y) = \sqrt{e^x + 3e^{y-1}}$.

   (a) (5 points) Find the total differential of f at $x = 0, y = 1$.

   (b) (5 points) Find the equation of the tangent plane to the graph $\{(x, y, z) : z = f(x, y)\}$ of f at the point $P = (0, 1, f(0, 1))$.

(c) (5 points) Use the equation from (b) to compute an approximation to the value of $f(.01, .99)$.

(d) (3 points) Is there a point Q on the graph at which the tangent plane to the graph is horizontal (i.e., parallel to the xy-plane)? If so, find such a point, and if not explain why.

(e) (3 points) Is there a point R on the graph where the tangent plane is vertical (i.e., perpendicular to the xy-plane)? If so, find such a point, and if not explain why.

6. (10 points) Let $w = w(x, y)$ be a differentiable function and let $x = s^2t$ and $y = s + 1/t$. Use the chain rule to express the partial derivatives w_s and w_t in terms of w_x, w_y, s, and t.

7. (10 points) Suppose $z(x, y)$ is a differentiable function satisfying the equation

$$x^2 - y^2 + z^2 - 2z = 4.$$

Find $\partial z/\partial x$ and $\partial^2 z/\partial x^2$ in terms of x, y, z.

8. Let $A(x, y)$ be the area of the triangle with vertices at the points $P = (0, 0), Q = (1, 2)$, and $R = (x, y)$.

   (a) (5 points) Derive a formula for $A(x, y)$.

   (b) (5 points) Identify all critical points of $A(x, y)$, and classify them as local minima, maxima, or saddle points, with justification.

(c) (6 points) Use Lagrange multipliers to find a point $R = (x, y)$ on the curve

$$xy = -1$$

which minimizes $A(x, y)$, and determine the minimum area. (hint: it might be easier to minimize A^2)