Problem 1

(a) 6pts

For a diatomic ideal gas near room temperature, $C_V = \frac{5}{2}R$, and $C_P = C_V + R = \frac{7}{2}R$.

(i) If the gas is expanded at constant pressure,

$$Q = nC_P\Delta T = \frac{7}{2}nR\Delta T$$

$$W = P\Delta V = nR\Delta T$$

$$\frac{W}{Q} = \frac{nR\Delta T}{\frac{7}{2}nR\Delta T} = \frac{2}{7}$$

(ii) If the gas is expanded at constant temperature,

$$\Delta E_{int} = 0 \ (\Delta T = 0)$$

$$Q = W$$

by the first law of thermodynamics. Hence,

$$\frac{W}{Q} = 1$$

Grading Rubric:

+3 Correct W/Q for the constant-pressure expansion

(+1 Attempt to use the first law of thermodynamics, but insufficient/wrong)

(+2 Correct approach, but wrong C_P)

+3 Correct W/Q for the constant-temperature expansion

(+1 Attempt to use the first law of thermodynamics, but insufficient/wrong)

(+2 Know $\Delta E = 0$)

(b) 6pts

Before heated, the initial volume of a liquid is $V_i = AH$.
After it is heated by an amount ΔT, its volume changes by $\Delta V = \beta V_i \Delta T$.
It is given that the cup has a constant area A, independent of temperature, so ΔH is:

$$\Delta H = \frac{\Delta V}{A} = \frac{\beta V_i \Delta T}{A} = \frac{\beta AH\Delta T}{A} = \beta H \Delta T$$

Grading Rubric:

+6 All correct

+2 Know ΔV is proportional to ΔT, but wrong expression for ΔV

+3 Write $\Delta V = V_i(1 + \beta\Delta T)$

+4 Know $\Delta V = \beta V_i \Delta T$

(c) 6pts

The rms speed of the nitrogen at the temperature T is: $v_{rms,N_2} = \sqrt{\frac{3kT}{m_{N_2}}}$. $(\overline{KE_{N_2}} = \frac{1}{2}m_{N_2}\overline{v^2} = \frac{3}{2}kT)$.
Given the Maxwell distribution of speeds $f(v)$, we can compute the fraction of the hydrogen molecules that have speed faster than v_{rms,N_2}:

$$\int_{v_{rms,N_2}}^{\infty} f(v)dv = \int_{v_{rms,N_2}}^{\infty} 4\pi\left(\frac{m_{H_2}}{2\pi kT}\right)^{3/2} v^2 e^{-m_{H_2}v^2/2kT}dv$$

where T = 300K and $v_{rms,N_2} = \sqrt{\frac{3kT}{m_{N_2}}}$.

Grading Rubric:

+6 All Correct

+3 Correct v_{rms,N_2}

(-1 Not specifying that m is the mass of the nitrogen)

(-1 Other minor mistakes)

+3 Correct integral

(-1 Wrong integration bounds)

(-1 Not specifying that m in f(v) is the mass of H_2)

(-1 Other minor mistakes)

2.

(a)

The net heat absorbed by the system is zero, so express the heats absorbed and gained in a single equation. Intuitively, the ice absorbs heat, melts, and absorbs some more heat, and the water releases some heat. So:

$$0 = m_i c_i (T_m - T_i) + m_w c_w (T_f - T_w) + m_i L + m_i c_w (T_f - T_m)$$

where T_m is the melting temperature, T_w and T_i are the initial temperatures of water and ice, and T_f is the final temperature we solve for. the m's are the masses, and the c's, L are the specific heats of ice, water, and the latent heat of fusion. Thus, cancelling out the masses since they are equal:

$$T_f = \frac{c_w(T_m + T_w) - c_i(T_m - T_w) - L}{2c_w}$$

To one significant digit, this results in about 4 degrees Celsius.

Rubric:

4 points for writing down that the net heat change is zero (3 points if incorrect method).

5 points for solving for the correct answer (okay to leave in terms of variables if variables are clearly defined). (4 points if initial equation is correct but they are off by a small error, such as a sign. 3 points if initial equation is correct but they made a large algebraic mistake. 1 point if initial equation is incorrect.)

(b)

The entropy is given by the sums of the entropy changes of the heat terms in part a:

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_m} \frac{m_i c_i}{T} + \int_{T_w}^{T_f} \frac{m_w c_w}{T} + \int_{T_m}^{T_f} \frac{m_i c_w}{T} + \frac{m_i L}{T_m}$$

This simplifies to:

$$\Delta S = m_i c_i \ln\left(\frac{T_m}{T_i}\right) + m_w c_w \ln\left(\frac{T_f}{T_w}\right) + m_w c_w \ln\left(\frac{T_f}{T_m}\right) + \frac{m_i L}{T_m}$$

Rubric:

4 points for writing down the entropy equation in terms of sums of individual entropy changes (3 points if incorrect method).

5 points for solving for the entropy via integration (okay to leave in terms of variables or if not simplified, or if they have the right answer symbolically but got part a wrong). (4 points if initial equation is correct but they are off by a small error, such as a sign. 3 points if initial equation is correct but they made a large mistake, such as plugging in temperatures in Celsius instead of Kelvin. 1 point if initial equation is incorrect.)

Physics 7B Midterm 1 Problem 3 Solution

Begin by noting that by definition of efficiency, $W_1 = e_1 Q_{\text{in},1}$ and $W_2 = e_2 Q_{\text{in},2}$, where $Q_{\text{in},1}$ is the heat input to engine 1, and $Q_{\text{in},2}$ is the heat input to engine 2. Therefore, the total work done, W, is given by $W = e_1 Q_{\text{in},1} + e_2 Q_{\text{in},2}$. The heat input to the composite heat engine is simply the heat input to engine 1, since engine 2 receives its heat input from the waste heat of engine 1. Therefore, the efficiency of the composite engine, given by $e = W/Q_{\text{in}}$, is:

$$e = \frac{e_1 Q_{\text{in},1} + e_2 Q_{\text{in},2}}{Q_{\text{in},1}}. \tag{1}$$

Since the heat input to engine 2 is simply the waste heat of engine 1, we have that $Q_{\text{in},2} = Q_{\text{out},1}$. By the first law, since the internal energy of the gas is unchanged after a complete cycle, $Q_{\text{in},1} - Q_{\text{out},1} - W_1 = 0$. Therefore, $Q_{\text{in},2} = Q_{\text{out},1} = Q_{\text{in},1} - W_1 = (1 - e_1)Q_{\text{in},1}$, where the last equality follows from the definition of efficiency. Inserting this expression for $Q_{\text{in},2}$ into the above equation for the efficiency of the composite engine gives:

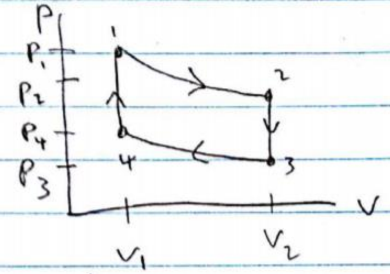
$$\begin{aligned} e &= \frac{e_1 Q_{\text{in},1} + e_2(1 - e_1)Q_{\text{in},1}}{Q_{\text{in},1}} \\ &= e_1 + (1 - e_1)e_2. \end{aligned} \tag{2}$$

Since the two engines are taken to be ideal, $e_1 = 1 - T_M/T_H$, and $e_2 = 1 - T_L/T_M$. The overall efficiency is therefore given by:

$$\begin{aligned} e &= (1 - \frac{T_M}{T_H}) + \left(1 - (1 - \frac{T_M}{T_H})\right)(1 - \frac{T_L}{T_M}) \\ &= 1 - \frac{T_M}{T_H} + \frac{T_M}{T_H}(1 - \frac{T_L}{T_M}) \\ &= 1 - \frac{T_M}{T_H} + \frac{T_M}{T_H} - \frac{T_M T_L}{T_H T_M} \\ &= 1 - \frac{T_L}{T_H}. \end{aligned} \tag{3}$$

a) $\gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}+1}{5/2} = 1 + \frac{2}{5} = \frac{7}{5}$

b)



c)

	W_{by}	Q_{in}	ΔS
1	$P_1 V_1 \ln(V_2/V_1)$	$P_1 V_1 \ln(V_2/V_1)$	$nR \ln(V_2/V_1)$
2	0	$\frac{5}{2} V_2 (P_3 - P_2)$	$\frac{5}{2} nR \ln(P_3/P_2)$
3	$P_3 V_2 \ln(V_1/V_2)$	$P_3 V_2 \ln(V_1/V_2)$	$nR \ln(V_1/V_2)$
4	0	$\frac{5}{2} V_1 (P_4 - P_1)$	$\frac{5}{2} nR \ln(P_1/P_4)$

d) Friction between the piston and the walls of the cylinder

Non-quasistatic execution of processes

Midterm 1 problem 5 Solution

March 1 2017

A spherical black body of radius R_B, at absolute temperature T_B, is surrounded by black body radiation at temperature T_0.

1 a)

[5 points] Find the rate of radiant heat loss of the spherical body. Express your answer in terms of the given parameters and any physics constants that are relevant.

In this problem, since the shield is black and the other bodies are called "black bodies," it is acceptable to state $\epsilon = 1$ at the beginning of the problem and omit it from the following equations.

Note that radiant heat loss refers to heat radiating out of the black body. The rate of heat flowing out, $\frac{dQ_B}{dt}$, equals the heat radiating out of the black body to its environment minus the heat absorbed by the blackbody.

$$\begin{aligned}\frac{dQ_B}{dt}\text{ radiated } &= \epsilon\sigma_b 4\pi R_B^2 T_B^4\\ \frac{dQ_B}{dt}\text{ absorbed } &= \epsilon\sigma_b 4\pi R_B^2 T_0^4\\ \frac{dQ_B}{dt} &= \epsilon\sigma_b 4\pi R_B^2 (T_B^4 - T_0^4)\end{aligned} \tag{1}$$

A note about minus sign convention: Imagine for example that you are the black body with T_B near 310 Kelvin sitting in a room with T_0 near 293 Kelvin. You're radiating more heat than you're absorbing, so $\frac{dQ_B}{dt}$ must be positive. You lose heat at a rate of about 100W-[heat you absorb]. It's incorrect to say that you lose heat at a rate of [heat you absorb]-100W because then you'd be loosing negative heat, which equals gaining heat and would violate the second law of thermodynamics. With heat questions, it's necessary to pay close attention to whether you're asked about the "change in heat" vs. "heat loss" vs. "heat gained" and answer accordingly.

2 b)

(b) [15 points] The spherical body is now surrounded by a thin spherical and concentric shell of radius RS, black on both sides. This shell is called a radiation shield. The spherical body is assumed to maintain its temperature T_B and the shell reaches a steady temperature T_S. The shell is surrounded by the black body radiation at temperature T_0. Express your answers to the questions below in terms of the given parameters and any physics constants that are relevant.

2.1 i

i) **[3 points] Find the new rate of radiant heat loss of the spherical body.** Now the blackbody radiates to an absorbing shell (or black body) of temperature T_S instead of T_0. Just substitute T_S in where you had T_0 before:

$$\frac{dQ'_B}{dt} = \epsilon\sigma_b 4\pi R_B^2 (T_B^4 - T_S^4) \tag{2}$$

2.2 ii

[3points] Find the rate [of] radiant heat loss of the spherical shell.

The "rate of radiant heat loss" refers to heat lost from the sphere to its surrounding region at T_0, or in the analogy of you being the black body, the rate of heat loss from your sweater. Thus the rate of heat radiating from the shell into to the surrounding blackbody radiation at T_0 is:

$$\frac{dQ_S}{dt} = \epsilon\sigma_b 4\pi R_S^2(T_S^4 - T_0^4) \tag{3}$$

2.3 iii

[4 points] Find the temperature of the shell, T_S, in terms of R_B, R_S, T_B, and T_0

Given that the temperature of the shell reaches a steady value, the heat radiating from the blackbody must equal the heat radiating away from the shell:

$$\begin{aligned} \frac{dQ_S}{dt} &= \frac{dQ'_B}{dt} \\ \epsilon\sigma_b 4\pi R_S^2(T_S^4 - T_0^4) &= \epsilon\sigma_b 4\pi R_B^2(T_B^4 - T_S^4) \\ \text{Solve for } T_S&: \\ T_S &= \left(\frac{R_B^2 T_B^4 + R_S^2 T_0^4}{R_S^2 + R_B^2}\right)^{\frac{1}{4}} \end{aligned} \tag{4}$$

2.4 iv

[5 points] Using your answers to the above, find the factor by which this radiation shield reduces the rate of cooling of the body (consider space between spheres evacuated, with no thermal conduction losses). Your answer should be independent of the temperatures $\mathbf{T}_B$, and T_0.

To find the factor, find k such that new rate=k*old rate, or k=(new rate)/(old rate). In the terms of the problem:

$$\begin{aligned} k &= \frac{\left(\frac{dQ'_B}{dt}\right)}{\left(\frac{dQ_B}{dt}\right)} \\ &= \frac{T_B^4 - T_S^4}{T_B^4 - T_0^4} \\ \text{Substitute in value from part iii) for } T_S^4&: \\ &= \frac{T_B^4 - \left(\frac{R_B^2 T_B^4 + R_S^2 T_0^4}{R_S^2 + R_B^2}\right)}{T_B^4 - T_0^4} \\ \text{which simplifies to the solution}&: \\ &= \frac{R_S^2}{R_S^2 + R_B^2} \end{aligned} \tag{5}$$