1a)

Writing differential form of potential $dV = \frac{dq}{4\pi\epsilon_0 r}$. (2 point)

Drawing a diagram, labelling distance to each point $r = \sqrt{z^2 + R^2}$, constant. (2 point)

Since we are given total charge Q and not linear charge density, we can do the integral simply

$$V = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq = \frac{Q}{4\pi\epsilon_0}\frac{1}{\sqrt{z^2+R^2}}. \quad \text{(4 point)}$$

1b)

Electric field given by $\vec{E} = E_0 \frac{\exp(-\kappa r)}{r^2}\hat{r}$. Note that dimensions of $[E_0] = [q/\epsilon_0]$. (1 point)

To find charge enclosed in radius $1/\kappa$, use Gauss's law and symmetry. (3 point)

Choosing gaussian sphere with radius $1/\kappa$ (2 point), we can immediately solve for Q_{enc}

$$(E_0 e^{-1}\kappa^2)\frac{4\pi}{\kappa^2} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow Q_{enc} = E_0(4\pi\epsilon_0)e^{-1}. \quad \text{(2 point)}$$

1c)

Multiple ways to do this. All will get +2 points.

Use differential form of Gauss's law $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$.

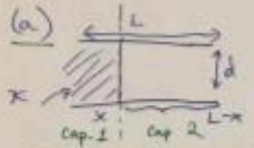
Plugging in for the given $\vec{E} = E(r)\hat{r}$, we find that (in spherical coordinates),

$$\frac{\rho}{\epsilon_0} = \vec{\nabla}\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E(r)) = \frac{2E(r)}{r} + E'(r) = -E_0\kappa\frac{\exp(-\kappa r)}{r^2}$$

PHYS 7B, Section 3 (Wurtele) Robert McGehee

Midterm #2 4/3/17 #2

(a)



This capacitor looks like 2 capacitors in parallel: capacitor 1 has the dielectric in between its plates and capacitor 2 does not. $C = \frac{\epsilon_0 A}{d}$ for parallel-plate capacitors and $C = \kappa C_0$ for capacitors with dielectrics, so:

$$C_1 = \kappa \frac{\epsilon_0 x L}{d}, \quad C_2 = \frac{\epsilon_0 (L-x)L}{d}$$

Since capacitors add in parallel, the total capacitance is

$$C = C_1 + C_2 = \boxed{\frac{\epsilon_0 L}{d}[\kappa x + L - x]}$$

(b) $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2$

$$\Rightarrow \boxed{U = \frac{1}{2}\frac{\epsilon_0 L}{d}[\kappa x + L - x]V^2}$$

(c) After the voltage is disconnected, the circuit is broken so that charges cannot flow anymore. So **charge does not change** if the dielectric is removed.

If the dielectric is removed, the capacitance is just $\frac{\epsilon_0 L^2}{d}$ (it looks like a single capacitor). Thus, the final voltage is

$V_f = \frac{Qd}{\epsilon_0 L^2}$ Also, $Q = \frac{\epsilon_0 L}{d}[\kappa x + L - x]V$

$$\Rightarrow \boxed{V_f = V\left[\frac{\kappa x + L - x}{L}\right]} = V\left[1 + \frac{x}{L}(\kappa - 1)\right]$$

$\Delta V = V_f - V = V\frac{x}{L}(\kappa - 1) > 0$ (since $\kappa > 1$), so voltage increases.

$$U_f = \frac{1}{2}C_f V_f^2 = \boxed{\frac{1}{2}\frac{\epsilon_0 L^2}{d}V^2\left[\frac{\kappa x + L - x}{L}\right]^2} = \frac{1}{2}\frac{\epsilon_0 L}{d}[\kappa x + L - x]V^2\left[1 + \frac{x}{L}(\kappa - 1)\right]$$

$\Delta U = U_f - U_i = \frac{1}{2}\frac{\epsilon_0 x}{d}V^2(\kappa - 1) > 0$, so energy increases.

(d) Since the voltage source is connected, the **voltage does not change** if the dielectric is removed. Again, $C_f = \frac{\epsilon_0 L^2}{d}$ if the dielectric is removed so:

$$Q_f = C_f V = \boxed{\frac{\epsilon_0 L^2}{d}V}, \quad Q_i = \frac{\epsilon_0 L}{d}[\kappa x + L - x]V$$

$$= Q_i \cdot \frac{L}{L + \kappa x - x}$$

$\Delta Q = Q_f - Q_i = \frac{\epsilon_0 L}{d}x[1 - \kappa]V < 0$ (since $\kappa > 1$), so charge decreases

$$U_f = \frac{1}{2}C_f V^2 = \boxed{\frac{1}{2}\frac{\epsilon_0 L^2}{d}V^2} = U_i\left[\frac{L}{\kappa x + L - x}\right]$$

$$U_i = \frac{1}{2}\frac{\epsilon_0 L}{d}[\kappa x + L - x]V^2$$

$\Delta U = U_f - U_i = \frac{1}{2}\frac{\epsilon_0 L}{d}x[1 - \kappa]V^2 < 0$ (since $\kappa > 1$), so energy decreases.

Problem 3a. Find the electric field at a point x along the line joining the wires.

Solution The electric field outside a cylinder of uniform charge Q and length l is:

$$\boldsymbol{E} = \frac{Q}{2\pi\epsilon_0 l r}\hat{r} \tag{1}$$

And thus at a point x between the wires, the net electric field is:

$$\boldsymbol{E}_{net} = \frac{-Q}{2\pi\epsilon_0 l}\left(\frac{1}{x} + \frac{1}{d-x}\right)\hat{x} \tag{2}$$

Note both fields point in the negative $\hat{x}$ (towards the negatively charged wire) at x.

Problem 3b. Find the potential at x.

Solution In general,

$$V(a) - V(b) = -\int_b^a \boldsymbol{E}\cdot\boldsymbol{ds}$$

We can't choose $V(\infty)$ to be finite since it diverges at infinity. Technically one would need to use a different electric field equation than (3) to show this since (3) is only valid for $R < x < d - R$. Regardless, the real electric field line integral diverges at infinity, so our problem remains. Thus, let us choose $b = R$ and $a = x$:

$$V(x) = V(R) - \int_R^x \frac{-Q}{2\pi\epsilon_0 l}\left(\frac{1}{x'} + \frac{1}{d-x'}\right)dx' \tag{3}$$

$$= V(R) + \frac{Q}{2\pi\epsilon_0 l}\ln\left(\frac{x(d-R)}{R(d-x)}\right) \tag{4}$$

Problem 3c. Find the capacitance per unit length of the two wires.

Solution The capacitance is

$$C = \frac{Q}{V} \tag{5}$$

Where V is the voltage difference between the two wires, that is $V(d-R) - V(R)$. Using our result from part b, we have:

$$V \equiv V(d-R) - V(R) \tag{6}$$

$$= \frac{Q}{2\pi\epsilon_0 l}\ln\left(\frac{(d-R)^2}{R^2}\right) \tag{7}$$

$$= \frac{Q}{\pi\epsilon_0 l}\ln\left(\frac{d-R}{R}\right) \tag{8}$$

And consequently, the capacitance is:

$$C = \frac{Q}{V} \tag{9}$$

$$= \frac{Q}{\frac{Q}{\pi\epsilon_0 l}\ln\left(\frac{d-R}{R}\right)} \tag{10}$$

$$= \frac{\pi\epsilon_0 l}{\ln\left(\frac{d-R}{R}\right)} \tag{11}$$

Just divide both sides by l to obtain the capacitance per unit length.

Midterm 2

Wednesday, April 5, 2017 9:11 PM

Problem 4

a) $r = r_0(1+z/L)$ think of each cylindrical slice as a resistor in series → their resistances add linearly

$$R = \int dR = \int_0^L \rho \frac{dz}{A} = \int_0^L \rho \frac{dz}{\pi r_0^2 (1+\frac{z}{L})^2} = \frac{\rho L}{\pi r_0^2}\left(\frac{1}{L} - \frac{1}{2L}\right) = \frac{\rho L}{2\pi r_0^2}$$

b) $P = I^2 R = I^2 \frac{\rho L}{2\pi r_0^2}$

c) Looking at a single cylindrical slice of length dz

its power dissipated is $I^2 \frac{\rho dz}{\pi r_0^2 (1+\frac{z}{L})^2}$ So $\frac{dP}{dz} = \frac{I^2 \rho}{\pi r_0^2 (1+\frac{z}{L})^2}$

d) Consider a volume of area dA and length dz through which a current density J passes through

The power dissipated in this volume is $(J dA)^2 \frac{\rho dz}{dA} = J^2 \rho dV$

So $\frac{dP}{dV} = J^2 \rho$

rubric

a) mention the slices are in series → 4

set up R integral correctly → 4

evaluate the integral correctly → 4

b, c, d answer correct full points (won't double penalize mistakes)

5) a) forces cancel pairwise (eg 1 o'clock balances 7 o'clock)

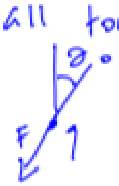
$F = 0$

b) $W = qV = \frac{12 q Q}{4\pi\epsilon_0 R} = \frac{3 q Q}{\epsilon_0 R}$

c) all forces cancel pairwise except "1 o'clock" charge

θ, $\theta = 2\pi$

c) all forces cancel pairwise except "1 o'clock" charge



$\theta = \frac{2\pi}{12}$

$F = \frac{qQ}{4\pi\epsilon_0 R^2}$