EXAM 1 (100 POINTS)

GS WESTON
PHY 7B
7/15/96

(10 POINTS EACH)

1. Calculate the probability that a molecule travels a distance less than one mean free path before it's next collision.

2. A steel tape is placed around the earth at the equator when the temperature is T_i. What will the clearance between the tape and the ground (assumed to be uniform) be if the temperature of the tape rises to T_f? Neglect the expansion of the earth. Express answer in terms of the radius of earth, Re, T_i, T_f, and alpha (steel). No numbers are necessay.

3. Prove that the slope (dP/dV) of the adiabatic curve passing through a point on the PV diagram is gamma times the slope of the isothermal curve passing through the same point.

(14 POINTS)

4. Suppose that each engine in Figure 1 is an ideal reversible heat engine. Engine 1 operates between temperatures T_h and T_m and engine 2 operates beween temperatures T_m and T_c, where $T_h > T_m > T_c$. Hint: You may assume that $e_1 = 1 - T_m/T_h$ and that $e_2 = 1 - T_c/T_m$.
   a) Show that $e_{net} = e_1 + (1 - e_1)e_2$.
   b) Show that $e_{net} = 1 - T_c/T_h$.

(16 POINTS EACH)

5. One mole of a monatomic ideal gas is taken from an initial state of pressure P_o and volume V_o to a final pressure, $P_f = 2P_o$, and final volume, $V_f = 2V_o$, two different ways. (1) It expands isothermally until it's volume is doubled, and then it's pressure is increased at constant volume to it's final state. (2) It is compressed isothermally until it's pressure is doubled, and then it's volume is increased at constant pressure to it's final state. Calculate the change in entropy $(S_f - S_i)$ for both ways in terms of P_o and V_o. Make sure you calculate for both (1) and (2).

6. A cylindrical silver rod of length, 1.17 m, and cross-sectional area, 4.76 cm^2 is insulated to prevent heat loss through it's surface. The ends are maintained at a temperature difference of 100 °C by having one end in a water-ice mixture and the other in boiling water and steam.
   a) Find the rate at which heat is transferred along the rod.
   b) Calculate the rate at which ice melts at the cold end.

(24 POINTS)

7. An ideal diatomic gas follows the cycle shown in Figure 2. The temperature of state 1 is 200 K.
   a) Calculate the temperature of the other three states.
   b) Find the net work done **on the gas**, net change in internal energy and the net heat added (Q_{in}) during one complete cycle.
   c) Calculate the efficiency of the cycle.

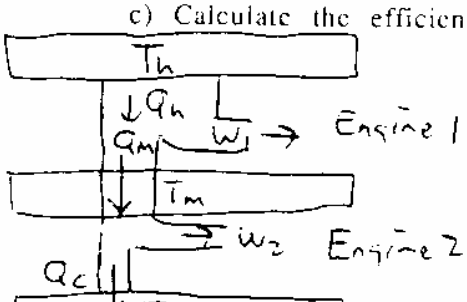

Figure 1

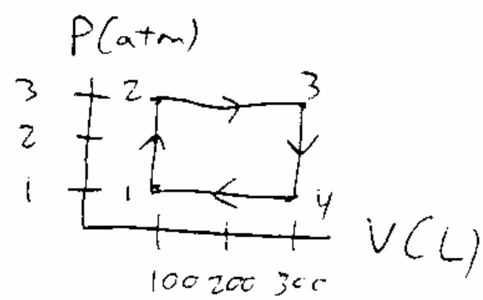

Figure 2

POSSIBLY USEFUL CONSTANTS

$e^1 = 2.72$ $e^{-1} = 0.37$
R = 0.0821 Liter-atm/mole-K (gas constant)
$c_v = 3R/2$ (monoatomic gas)
$c_p/c_v = 1.67$ (gamma for monoatomic gas)

k = 428 W/m-K (thermal conductivity of silver)
Heat of Fusion (Water) = 3.33 x 10^3 J/Kg
$c_v = 5R/2$ (diatomic gas)
$c_p/c_v = 1.4$ (gamma for diatomic gas)

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Exam I Solutions

① Probability for Molecular Collision

$$\text{Prob} = 1 - I/I_0 = 1 - e^{-r/\lambda} = 1 - e^{-1} = 1 - 0.37 = \boxed{0.63}$$

② Expansivity of Steel

at T_i $L = 2\pi R_E$, at T_f $L' = L + \Delta L = 2\pi(R_E + h)$

$$\Delta L = 2\pi h \implies h = \Delta L / 2\pi = \alpha L \Delta T / 2\pi = \alpha R_E \Delta T$$

$$h = \alpha R_E \Delta T = \boxed{\alpha R_E (T_f - T_i) = h}$$

③ Adiabatic & Isothermal Processes

$PV = \text{const}$ for Isothermal Process $\Rightarrow$ $P = C/V$ (C ← constant)

$$\Rightarrow dP/dV = -C/V^2 = -P/V \quad (\text{isothermal})$$

$PV^\gamma = \text{const}'$ for Adiabatic Process $\Rightarrow$ $P = C'/V^\gamma$

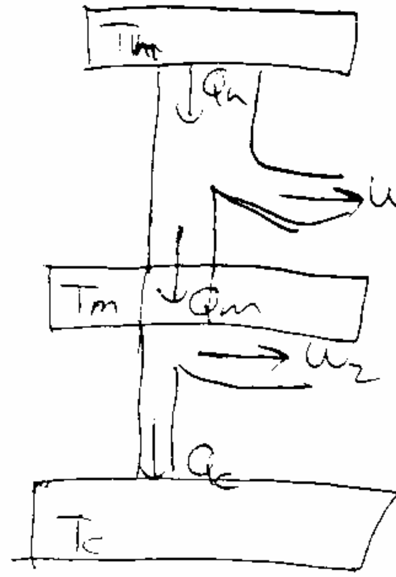
$$\Rightarrow dP/dV = -\gamma C'/V^{\gamma+1} = -\gamma P/V \quad (\text{adiabatic})$$

$$\Rightarrow (dP/dV)_{\text{adiabatic}} = \gamma (dP/dV)_{\text{isothermal}}$$

④ Two Reversible Heat Engines in "Series"

$$e_1 = \frac{W_1}{|Q_H|} = \frac{|Q_h|-|Q_m|}{|Q_h|} = 1 - \frac{|Q_m|}{|Q_h|} = 1 - \frac{T_m}{T_h}$$

$$e_2 = \frac{W_2}{|Q_m|} = \frac{|Q_m|-|Q_c|}{|Q_m|} = 1 - \frac{|Q_c|}{|Q_m|} = 1 - \frac{T_c}{T_m}$$



These were assumed in hint.

$$e_{net} = \frac{W_{tot}}{|Q_h|} = \frac{W_1+W_2}{|Q_h|} = \frac{W_1}{|Q_h|} + \frac{W_2}{|Q_h|}$$

$$\Rightarrow e_{net} = e_1 + \frac{|Q_m|}{|Q_h|} e_2, \quad \frac{|Q_m|}{|Q_h|} = 1 - e_1 \Rightarrow \boxed{e_{net} = e_1 + (1-e_1)e_2}$$

b) $e_{net} = (1 - T_m/T_h) + (T_m/T_h)(1 - T_c/T_m) = 1 - T_c/T_m$ Q.E.D.

⑤ Entropy Change , $P_oV_o = nRT_o$, $P_fV_f = nRT_f \Rightarrow T_f = 4T_o$

$dS = dQ/T$ since $P_f = 2P_o$, $V_f = 2V_o$

(1) $$\Delta S = \Delta S_T + \Delta S_V = \frac{Q_T}{T_c} + \int \frac{dQ_V}{T} = \frac{-W_T}{T_c} + \int \frac{nC_v dT}{T}$$

$$\Delta S = nR \ln(V_f/V_o) + nC_v \ln(T/T_c) = \frac{P_oV_o}{T_o}\ln 2 + C_v \ln 4$$

$$\Delta S = R\ln 2 + (\tfrac{3}{2}R)(2)\ln 2 = \boxed{4R\ln 2 = \Delta S}$$

(2) $$\Delta S = \Delta S_T + \Delta S_P = \frac{Q_T}{T_c} + \int \frac{dQ_P}{T} = \frac{nRT_c}{T_c}\ln\frac{V_f}{V_o} + \int \frac{nC_p dT}{T}$$

$$\Delta S = -\frac{P_oV_c}{T_c}\ln(\tfrac{1}{2}) + \tfrac{5}{2}R\ln(T/T_c) = -R\ln 2 + \tfrac{5}{2}(2)R\ln 2 = \boxed{4R\ln 2 = \Delta S}$$

⑥ Heat Conduction & Heat of Transformation

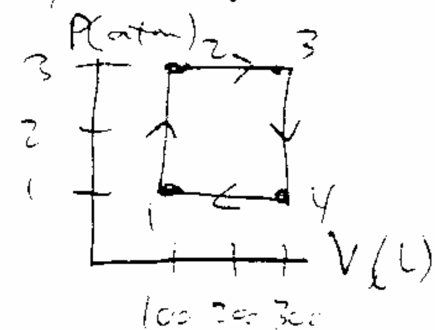
a) $H = Q/\Delta t = -kA\, dT/dx$

$|H| = kA\, dT/dx = (428\ W/m\cdot K)(4.76\times 10^{-4} m^2)(100K)/1.17m$

$\boxed{|H| = 17.4\ W}$

b) $R = m/\Delta t = \frac{(Q/L_f)}{\Delta t} = \frac{|H|}{L_f} = \frac{17.4\ J/s}{333\times 10^3\ J/kg} = \boxed{5.23\times 10^{-5}\ \frac{kg}{s}}$

⑦ Heat Engine - 1st Law of Thermodynamics



a) $nR = P_1V_1/T_1 = 0.5$

$\Rightarrow T_2 = P_2V_2/nR = \boxed{600K = T_2}$

$T_3 = P_3V_3/nR = \boxed{1800K = T_3}$

$T_4 = P_4V_4/nR = \boxed{600K = T_4}$

b) $\boxed{W = -400\ L\text{-}atm}$ (area under curve), $\boxed{\Delta E_{int} = 0}$ ← State function

$Q_{in} = Q_{12} + Q_{23} = nC_v(600K - 200K) + nC_p(1800K - 600K)$

$C_v = 5/2 R,\ C_p = 7/2 R \Rightarrow Q_{in} = \frac{5}{2}nR(400K) + \frac{7}{2}nR(1200K) = \boxed{2600\ L\text{-}atm}$

c) $e = W/Q_{in} = \boxed{0.154 = e}$