1 a When pulled slowly, the top string breaks. The string breaks when the tension exceeds a certain value. Since we are pulling slowly, the mass does not accelerate much, so N2L gives $T_1 - T_2 - mg = 0$ and $T_2 = F$, so $T_1 = T_2 + mg > T_2$. As F increases T_1 will exceed the threshold first.

If pulled quickly there is no time for the tension to travel up the string before it snaps. Or, the inertia of the mass prevents it (briefly) from responding to the pull. Either way, the bottom string breaks first.

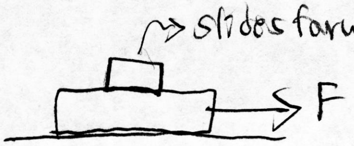
Common errors: In the slow case, you need to explain that $T_1 > T_2$ otherwise they would break at the same time.

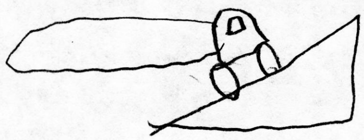
1 b The static coeff of friction is larger than the kinetic coeff of friction because surfaces aren't perfectly smooth. The "hills" in one material lock into the "valleys" of the other material. When they are static, they fall deeper in place and are harder to separate. If they are moving, the hills and valleys collide but don't lock as much.

Common errors: Newton's first law does not explain this effect. Newton's first law tells us if $F=0$ then $\Delta v = 0$. It is "harder" to make an object accelerate than to not

but that difference can be very small. If Newton's first law were the complete explanation, then in a vacuum, $\mu_s > \mu_k$ even though both are 0.
There were also tautological arguments: "$\mu_s > \mu_k$ since more force is needed to start an object moving than keep it moving because $f_s \le \mu_s N$ so max $f_s > f_k = \mu_k N$."

1 c No. There are many examples:



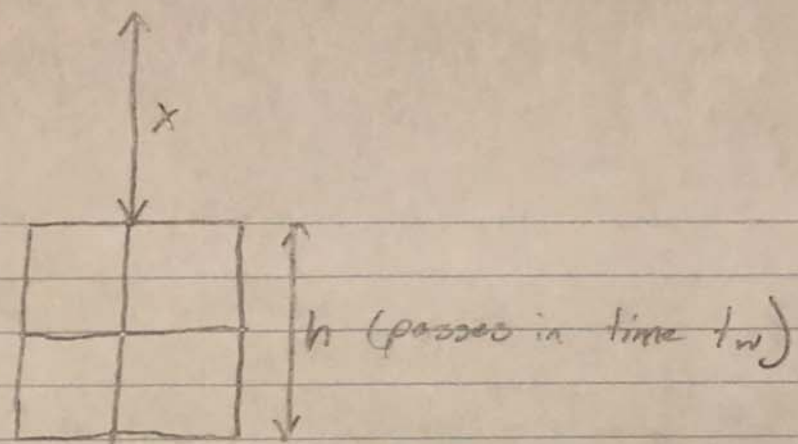


car going in circle has friction in perpendicular direction

Friction opposes relative motion.

Common errors: friction only opposes relative motion of two surfaces not absolute motion.
Some got the correct answer "no" but provided an unclear or incorrect example.

2a)



If $t=0$ is when the projectile crosses the top of the window, then we have

$$h = \frac{1}{2} g t_w^2 + v_0 t_w$$

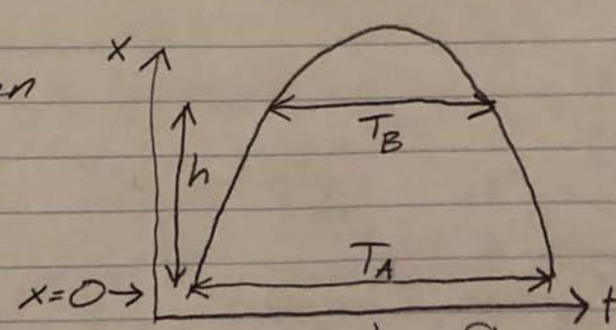
$$v_0 = \frac{h}{t_w} - \frac{1}{2} g t_w$$

Then, because it begins at rest and travels a distance h, at which it has final velocity v_0,

$$2gx = v_0^2$$

$$\boxed{x = \frac{v_0^2}{2g} = \frac{\left(\frac{h}{t_w} - \frac{1}{2} g t_w\right)^2}{2g}}$$

2b) This parabola is given by



$$x(t) = \frac{-1}{2} g t^2 + v_0 t,$$

and if we let $x=0$ be the height that is crossed second after time T_A, then

$$x(0) = 0 \quad \text{and} \quad x(T_A) = 0.$$

So

$$0 = -\frac{1}{2} g T_A^2 + v_0 T_A.$$

As $T_A \neq 0$, we divide by T_A and solve:

$$T_A = \frac{2v_0}{g} \qquad \left(T_A^2 = \frac{4v_0^2}{g^2}\right).$$

Next at two times, which we call τ_+ and τ_-, $x(\tau_\pm) = h$. So

$$x(\tau_\pm) = h = -\frac{1}{2} g \tau_\pm^2 + v_0 \tau_\pm$$

$$0 = \frac{1}{2} g \tau_\pm^2 - v_0 \tau_\pm + h$$

Solving this quadratic yields

$$\tau_\pm = \frac{v_0 \pm \sqrt{v_0^2 - 2gh}}{g}$$

Now, $T_B = \tau_+ - \tau_-$, so

$$T_B = \frac{2\sqrt{v_0^2 - 2gh}}{g}$$

We substitute $v_0^2 = \frac{T_A^2 g^2}{4}$ to get

$$\frac{T_B^2 g^2}{4} = \frac{T_A^2 g^2}{4} - 2gh$$

$$2h = \frac{1}{4} g \left(T_A^2 - T_B^2\right)$$

$$\boxed{g = \frac{8h}{T_A^2 - T_B^2}}$$

(There are many other approaches!)

7A Spring Lecture 2 and 3 Question 3 Solution

Man-Yat CHU (Energy)

February 24, 2018

1 Part a

Shown in figure 1 is the free body diagram of the mass. *Note that centrifugal or centripetal forces are NOT real forces, thus not included in the diagram.*
It is not advised to draw the components of the forces on the diagram.
Also, the length of arrows does not need to be in scale.
One can use another coordinate system, like x y aligned with the two tension, it will give the same result.

2 Part b

To find the tension, we can set up the Newtons second law for each orthogonal direction, namely x and y.

$$\text{In the x direction: } -T_{upper}\cos(45) - T_{lower}\cos(45) = ma_x \tag{1}$$

$$\text{In the y direction: } T_{upper}\sin(45) - T_{lower}\sin(45) - mg = ma_y. \tag{2}$$

If we count the no. of variables, we got T_{upper}, T_{lower}, a_x and a_y, that means we have 2 equation and 4 unknowns at this point, which means we need more constraints.But we know that the mass is in a uniform circular motion. From which we know that $a_y = 0$ and $a_x = \omega^2 R$, where ω is the angular velocity given and R is the radius of curvature for the circular motion. In the case,

$$R = l\cos(45). \tag{3}$$

On plugging in, we have

$$\text{In the x direction: } -T_{upper}\cos(45) - T_{lower}\cos(45) = -m\omega^2 R \tag{4}$$
$$-\frac{T_{upper}}{\sqrt{2}} - \frac{T_{lower}}{\sqrt{2}} = -\frac{m\omega^2 l}{\sqrt{2}} \tag{5}$$

$$\text{In the y direction: } T_{upper}\sin(45) - T_{lower}\sin(45) - mg = 0 \tag{6}$$
$$\frac{T_{upper}}{\sqrt{2}} - \frac{T_{lower}}{\sqrt{2}} - mg = 0. \tag{7}$$

The rest is just solving the system of equation. To solve it, we add the x y equation to eliminate T_{upper}

$$-\frac{2T_{lower}}{\sqrt{2}} - mg = -\frac{m\omega^2 l}{\sqrt{2}} \tag{8}$$
$$T_{lower} = \frac{m\omega^2 l}{2} - \frac{\sqrt{2}mg}{2} \tag{9}$$

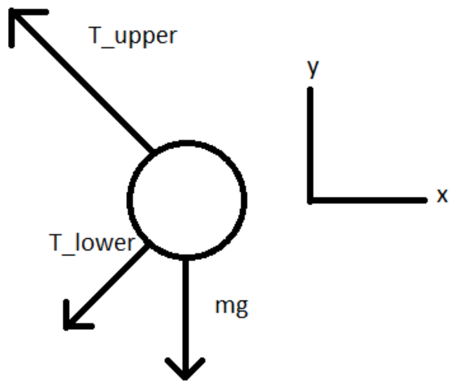


Figure 1: Freebody diagram for question 3

Then we plug in the T_{lower} back to y equation to get

$$\frac{T_{upper}}{\sqrt{2}} - \frac{T_{lower}}{\sqrt{2}} - mg = 0 \tag{10}$$

$$\frac{T_{upper}}{\sqrt{2}} - \frac{m\omega^2 l}{2\sqrt{2}} + \frac{mg}{2} - mg = 0 \tag{11}$$

$$T_{upper} = \frac{\sqrt{2}mg}{2} + \frac{m\omega^2 l}{2}. \tag{12}$$

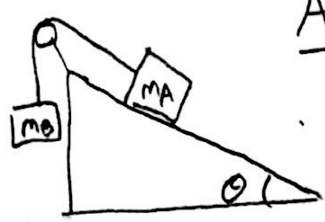
And thus we found the two tensions.
Note that many students used $a_r = v^2/R$ or even $a_r = \omega^2/R$, we are not given v and the second equation is simply wrong. Also R is not l given.
There is no need to put unit in the final answer for symbolic questions, th units are hidden in m and g and etc.

Score A + Score B

25

4)

$g \downarrow$



A: Given: m_A, θ, μ_s

Find: m_B range to keep system at rest (min and max)

(B) (A) m_B Max (A) m_B min

M_B Max Case:

$$\begin{cases} ① \ \Sigma F_{Bx} = 0 \\ ② \ \Sigma F_{By} = T_B - m_B g = m_B a_{By} \end{cases}$$

$$\begin{cases} ③ \ \Sigma F_{Ax'} = F_f - T_A + m_A g \sin(\theta) = m_A a_{Ax} \\ ④ \ \Sigma F_{Ay'} = N_A - m_A g \cos(\theta) = m_A a_{Ay} \end{cases}$$

- Since the cord is massless we know that $T_A = T_B = T$
- 3 equations, 6 unknowns

⇒ We know that $a_{By} = a_{Ax} = a_{Ay} = 0$ since we want to find when system is at rest

⇒ We know that static friction is $F_f = N\mu_s$ since we are at the limit just before system starts to move

② $T - m_B g = 0 \Rightarrow \boxed{T = m_B g}$

③ $N_A \mu_s - T + m_A g \sin(\theta) = 0$

④ $N_A - m_A g \cos(\theta) = 0 \Rightarrow \boxed{N_A = m_A g \cos(\theta)}$

$m_A g \cos(\theta) \mu_s - m_B g + m_A g \sin(\theta) = 0$

$m_A g \cos(\theta) \mu_s + m_A g \sin(\theta) = m_B g$

$$\boxed{m_{B\,max} = m_A(\sin(\theta) + \mu_s \cos(\theta))}$$

is the max mass of B so that A does not start to slide up the plane

knowing $M_{B\,max}$ is when F_f points down

* Likewise for m_B min Case:

② $T = m_B g$

③ $-N_A \mu_s - T + m_A g \sin(\theta) = 0$

④ $N_A = m_A g \cos(\theta)$

$-m_A g \cos(\theta) \mu_s - m_B g + m_A g \sin(\theta) = 0$

Score A

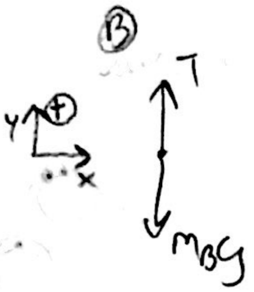
15 pts.

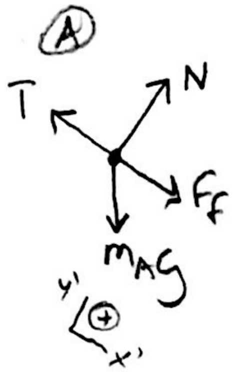
knowing $M_{B\,min}$ is when F_f points up

$$m_{B\,min} = m_A(\sin(\theta) - \mu_s \cos(\theta))$$ is the min mass of B so that A does not slide down the incline plane

B: Given: μ_K, m_A, m_B, θ $m_B >> m_A$ (A) moves up (B) moves down

Find: a_B, a_A





$$\text{①}\ \sum F_{Bx} = 0$$

$$\text{②}\ \sum F_{By} = T - m_B g = m_B a_{By}$$

$$\text{③}\ \sum F_{Ax'} = F_f - T + m_A g \sin(\theta) = m_A a_{Ax}$$

$$\text{④}\ \sum F_{Ay'} = N - m_A g \cos(\theta) = m_A \cancel{a_{Ay}} = 0$$

since mass A is in contact with plane entire time

We have 3 eqs. 5 unknowns

$\Rightarrow$ $F_f = N\mu_K$ since dynamic friction

$\Rightarrow$ since the rope of the system does not change length we know that $|\vec{a}_B| = |\vec{a}_A|$

$$a_{By} = a_{Ax}$$

② $T = m_B(g + a_{By})$

③ $N\mu_K - T + m_A g \sin(\theta) = m_A(+a_{By})$

④ $N = m_A g \cos(\theta)$

Score 10/10

$$m_A g \cos(\theta)\mu_K - m_B(g + a_{By}) + m_A g \sin(\theta) = +m_A a_{By}$$

$$m_A g \cos(\theta)\mu_K - m_B g + m_A g \sin(\theta) = m_B a_{By} + m_A a_{By}$$

$$a_{Ax} = \boxed{a_{By} = \frac{m_A g \cos(\theta)\mu_K - m_B g + m_A g \sin(\theta)}{(m_B + m_A)}}$$

downwards

a_{Ax} ↓ up the incline

a_{By} ↓ down

$m_B g > m_A g(\sin(\theta) + \mu_K \cos(\theta))$

$$\boxed{a_{System} = \frac{|m_A g(\sin(\theta) + \mu_K \cos(\theta)) - m_B g|}{(m_B + m_A)}}$$

final answer