Physics 110A (Spring 04)

Final Examination (Tuesday, May 18)

5 problems with total of 80 points.

Write clearly. **Erase what you do not want to be graded.**

Problem 1 (15 points)

An electric dipole moment $\mathbf{p} = p\hat{\mathbf{z}}$ is placed at the center of the hollow conductor sphere of inner and outer radii a and b. The sphere has total net charge Q. Fig.1.

(a) Compute the total electrostatic energy that is stored in the region outside the sphere ($r > b$). (5 pts)

(b) Suppose that a point charge q is placed at distance d from the outer surface of the hollow sphere. Fig.1b. Compute the electric force that acts on the point charge. (5 pts)

(c) Compute the surface charge density at the top of the inner surface (point A in Fig.1). (5 pts)

†The dipole moment is important in some cases but irrelevant in others. If an electric dipole moment is placed in a completely empty space, it would create the potential $V(\mathbf{r}) = \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\epsilon_0 r^2}$.

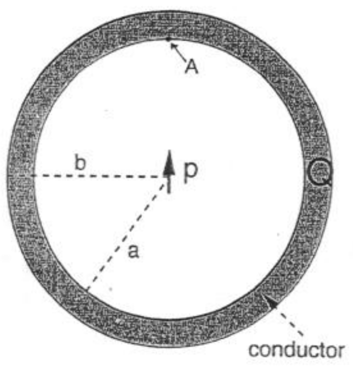

Fig.1

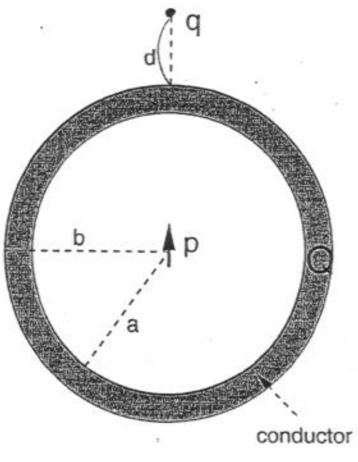

Fig.1b

Problem 2 (15 points)

A long bar of permanent magnet (radius of cross section a) is inserted half way into the densely wound long solenoid (radius of cross section a, length l and n turns per unit length) in which a steady current I flows counterclockwise. (Fig.2)
The magnet has uniform magnetization $\mathbf{M} = M\hat{\mathbf{z}}$ upward along the axis.
Assume that the permanent magnetization $\mathbf{M}$ is so strong (saturated) that it is not affected by the magnetic field generated by the solenoid current.
You may ignore the complications at the ends of the solenoid length.

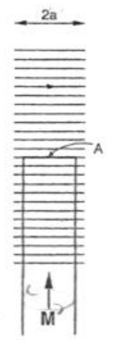

Fig.2

(a) Compute the magnetic $\mathbf{B}$ field at the top center of the magnet (point A in Fig.2) including the $\mathbf{B}$ fields from all sources. (5 pts)
$\dagger \int \frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x}{a^2\sqrt{x^2+a^2}}$.

(b) Does the magnetic force act to pull the magnet up (to $+\hat{\mathbf{z}}$) or push it down (to $-\hat{\mathbf{z}}$) ? First pick your answer from below:

(A) Pull up. (B) Push down.

Then compute the net pulling or pushing force on the magnet. (5 pts)

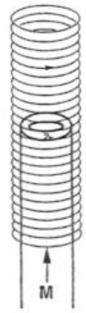

Fig.2c

(c) Suppose that a uniformly charged ring (radius $\frac{1}{2}a$ and positive line charge density λ) is placed on the top of the magnet and then the solenoid current is turned off. (Fig.2c) Assume that there is no friction between the ring and the top surface of the magnet.

To which direction does the ring spin as seen from above ?

(C) Clockwise. (D) Counterclockwise.

Then compute how much spinning angular momentum the ring acquires by the time the solenoid current going out. The current goes away slowly. (5 pts)

Problem 3 (10 points)

A surface current flows uniformly upward on the inner wall of long coaxial cylinders (length l, inner and outer radii of circular cross sections a and b) and returns uniformly on the outer wall. The conductivity of the cylinders is so large that you may ignore their electric resistances. (Fig.3)

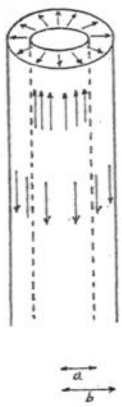

Fig.3

(a) The current $I(t)$ slowly diminishes with time. Compute in $O(I|\frac{dI}{dt}|)$ the electromagnetic energy flows at the inner wall per unit height. (5 pts)
(- sign)

(b) What supplies the energy flow of part(a) and where does it go ? Compute the relevant energies and demonstrate balance of energies (energy conservation). (5 pts)

Problem 4 (20 points)

A left circularly polarized electromagnetic wave of angular frequency ω is normally incident from vacuum upon an infinite planar boundary to a conductive medium. Let the electric field of the incident wave be given in the complex notation as

$$\mathbf{E}_I(\mathbf{r},t) = \mathbf{E}_{I0} e^{ikz-i\omega t}; \qquad \mathbf{E}_{I0} = \frac{\hat{\mathbf{x}} + i\hat{\mathbf{y}}}{\sqrt{2}} E_{I0}.$$

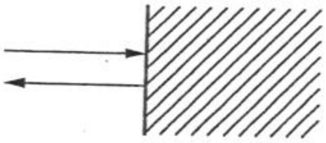

Fig. 4a

(a) Consider first the case of a perfect conductor at $z \geq 0$. (Fig.4a) Find the **E** and **B** fields of the reflected wave. Then answer how the reflected wave is polarized:

(E) Linearly polarized. (F) Elliptically polarized.
(G) Left-circularly polarized. (H) Right-circularly polarized. (5 pts)

(b) Find the surface charge density and the surface current density on the boundary surface in part(a). Then compute the time-averaged force per unit area of the boundary surface as the force that acts on your surface charge and/or current density. (5pts)

(c) If the conductor is not perfect but slightly imperfect, the wave partially penetrates into the imperfect conductor and attenuates as it propagates. This attenuation is described with a complex wave number vector $\mathbf{k}_T = (k_{Tr} + ik_{Ti})\hat{\mathbf{z}}$. Suppose that 99% of the incident energy flow is reflected back at the boundary. (Fig.4c) Obtain the conductivity σ in terms of the wave number k of the incident wave, μ_0 and c. The permiability of the imperfect conductor is μ_0 and make the good-conductor approximation $k_{Tr} \simeq k_{Ti}$ $(\simeq \sqrt{\frac{\sigma\omega\mu_0}{2}})$. (5pts)

†The relations for the reflection/transmission amplitudes hold even for complex k_T.

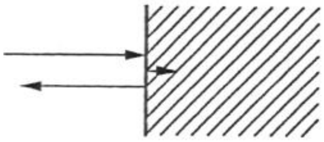

Fig.4c

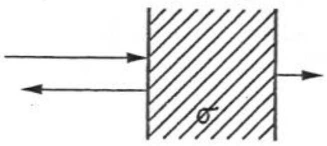

Fig.4d

(d) If the imperfect conductor of part(c) has a finite thickness d, part of the incident wave would pass through the imperfect conductor and come out of the other side. See Fig.4d. Let the **E**-fields of the incident, reflected and trasmitted waves be given in the complex notation as

$$\mathbf{E}_{I0} e^{ikz-i\omega t}, \qquad \mathbf{E}_{R0} e^{-ikz-i\omega t}, \qquad \mathbf{E}_{T0} e^{ikz-i\omega t}.$$

Find the net radiation pressure that the imperfect conductor receives by including the effects of all three waves. Express the pressure in terms of $\mathbf{E}_{I0}$, $\mathbf{E}_{R0}$, $\mathbf{E}_{T0}$ and fundamental physical constants. (5 pts)

Problem 5 (20 points) [Electric dipole radiation]

A point charge q (mass m) hangs at the end of a spring (spring constant k and natural length L).

(a) The spring is stretched by length l from the equilibrium and released so that the charge q oscillates vertically up and down with amplitude l. Fig.5a. Compute the followings: (5 pts)

(i) The ratio of the time-averaged radiation power emitted to the <u>horizontal</u> direction and the time-averaged radiation power emitted <u>45° off</u> the horizontal plane.

(ii) The time-averaged radiation power $\langle P \rangle$ integrated over all directions.

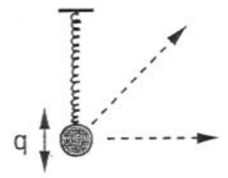

Fig.5a

(b) As the charge q emits radiation, the energy is slowly lost from the motion of the charge and the oscillation gradually damps away. Compute how much time it takes for the oscillation amplitude of part(a) reduces to a half of the original amplitude l. You may approximate, if necessary, that the energy loss is so slow that the motion of the charge is almost periodic. (5 pts)

(c) Consider the case that the point charge q moves on a horizontal circular orbit under the spring force and gravity. The spring points with angle α away from the the vertical direction, as shown in Fig.5c. Find the power of the emitted radiation. (5 pts)

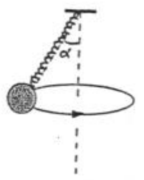

Fig.5c

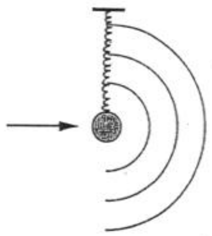

Fig.5d

(d) Consider the situation where a plane electromagnetic wave of angular frequency ω keeps coming horizontally on the poing charge q. The polarization of the incident wave is along the vertical direction. The point charge oscillates with the same frequency ω in response to the incident wave. Find the (total) scattering cross section of the incident wave off the charge q. (5 pts)