Problem 1

Since the acceleration changes at $t = t_1$, we can't just go directly to our regular kinematic equations to solve for the plane's velocity and distance at $t = t_2$. There are several ways that we can approach this. Here a few:

1) Since the acceleration *is* constant for $0 < t < t_1$ and $t_1 < t < t_2$, we can use the usual kinematic equations to write down the change in velocity/position over each interval, then add them (plus any initial velocity/position). Note that in the displacement equation $\Delta x(t) = v_0 t + a*t^2$ the initial velocity v_0 corresponds to the velocity *at the beginning of the interval*, and similarly the time t is actually the amount of time that has passed since then. So if you want to write down an expression for the plane's motion between t_1 and t_2, you have to use $t \rightarrow (t_2 - t_1)$ and $v_0 = v(t_1)$
2) Draw a graph of the acceleration/velocity, then calculate the area under the curve between t = 0 to t = t2. If using this method, *don't forget to add the initial velocity v_0*
3) Start with the definitions for velocity and acceleration, rearrange to get dv = a dt and dx = v dt, then integrate from t = 0 to $t = t_2$. Since the function you are integrating changes during the integral, you will have to break it up into the integral from t = 0 to $t = t_1$ plus the integral from $t = t_1$ to $t = t_2$

Using method 1) for parts (a) and (b):

(a) Let $v_1(t)$ be the velocity of the plane for $0 < t < t_1$, and $v_2(t)$ the velocity of the plane for $t_1 < t < t_2$. If v_0 is the initial velocity, and a_0 is the acceleration during that time interval, then $v_1(t) = v_0 + a_0 t$. And if $2a_0$ is the acceleration during the second time interval, then $v_2(t) = v(t_1) + 2a_0(t - t_1)$. The first term is the "initial velocity" when the plane starts to accelerate at $2a_0$, equal to the final velocity of the first time interval or $v(t_1) = v_0 + a_0 t_1$

At $t = t_2$ the instantaneous velocity is therefore $\boxed{v_2(t_2) = v_0 + a_0 t_1 + 2a_0(t_2 - t_1)}$.

(b) The equations for displacement are $\Delta x_1(t) = v_0 t + a_0/2*t^2$ and $\Delta x_2(t) = v(t_1)*(t - t_1) + a_0*(t - t_1)^2$, again defined over the time intervals $0 < t < t_1$ and $t_1 < t < t_2$, respectively. These correspond to the change in position over each interval, so the total distance the plane has traveled at $t = t_2$ is the sum $\Delta x_1 + \Delta x_2$

At $t = t_2$ the distance traveled is therefore $\boxed{x_2(t) = v_0 t_1 + a_0/2*t_1^2 + (v_0 + a_0 t_1)*(t_2 - t_1) + a_0*(t_2 - t_1)^2}$.

In the next two parts, we are asked to find the velocity/acceleration of the person *relative to the ground*. To do this we must add the velocity/acceleration vectors for the person relative to the plane, and the plane relative to the ground. Since the person walks from the back of the plane towards the front, the motion is in the same direction and we can simply add the magnitudes.

We can calculate the velocity of the person relative to the plane by taking the derivative of the distance function we are given, then take the derivative again to get acceleration. The velocity/acceleration of the plane relative to the ground is just what we used in the first two parts.

(c) In part (a) we found the velocity of the airplane relative to the ground to be $v_{ag}(t) = v_1(t) = v_0 + a_0 t$. Taking the derivative of the distance $D = b*t^3$ gives us the velocity of the person inside the airplane, which is $v_{pa}(t) = 3b*t^2$. Their sum is the velocity of the person relative to the ground, $v_{pg}(t)$.

At $t = t_1$, we get that the instantaneous velocity is $\boxed{v_{pg}(t_1) = v_0 + a_0 t_1 + 3b*t_1^2}$

(d) The instantaneous acceleration of the person relative to the plane is just the derivative of the velocity $v_{pg}(t)$. So the acceleration of the person relative to the ground is $a_{pg}(t) = 6b*t + a_0$

The instantaneous acceleration we get from this is $\boxed{a_{pg}(t_1/2) = 3b*t_1 + a_0}$

Reinsch Problem 2

A) → What do we know

$$\left.\begin{aligned} x_0 &= 0 \\ y_0 &= h \\ V_0 &= (V_{x0}, V_{y0}) \\ a_x &= 0 \\ a_y &= -g \end{aligned}\right\} \text{For ball}$$

So:

$$x(t) = x(0) + V_{x0}\, t + \tfrac{1}{2} a_x t^2$$

$$\Rightarrow \boxed{x(t) = V_{x0}\, t}$$

$$y(t) = y(0) + V_{y0}\, t + \tfrac{1}{2} a_y t^2$$

$$\Rightarrow \boxed{y(t) = h + V_{y0}\, t - \tfrac{1}{2} g t^2}$$

B) → Say the catch is made at time t_f: $y(t_f) = 2h$

$$y(t_f) = h + V_{y0} t_f - \tfrac{1}{2} g t_f^2 = 2h$$

$$\tfrac{1}{2} g t_f^2 - V_{y0} t_f + h = 0$$

$$t_f = \frac{V_{y0} \pm \sqrt{V_{y0}^2 - 2gh}}{g}$$

choose + solution (we want the later time)

$$\boxed{t_f = \frac{V_{y0} + \sqrt{V_{y0}^2 - 2gh}}{g}}$$

→ For t_f to be real, $V_{y0}^2 \geq 2gh$

$$\boxed{V_{y0} \geq \sqrt{2gh}}$$

C) → What is $x(t_f)$? This is where the ball is caught.

$$x(t_f) = \frac{V_{x0}}{g}\left(V_{y0} + \sqrt{V_{y0}^2 - 2gh}\right)$$

→

→ The ball player starts running (accelerating) at $t=0$. What is the acceleration?

→ If $x_{0,runner} = 0$ and $v_{0,runner} = 0$,

then $x(t)_{runner} = \frac{1}{2} a_{runner} t^2$

→ Set the x position of the ball and the runner equal at t_f

$$v_{x0} \cancel{t_f} = \frac{1}{2} a_R t_f^{\cancel{2}}$$

$$\boxed{a = \frac{2 v_{x0}}{t_f} = \frac{2g\, v_{x0}}{v_{y0} + \sqrt{v_{y0}^2 - 2gh}} = \frac{v_{x0}}{h}\left(v_{y0} - \sqrt{v_{y0}^2 - 2gh}\right)}$$

Problem 3

Box sliding down an incline, has mass m

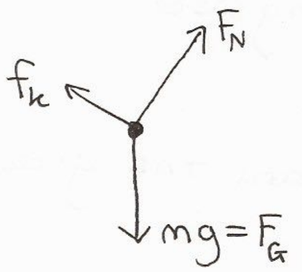
Incline makes angle θ with the horizontal

Coefficient of kinetic friction is μ_k

a) Assume the downhill direction is to the lower right

this means we have:

Draw a free body diagram:



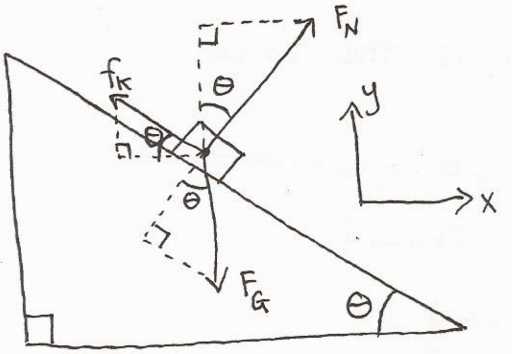
where f_k = force due to kinetic friction

F_N = normal force

$F_G = mg$ = weight of box

b) Using a coordinate system xy with the x-axis horizontal and pointing to the right, and the y-axis pointing vertically upwards. Calculate the x and y components of the forces in the free body diagram in part (a) and of the acceleration vector

First, draw the forces and their components with this coordinate system



Using the diagram on the left, we can figure out the x and y components of the three forces acting on the box, which are F_G, F_N, and f_k

We will start with F_G

In vector form, we have

$\vec{F}_G = -F_G\,\hat{y}$, where $F_G = mg$

$\vec{F}_G = -mg\,\hat{y}$

Thus, the x-component of F_G is zero, and the y-component of F_G is $-mg$

Next, we look at F_N

In vector form, we have

$\vec{F}_N = F_N \sin\theta\,\hat{x} + F_N \cos\theta\,\hat{y}$, where $F_N = mg\cos\theta$

$\vec{F}_N = mg\cos\theta\sin\theta\,\hat{x} + mg\cos^2\theta\,\hat{y}$

Thus, the x-component of F_N is $mg\cos\theta\sin\theta$, and the y-component of F_N is $mg\cos^2\theta$

We now look at f_k

In vector form, we have

$\vec{f}_k = -f_k\cos\theta\,\hat{x} + f_k\sin\theta\,\hat{y}$, where $f_k = \mu_k F_N$

$= \mu_k mg\cos\theta$

$\vec{f}_k = -\mu_k mg\cos^2\theta\,\hat{x} + \mu_k mg\cos\theta\sin\theta\,\hat{y}$

Thus, the x-component of f_k is $-\mu_k mg\cos^2\theta$, and the y-component of f_k is $\mu_k mg\cos\theta\sin\theta$

Lastly, find the box's acceleration, $\vec{a}$

Use Newton's 2nd Law and the vector form of the forces

$$\Sigma\vec{F} = m\vec{a} = \vec{F}_G + \vec{F}_N + \vec{f}_k$$

$$m\vec{a} = -mg\hat{y} + mg\cos\theta\sin\theta\,\hat{x} + mg\cos^2\theta\,\hat{y} - \mu_k mg\cos^2\theta\,\hat{x} + \mu_k mg\cos\theta\sin\theta\,y$$

$$m\vec{a} = mg(\cos\theta\sin\theta - \mu_k\cos^2\theta)\,\hat{x} + mg(\underbrace{\cos^2\theta - 1}_{=-\sin^2\theta} + \mu_k\cos\theta\sin\theta)\,\hat{y}$$

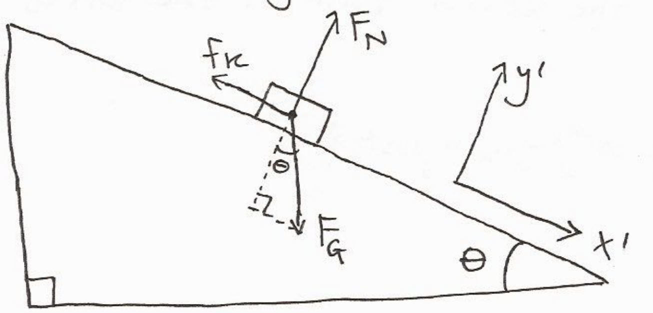
$$m\vec{a} = mg\cos\theta(\sin\theta - \mu_k\cos\theta)\,\hat{x} + mg\sin\theta(\mu_k\cos\theta - \sin\theta)\,\hat{y}$$

$$\Rightarrow \vec{a} = g\cos\theta(\sin\theta - \mu_k\cos\theta)\,\hat{x} + g\sin\theta(\mu_k\cos\theta - \sin\theta)\,\hat{y}$$

Thus, the x-component of $\vec{a}$ is $g\cos\theta(\sin\theta - \mu_k\cos\theta)$, and the y-component of $\vec{a}$ is $g\sin\theta(\mu_k\cos\theta - \sin\theta)$

c) Different coordinate system with axes x' and y'.
The x' axis is parallel to the surface of the slope and pointing down the slope, and the y' axis is perpendicular to the slope and pointing to the upper right.
Calculate the x' and y' components of the forces in the free body diagram in part (a) and of the acceleration vector.

We first draw the forces and their components with this new coordinate system



We will use this diagram to determine the x' and y' components of F_G, F_N, f_K

Starting with F_G

In vector form, we have

$\vec{F}_G = F_G \sin\theta\, \hat{x}' - F_G \cos\theta\, \hat{y}'$, where $F_G = mg$

$\vec{F}_G = mg\sin\theta\, \hat{x}' - mg\cos\theta\, \hat{y}'$

Thus, the x'-component of F_G is $mg\sin\theta$, and the y'-component of F_G is $-mg\cos\theta$

Next, look at F_N

In vector form, we have

$\vec{F}_N = F_N \hat{y}'$, where $F_N = mg\cos\theta$

$\vec{F}_N = mg\cos\theta\, \hat{y}'$

Thus, the x'-component of F_N is zero, and the y'-component of F_N is $mg\cos\theta$

Look at f_k

In vector form, we have:

$\vec{f}_k = -f_k\,\hat{x}'$, where $f_k = \mu_k F_N$

$\vec{f}_k = -\mu_k mg\cos\theta\,\hat{x}'$ $\qquad f_k = \mu_k mg\cos\theta$

Thus, the x'-component of $\vec{f}_k$ is $-\mu_k mg\cos\theta$, and the y'-component of $\vec{f}_k$ is zero.

Finally, find the box's acceleration $\vec{a}$

Use Newton's 2nd Law and the vector form of the forces

$$\sum\vec{F} = m\vec{a} = \vec{F}_G + \vec{F}_N + \vec{f}_k$$

$$m\vec{a} = mg\sin\theta\,\hat{x}' - \cancel{mg\cos\theta\,\hat{y}'} + \cancel{mg\cos\theta\,\hat{y}'} - \mu_k mg\cos\theta\,\hat{x}'$$

$$m\vec{a} = mg(\sin\theta - \mu_k\cos\theta)\,\hat{x}'$$

$$\Rightarrow \vec{a} = g(\sin\theta - \mu_k\cos\theta)\,\hat{x}'$$

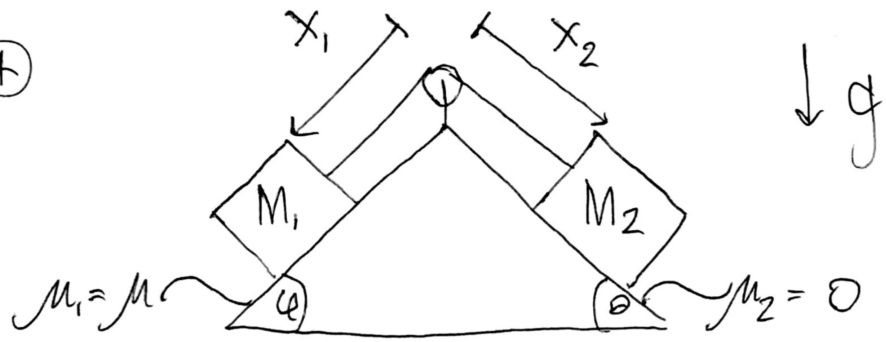
Thus, the x'-component of $\vec{a}$ is $g(\sin\theta - \mu_k\cos\theta)$, and the y'-component of $\vec{a}$ is zero

Note: The magnitudes of the accelerations found in parts (b) and (c) will be equal.

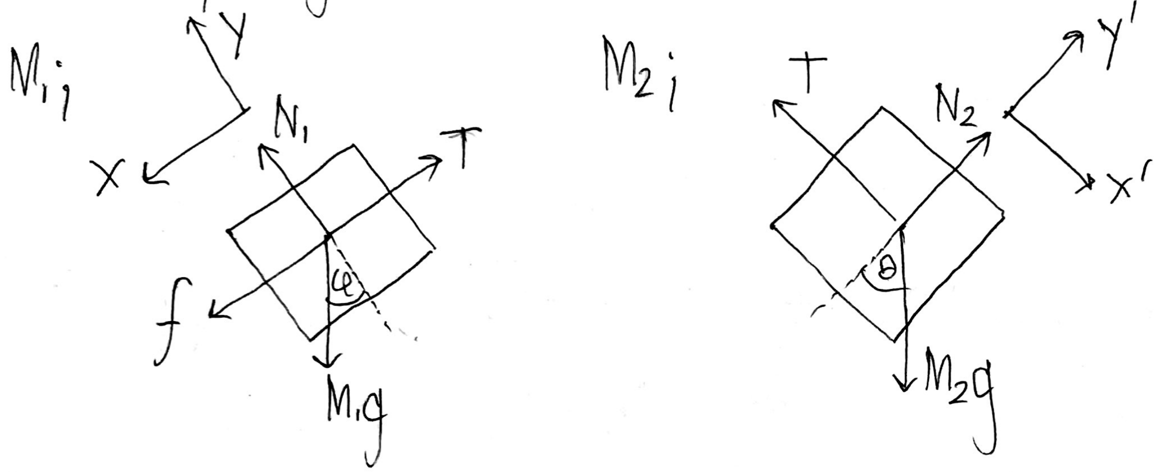
Common Errors

The majority of errors in this problem had to do with part b. For part a, nearly everyone had the correct free body diagram. However, a large amount of people had the wrong normal force for part b. Remember, the normal force will be the same regardless of coordinate system. Many people also did not put their answers in terms of the given variables m, μ_k, θ, instead, for example, leaving the normal force just as $\vec{F}_N = F_N \sin\theta\,\hat{i} + F_N \cos\theta\,\hat{j}$. A fair amount of people did not finish the entire problem. Some did not include the force components, some did not include the acceleration components, and some people left out an entire part of the problem. Other common errors include not having negative signs in their components (we did not ask for magnitudes, so there will be negatives for some components), as well as completely neglecting some forces when calculating force components.

Problem ④



a) Free-body diagrams



Here we assume that M_1 is going up, therefore the friction on M_1 points downhill. On the other hand, the friction on M_2 vanishes because $\mu_2 = 0$. The tension is constant throughout the whole string since we assume it is massless. The coordinate systems for M_1 and M_2 are depicted in the diagrams.

b) Newton's 2nd law

M_2; x; $M_2 g \sin\theta - T = M_2 \ddot{x}_2$ (Define $\ddot{x}_2 \equiv \frac{d^2 x_2}{dt^2}$) ——— ①

y; $N_2 - M_2 g \cos\theta = 0$ ——— ②

M_1; x; $f - T + M_1 g\sin\alpha = M_1\ddot{x}_1$

$$\mu_1 N_1 - T + M_1 g\sin\alpha = M_1\ddot{x}_1 \quad (f = \mu_1 N_1) \qquad (3)$$

y; $$N_1 - M_1 g\cos\alpha = 0 \qquad (4)$$

Notice that there are 5 variables: $\ddot{x}_1, \ddot{x}_2, T, N_1$, and N_2, but we only have 4 independent equations. The last equation can be derived from the constraint.

$$\text{String length} = \text{constant} = x_1 + x_2$$

Take the second time derivative of both sides, we get

$$0 = \ddot{x}_1 + \ddot{x}_2 \Rightarrow \ddot{x}_1 = -\ddot{x}_2 \qquad (5)$$

from (2) & (4) $N_1 = M_1 g\cos\alpha$, $N_2 = M_2 g\cos\theta$ (6)

Substitute (5) & (6) into (1) & (3)

$$\mu_1 M_1 g\cos\alpha - T + M_1 g\sin\alpha = M_1\ddot{x}_1 \qquad (7)$$

$$-T + M_2 g\sin\theta = -M_2\ddot{x}_1 \qquad (8)$$

(6) $\times M_2$ + (7) $\times M_1$ (Eliminate $\ddot{x}_1$)

$$-T(M_1+M_2) + M_1M_2 g(\sin\alpha + \sin\theta) + M_1M_2 g\cos\alpha\,\mu_1 = 0$$

$$\therefore T = \frac{M_1M_2 g}{(M_1+M_2)}\left\{\sin\alpha + \sin\theta + \mu\cos\alpha\right\}$$

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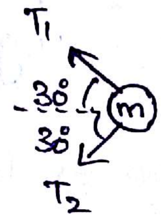
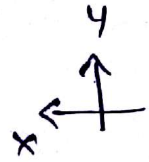
Midterm 1 - Section 1

Solution for ⑤

by Sai Neha Santpur

a) Free body diagram :





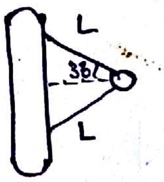
y- component: $(\vec{F}_{net})_y = 0 \Rightarrow T_1 \sin 30° - T_2 \sin 30° = 0$

$\Rightarrow \frac{T_1}{2} - \frac{T_2}{2} = 0 \Rightarrow T_1 = T_2$ — ①

X- component: $(\vec{F}_{net})_x = \frac{mv^2}{r} \Rightarrow T_1 \cos 30° + T_2 \cos 30° = \frac{mv^2}{r}$

$\Rightarrow 2T_1 \cos 30° = \frac{mv^2}{r} \Rightarrow 2T_1 \cdot \frac{\sqrt{3}}{2} = \frac{mv^2}{r}$ $(\because T_1 = T_2)$

$\Rightarrow T_1\sqrt{3} = \frac{mv^2}{r} \Rightarrow T_1 = \frac{mv^2}{\sqrt{3}r}$



$r = \cos 30° L = \frac{\sqrt{3}L}{2}$

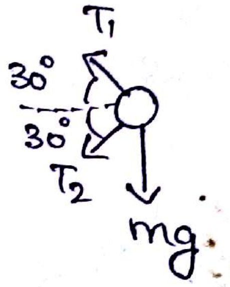
$\therefore T_1 = T_2 = \frac{mv^2}{\sqrt{3}\left(\frac{\sqrt{3}L}{2}\right)} = \frac{2mv^2}{3L}$

$\Rightarrow \boxed{T_1 = T_2 = \frac{2mv^2}{3L}}$

b) Minimum value for v such that the lower string doesn't slack $\Rightarrow T_2 > 0$

(Or consider the limiting case $T_2 = 0$ for the rest of the problem)

Free body diagram:



Y component : $(\vec{F}_{net})_y = 0 \Rightarrow T_1 \sin 30° - T_2 \sin 30° - mg = 0$

$\Rightarrow \frac{T_1}{2} - \frac{T_2}{2} - mg = 0 \Rightarrow T_1 - T_2 = 2mg$ - ②

X component : $(\vec{F}_{net})_x = \frac{mv^2}{r} = \frac{mv^2}{\left(\frac{\sqrt{3}L}{2}\right)} = \frac{2mv^2}{\sqrt{3}L}$

$\Rightarrow T_1 \cos 30° + T_2 \cos 30° = \frac{2mv^2}{\sqrt{3}L}$

$\Rightarrow (T_1 + T_2)\frac{\sqrt{3}}{2} = \frac{2mv^2}{\sqrt{3}L} \Rightarrow T_1 + T_2 = \frac{4mv^2}{3L}$ - ③

From equations ② and ③,

$2T_2 = \frac{4mv^2}{3L} - 2mg \Rightarrow T_2 = \frac{2mv^2}{3L} - mg$

Now using $T_2 > 0$, we have $\frac{2mv^2}{3L} > mg$

$\Rightarrow v^2 > \frac{3gL}{2} \Rightarrow v > \sqrt{\frac{3gL}{2}}$

The minimum value of v for which the string doesn't slack is $\boxed{V_{min} = \sqrt{\frac{3gL}{2}}}$

c) Free body diagram:

x Component: $(\vec{F}_{net})_x = 0 \Rightarrow T_1 \sin 30° - T_2 \sin 30° = 0$

$\Rightarrow \quad T_1 = T_2 \quad$ — (4)

y Component: $(\vec{F}_{net})_y = -\frac{mv^2}{r} = -\frac{mv^2}{\left(\frac{\sqrt{3}L}{2}\right)} = -\frac{2mv^2}{\sqrt{3}L}$

$\Rightarrow \quad -T_1 \cos 30° - mg - T_2 \cos 30° = -\frac{2mv^2}{\sqrt{3}L}$

$\Rightarrow \quad 2T_1 \cos 30° + mg = \frac{2mv^2}{\sqrt{3}L} \quad (\because T_1 = T_2)$

$\Rightarrow \quad 2T_1 \cdot \frac{\sqrt{3}}{2} + mg = \frac{2mv^2}{\sqrt{3}L} \Rightarrow T_1\sqrt{3} + mg = \frac{2mv^2}{\sqrt{3}L}$

$\Rightarrow \quad T_1\sqrt{3} = \frac{2mv^2}{\sqrt{3}L} - mg \Rightarrow T_1 = \frac{2mv^2}{3L} - \frac{mg}{\sqrt{3}}$ — (5)

Both the strings do not slack $\Rightarrow T_1 = T_2 > 0$.
(Or you can use the limiting case of $T_1 = T_2$ to find the minimum velocity)

$T_1 = T_2 > 0 \Rightarrow \frac{2mv^2}{3L} - \frac{mg}{\sqrt{3}} > 0 \Rightarrow \frac{2mv^2}{\sqrt{3}L} > mg \Rightarrow v^2 > \frac{\sqrt{3}gL}{2}$

$\Rightarrow \quad v > \sqrt{\frac{\sqrt{3}gL}{2}}$

The minimum value of v = $\boxed{v_{min} = \sqrt{\frac{\sqrt{3}gL}{2}}}$

d) $V = 2V_{min}$ (V_{min} obtained in part c)

Using equations (4) and (5),

$$T_1 = T_2 = \frac{2mv^2}{3L} - \frac{mg}{\sqrt{3}} = \frac{2m}{3L}\left(4V_{min}^2\right) - \frac{mg}{\sqrt{3}}$$

$$= \frac{8m}{3L}\frac{\sqrt{3}gL}{2} - \frac{mg}{\sqrt{3}} = \frac{4mg}{\sqrt{3}} - \frac{mg}{\sqrt{3}}$$

$$= \frac{3mg}{\sqrt{3}} = \sqrt{3}mg$$

$$\therefore \boxed{T_1 = T_2 = \sqrt{3}mg}$$

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