Professor R. Ramesh's Midterm 1: Monday, October 4 2005

1 Problem 1

A microwave source emits pulses of 20 GHz radiation, with each pulse lasting 1.0ns. A parabolic reflector ($R = 6.0cm$) is used to focus these into a parallel beam of radiation (see figure). The average power of each pulse is 25kW.

a) What is the wavelength of these microwaves?

b) What is the total energy contained in each pulse?

c) Compute the average energy density inside each pulse.

d) Determine the amplitude of the electric and magnetic fields in these microwaves.

e) If this pulsed beam strikes an absorbing surface, compute the force exerted on the surface during the 1.0ns duration of each pulse.

Three points each, all or nothing, with the following exceptions:

a) $\lambda = \frac{c}{f} = 1.50cm$

b) $U = P \cdot \Delta t = (2.5 \times 10^3 W)(10^{-9} s) = 25.0 \times 10^{-6} J$

c) $u_{av} = \frac{U}{Vol} = \frac{25 \times 10^{-6} J}{\pi (0.06)^2 (3 \times 10^8 m/s)(10^{-9} s)} = 7.37 \times 10^{-3} J/m^3$

d) $E_m = \sqrt{2U_{av}/\epsilon_0} = 4.08 \times 10^4 V/m$ *2/3 for missing $\sqrt{2}$*

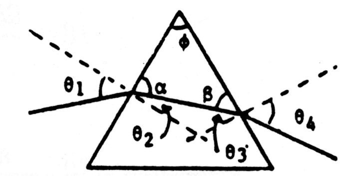
$B_m = E_m/c = 1.36 \times 10^{-4} T$

e) $F = pA = (\frac{cu_av}{c})A = 8.33 \times 10^{-5} N$

Points may be deducted for missing units.

2 Problem 2

A light ray is incident on a prism and refracted at the surface A (see figure). Let Φ represent the apex angle of the prism, and n its index of refraction. Find in terms of n and Φ the smallest angle of incidence that will allow light to be transmitted through side B.



We see that $\theta_2 + \alpha = \pi/2$, and $\theta_3 + \beta = \pi/2$, so $\theta_2 + \theta_3 + \alpha + \beta = \pi$. Also from the figure we see $\alpha + \beta + \phi = \pi$, therefore $\phi = \theta_2 + \theta_3$. *5 points*

Applying Snell's law at the first and second refracting surfaces, we find:

$$\theta_2 = sin^{-1}(\frac{sin\theta_1}{n})$$

2 points

$$\theta_3 = sin^{-1}(\frac{sin\theta_4}{n})$$

2 points

The limiting case for light to be transmitted is that $\theta_4 \to \pi/2$. In this case, we have:

$$\theta_3 = sin^{-1}(\frac{1}{n})$$

Recalling that $sin(a+b) = sinacosb + sinbcosa$,

$$\begin{aligned} sin(\theta_2) &= sin(\phi - \theta_3) = sin(\phi)cos(\theta_3) - sin(\theta_3)cos(\phi). \\ &= sin(\phi)\sqrt{1 - sin^2(\theta_3)} - sin(\theta_3)cos(\phi) \\ &= sin(\phi)\sqrt{1 - \frac{1}{n^2}} - \frac{1}{n}cos(\phi) \end{aligned}$$

Then

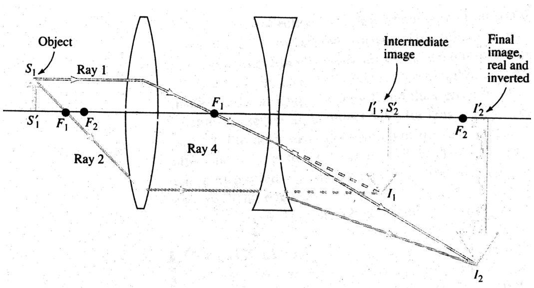
$$\begin{aligned} sin(\theta_1) &= nsin(\theta_2) \\ &= n\left[sin(\phi)\sqrt{1 - \frac{1}{n^2}} - \frac{1}{n}cos(\phi)\right] \end{aligned}$$

6 points

$$\boxed{\theta_1 = sin^{-1}[\sqrt{n^2 - 1}\sin\phi - \cos\phi]}$$

3 Problem 3

Lens 1 in the figure is converging with a focal length 22cm. An object is placed 32cm to its left. Lens 2, which is diverging with a focal length 57cm, lies 41 cm to the right of lens 1. Describe the position, orientation and magnification of the final image. Draw a ray diagram and compare to your calculated result.



Generally, points are given for the second image if it is consistent with the first. The diagram should show relevant features such as optic axis and focal points, and the standard rays used. *2 points for each image.*

Using the lens equation, with $f_1 = +22cm$ (converging), $o_1 = +32cm$ (real):

$$\begin{aligned} \frac{1}{o_1} + \frac{1}{i_1} &= \frac{1}{f_1} \\ \frac{1}{i_1} &= \frac{1}{+22cm} - \frac{1}{+32cm} \\ &= \frac{1}{+70.4cm} \end{aligned}$$

So $i_1 = 70.4cm$ to the right of lens 1. Since the lenses are only separated by 41cm, the image of the first lens makes a *virtual* object for the second lens at $o_2 = 41cm - 70.4cm = -29.4cm$. *2 points* *2 points*

$$\begin{aligned} \frac{1}{o_2} + \frac{1}{i_2} &= \frac{1}{f_2} \\ \frac{1}{i_2} &= \frac{1}{-57cm} - \frac{1}{-29.4cm} \\ &= \frac{1}{+60.7cm} \end{aligned}$$

3 points

So the final image is real, 60.7cm to the right of the second lens. The total magnification is found by applying the magnification formula twice:

$$\begin{aligned} M &= m_1 \cdot m_2 \\ &= \left(-\frac{i_1}{o_1}\right)\left(-\frac{i_2}{o_2}\right) \\ &= \left(-\frac{70.4}{32cm}\right)\left(-\frac{60.7cm}{-29.4cm}\right) \\ &= -4.5 \end{aligned}$$

2 points

And the image is real and inverted, as shown in the diagram.

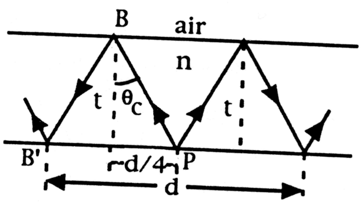
4 Problem 4

The phenomena of total internal reflection can be used to measure the index of refraction of a material via Pfund's method, as follows. A slab of thickness t is painted on one side to serve as a screen. A small hole scraped in the paint serves as a source of light. Rays striking the opposite surface will emerge if the angle is less than critical. Thus on the painted screen there will be a dark circle of diameter d, and outside of this a bright halo.

a) Derive a formula for n in terms of d and t.

b) What is the diameter of the dark circle if $n = 1.52$ and $t = 0.600cm$?

c) If white lights is used, the critical angle depends on color caused by dispersion. Is the inner edge of the halo tinged red or violet? Explain.



a) The dark circle is formed because light hitting the clear surface will escape if the incident angle is less than θ_c. The condition for total internal reflection is: *4 points*

$$\begin{aligned} n_{air} sin(\pi/2) &= n_{material} sin(\theta_c) \\ \text{or } sin\theta_c &= \frac{1}{n_m} \end{aligned}$$

From the diagram, $tan\theta_c = \frac{d/4}{t}$ or: *4 pts*

$$sin\theta_c = \frac{d/4}{\sqrt{t^2 + (d/4)^2}}$$

Then *2 pts*

$$\boxed{n = \frac{\sqrt{(4t)^2+d^2}}{d}}$$

b) Rearranging gives: *2 pts given if consistent with part a*

$$\begin{aligned} dn &= \sqrt{(4t)^2 + d^2} \\ d^2n^2 &= 16t^2 + d^2 \\ d^2(n^2 - 1) &= 16t^2 \end{aligned}$$

$$d = \frac{4t}{\sqrt{n^2 - 1}} = \frac{4 \cdot 0.600cm}{\sqrt{1.52^2 - 1}} = 2.10cm$$

c) See Giancoli page 825 for description of prisms and dispersion. Light of shorter wavelengths is refracted more strongly than light of longer wavelenth, therefore θ_c is smaller for violet, and closer to $\pi/2$ for red. A smaller θ_c results in a smaller ring and violet will be on the inside. *3 pts*

5 Problem 5

Light falls normally on a soap bubble and is reflected back. If the bubble's walls have a thickness t and index of refraction n, express the condition for constructive interference of the reflected light in terms of the incident wavelength λ, n, t. If $t = 400nm$ and $n = 1.3$, what color or colors will interfere constructively?

a) The interference pattern arises due to a phase difference between the light reflected from the front and back surfaces of the bubble. If the phase difference is $(2m-1)\pi$ for some integer m, then there is destructive interference (no light seen), while if the phase difference is $2m\pi$ there will be a bright spot.

The ray directly reflected picks up a phase $\phi_1 = \pi$ because the index of refraction of soap is higher than that of air.

The ray reflecting off the back surface picks up a phase due only to the extra path length traveled. Fractions of a wavelength traveled will give fractions of 2π as :

$$\frac{\phi_2}{2\pi} = \frac{2t}{\lambda_n} = \frac{2tn}{\lambda}$$

Thus, the condition for constructive interference is:

$$\Delta\phi = 2\pi\frac{2tn}{\lambda} - \pi = 2m\pi$$

$$\boxed{4tn = (2m-1)\lambda, \quad m = 1, 2, 3, \ldots}$$

12 pts

b) To find the colors appearing, we invert the expression above to isolate λ:

$$\lambda = \frac{4nt}{2m-1} = \frac{4(1.3)(400nm)}{2m-1} \approx \frac{2100nm}{2m-1}$$

For the values m = 1 through 4, the wavelengths obtained are:

$$\lambda = 2080nm, 693nm, 416nm, 300nm$$

Only red (693 nm) and blue (416 nm) are part of the visible spectrum. *3 pts*

6 Problem 6

In a 2-slit experiment a piece of glass with an index of refraction n and thickness L is placed in front of the upper slit.

a) Describe qualitatively what happened to the interference pattern.

b) Given that the result for the 2-slit pattern without the glass is given by: $I_\theta = I_0 cos^2(\frac{\pi d sin(\theta)}{\lambda})$, (where d is the distance between the slits and θ is the usual angle as measured from the center), give an expression for the intensity of light at points on a screen as a function of n, L, θ.

c) Write down an expression for values of θ that locate the interference maxima. Compare to results of part b.

a) The addition of the glass causes one of the rays to pick up an extra (constant) phase, causing the whole pattern to shift *3 pts*

without changing the separation between successive maxima . *-1 if wrong.*

Because the glass is in front of the *upper* slit, the pattern will shift upwards. *1pt*

b) The usual phase is

$$\delta_{old} = \frac{2\pi}{\lambda} d \sin\theta$$

The new phase is simply

$$\delta_{new} = \delta_{old} \pm (n-1)L\frac{2\pi}{\lambda}$$

2 pts

The intensity pattern will then be:

$$I(\theta) = I_0 \cos^2\left(\frac{\delta_{new}}{2}\right) \text{ or } I_0 \cos^2\left(\frac{\delta + \Delta\phi_{new}}{2}\right)$$

4 pts

$$\boxed{I(\theta) = I_0 \cos^2\left(\frac{\pi}{\lambda}(d\sin\theta + (n-1)L)\right)}$$

c) Condition for constructive interference is :

$$\delta_{new} = 2\pi m, \quad m = 0, 1, 2, \ldots$$

4 pts

$$\frac{d\sin\theta}{\lambda} \pm (n-1)\frac{L}{\lambda} = 2\pi m$$

2 pts

1 point partial credit if using $\delta_{new} = \frac{2\pi}{\lambda}(dsin\theta \pm nL)$, full credit if using $\frac{L}{cos\theta}$ instead of L

To located the interference maxima, solve for theta: *1 pt*

$$\sin\theta = \frac{\lambda m \mp (n-1)L}{d}$$

2 pts for using correct δ_{new}