

# Midterm Exam

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Answer all questions. Partial credit generously given; show what you know!

1. A BART train can carry 200 passengers. About 5 percent of passengers bring a bike on board. (5 pts for each part; 30 total)
  - a. What is the expected value of the number of bikes on a fully loaded (i.e. with 200 passengers) BART train)?
  - b. What is the variance of the number of bikes on a fully loaded (i.e. with 200 passengers) BART train?
  - c. Write an (exact) arithmetic expression for the probability that a fully loaded train has exactly 20 bikes on board. Assume the train has 200 passengers and passenger choices of whether to bring a bike on board are independent.
  - d. Write an approximate arithmetic expression for the probability described in c), which does not contain any factorial terms. (Do not use the normal distribution.)
  - e. What is the approximate PDF of the total number of bikes carried by 40 fully loaded trains?
  - f. What is the approximate probability that this total number carried by 40 fully loaded trains is less than 420?

2. There is a 30% chance of a major (magnitude 6.7 or more) earthquake on the Hayward fault over the next 30 years. There is a 20% chance of such an earthquake on the San Andreas fault. Assume that there is no probability of more than one earthquake on either fault.
- a. Let  $H$  be the event of a major earthquake on the Hayward fault, and  $S$  be the event of a major earthquake on the San Andreas fault.
- What is  $P(H)$ ?  $P(S)$ ? (5 pts)
  - Assuming  $H$  and  $S$  are independent, what is (4 pts each):
    - $P(H \cup S)$  ?
    - $P(HS)$  ?
    - $P(H | S)$  ?
    - $P(HS | H)$  ?
  - Assuming  $H$  and  $S$  are mutually exclusive, what is (4 pts each):
    - $P(H \cup S)$  ?
    - $P(HS)$  ?
    - $P(H | S)$  ?
    - $P(HS | H)$  ?
- b. A house is situated in a location such that if there is a major earthquake on the Hayward fault, the probability that it is seriously damaged is .5, while if there is a major earthquake on the San Andreas fault, the probability that it is seriously damaged is .2. Assuming that  $H$  and  $S$  are mutually exclusive (5 pts each):
- What is the probability that the house is damaged as a result of a major earthquake on one of these two faults over the next 30 years.
  - If the house is damaged in a major earthquake, what is the probability that the quake was on the Hayward fault?

3. A sadistic professor institutes the policy of random quizzes at the beginning of each class. In every class, he throws a die and if the result is a 1 there is a quiz, and the result if other than a 1 there is no quiz. He has two different dice, one with the numbers 1-6, and another with the numbers 1-3, each on two different faces. He uses the same die in every throw.
  - a. Let  $X$  be the first class when there is a quiz. In other words if the first class of the semester has a quiz then  $X=1$ , if the first class has no quiz but the second class has a quiz, then  $X=2$ , etc. Assume the professor uses the die with the numbers 1-6, what is the PMF of  $X$ ? (7 pts)
  - b. Suppose the first quiz is in the fifth class (4 pts each).
    - i. What is the value of the likelihood function if he is using the die with the numbers 1-3?
    - ii. What is the value of the likelihood function if he is using the die with the numbers 1-6?
    - iii. Based on the results above, which is the better estimate of the quiz probability,  $1/3$  or  $1/6$ ?
    - iv. What is the MLE for the quiz probability, assuming the probability could have any value—not just  $1/3$  or  $1/6$ ?