1. a) Using the linear expansion formula, we have

$$R = R_0 + \Delta R = R_0 + R_0 \alpha_s (T - T_a)$$

$$R = R_0 \left(1 - \alpha_s (T_a - T) \right)$$

As the temperature is lowered, the thermal energy of the atoms decreases and the average distance between them falls. Consequently, the diameter of the hole must shrink to bring the atoms on the perimeter into closer proximity (same number of atoms with a smaller radius $\Rightarrow$ smaller interatomic distance)

b) Rivet radius at temperature T

$$r(T) = r_0 + \alpha_r r_0 (T - T_a)$$

Given $r_0 = 1.01 R_0$, we require $r(T) = R_0$; solving for T gives

$$R_0 = r(T) = r_0 \left(1 + \alpha_r (T - T_a) \right)$$

$$\frac{R_0}{r_0} = 1 + \alpha_r (T - T_a)$$

$$\frac{R_0}{1.01 R_0} = 1 + \alpha_r (T - T_a)$$

$$\frac{100}{101} - 1 = \alpha_r (T - T_a)$$

$$\frac{-1}{101} = \alpha_r (T - T_a)$$

$$\frac{-1}{101 \alpha_r} = T - T_a$$

$$T_a - \frac{1}{101 \alpha_r} = T$$

c) According to the ideal gas law, the temperature and pressure for a fixed number of particles and constant volume vary as follows.

$$\frac{P_1}{T_1} = \frac{Nk}{V} = \frac{P_2}{T_2}$$

The new pressure, then, is

$$P = \frac{T}{T_a} P_a$$

As the pressure times the area of the lid gives a force, and the inner and outer pressures are opposed, the net force on the lid is

$$F_1 = \underset{\substack{\uparrow \\ \text{inner}}}{PA} - \underset{\substack{\uparrow \\ \text{outer}}}{P_a A} = A\left(\frac{T}{T_a} P_a - P_a\right)$$

$$F_1 = AP_a\left(\frac{T}{T_a} - 1\right)$$

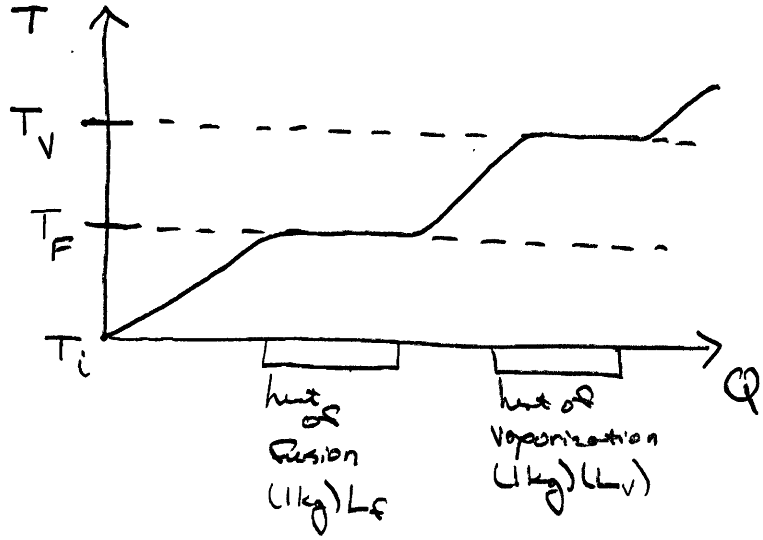
d) When the vessel is removed from the stove, it will cool from T to the ambient temperature. Using the same strategy as in c), we calculate F_2.

$$\frac{P}{T_a} = \frac{P_a}{T} \Rightarrow P = \frac{T_a}{T} P_a$$

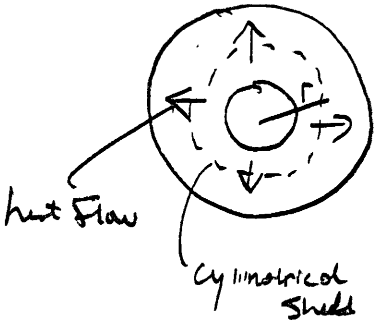
$$F_2 = P_a A - PA = P_a A - \frac{T_a}{T} P_a A$$

$$F_2 = P_a A\left(1 - \frac{T_a}{T}\right)$$

2. a) Taking into account the phase changes, we have



b)



heat flow is radial, so the usual assumptions of symmetry give the following result, in which $\frac{dQ}{dt}$ is constant for any cylindrical surface

$$\frac{dQ}{dt} = -kA\frac{dT}{dr} = -k\,(2\pi r h)\frac{dT}{dr}$$

$$\frac{dQ}{dt} = -(2\pi hk)\,r\frac{dT}{dr}$$

$$\frac{dQ}{dt}\int_{R_1}^{R_2}\frac{1}{r}\,dr = -2\pi hk\int_{T_1}^{T_2} dT$$

$$\frac{dQ}{dt}\ln\frac{R_2}{R_1} = -2\pi hk\,(T_2 - T_1)$$

$$\frac{dQ}{dt} = \frac{2\pi hk\,(T_1 - T_2)}{\ln\frac{R_2}{R_1}}$$

The sign is positive to reflect that heat flows from the hot inner surface out.

c) Using the preceding equations, we may write

$$\frac{2\pi h k (T_1 - T_2)}{\ln \frac{R_2}{R_1}} = \frac{dQ}{dt} = -(2\pi h k)\, r \frac{dT}{dr}$$

$$\frac{T_1 - T_2}{\ln \frac{R_2}{R_1}} = -r \frac{dT}{dr}$$

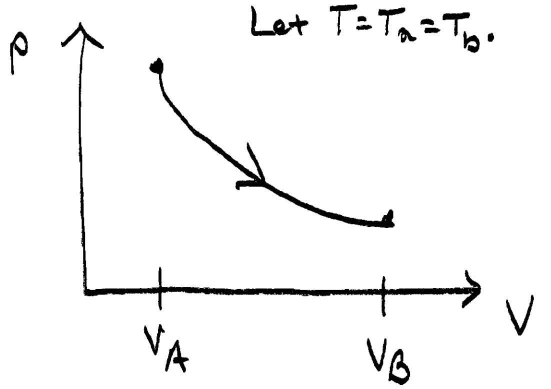
$$-\frac{(T_1 - T_2)}{\ln \frac{R_2}{R_1}} \int_{R_1}^{R} \frac{1}{r}\, dr = \int_{T_1}^{T} dT$$

$$-\frac{(T_1 - T_2)}{\ln \frac{R_2}{R_1}} \ln \frac{R}{R_1} = T(R) - T_1$$

$$T_1 - \frac{(T_1 - T_2)}{\ln \frac{R_2}{R_1}} \ln \frac{R}{R_1} = T(R)$$

d) As entropy is a state variable, we may determine the change in entropy using any arbitrary path between a & b. As $T_b = T_a$, we know the two paths may be connected by an isotherm, so we calculate ΔS by using an isotherm between a & b.

Let $T = T_a = T_b$.



isotherm: $P = \frac{nRT}{V}$

As the temperature doesn't change, $\Delta E = 0$ for the process. By the first law, then, $dQ = dW$ here, and we have

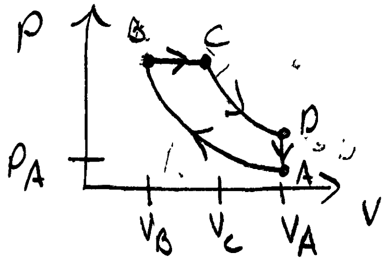
$$\Delta S = \int \frac{dQ}{T} = \int \frac{dW}{T} = \int \frac{P\,dV}{T} = \int_{V_a}^{V_b} \frac{nRT}{V} \frac{1}{T} dV$$

$$= nR \int_{V_a}^{V_b} \frac{dV}{V}$$

$$\Delta S = nR \ln \frac{V_b}{V_a}$$

e) For an adiabat, by definition $dQ = 0$, so the entropy change is zero. If such a process is isentropic, we may infer that the word refers to a process with no change in entropy.

3. a)



Aside from the fact that we know the direction from the description (as we are told about compressions, expansions, etc.), the cycle must run clockwise to produce a positive net work as determined by the sign of the enclosed area & the usual rules of calculus.

b) $A \Rightarrow B$ adiabat (we assume $\gamma = \frac{7}{5}$ since the gas is diatomic)

$$pV^\gamma = p_A V_A^\gamma \Rightarrow p = \frac{p_A V_A^\gamma}{V^\gamma}$$

$$W_{AB} = \int_{V_A}^{V_B} p\,dV = \int_{V_A}^{V_B} p_A \frac{V_A^\gamma}{V^\gamma}\,dV = p_A V_A^\gamma \frac{1}{1-\gamma}\left[\frac{1}{V^{\gamma-1}}\right]_{V_A}^{V_B}$$

$$= \frac{p_A V_A}{1-\gamma} V_A^{\gamma-1}\left[\frac{1}{V_B^{\gamma-1}} - \frac{1}{V_A^{\gamma-1}}\right]$$

$$W_{AB} = \frac{p_A V_A}{1-\gamma}\left[\left[\frac{V_A}{V_B}\right]^{\frac{7}{5}-1} - 1\right]$$

$B \Rightarrow C$ isobar — constant pressure p_B, which we calculate using adiabatic endpoint

$$p_B V_B^\gamma = p_A V_A^\gamma \Rightarrow p_B = p_A\left(\frac{V_A}{V_B}\right)^\gamma$$

$$W_{BC} = \int_{V_B}^{V_C} p_B\,dV = p_B \int_{V_B}^{V_C} dV = p_B(V_C - V_B) = p_A\left(\frac{V_A}{V_B}\right)^{7/5}(V_C - V_B)$$

$C \to D$ adiabat; proceeding as before,

$$PV^\gamma = P_B V_C^\gamma \Rightarrow P = P_B \frac{V_C^\gamma}{V^\gamma} = P_A \frac{V_A^\gamma}{V_B^\gamma} \frac{V_C^\gamma}{V^\gamma}$$

$$W_{CD} = \int_{V_C}^{V_A} P\,dV = P_A \frac{V_A^\gamma}{V_B^\gamma} V_C^\gamma \int_{V_C}^{V_A} \frac{dV}{V}$$

$$= P_A \frac{V_A^\gamma}{V_B^\gamma} \frac{V_C^\gamma}{1-\gamma} \left[\frac{1}{V^{\gamma-1}}\right]_{V_C}^{V_A}$$

$$= P_A \frac{V_A^\gamma}{V_B^\gamma} \frac{V_C^\gamma}{1-\gamma} \left[\frac{1}{V_A^{\gamma-1}} - \frac{1}{V_C^{\gamma-1}}\right]$$

$$= P_A \left(\frac{V_A}{V_B}\right)^{7/5} \frac{V_C}{1-\frac{7}{5}} \left[\left(\frac{V_C}{V_A}\right)^{\frac{7}{5}-1} - 1\right]$$

$D \Rightarrow A$ isochor; $dW = 0$, so no work done

$$W_{DA} = 0$$

Net work: $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$$= \frac{P_A V_A}{1-\frac{7}{5}}\left[\left(\frac{V_A}{V_B}\right)^{\frac{7}{5}-1} - 1\right] + P_A\left(\frac{V_A}{V_B}\right)^{7/5}(V_C - V_B)$$

$$+ P_A \left(\frac{V_A}{V_B}\right)^{7/5} \frac{V_C}{1-\frac{7}{5}} \left[\left(\frac{V_C}{V_A}\right)^{\frac{7}{5}-1} - 1\right].$$

c) No heat is exchanged across the adiabats $A \Rightarrow B$ & $C \Rightarrow D$. The remaining processes have heat exchanges which may be found using the appropriate specific heats.

$B \Rightarrow C$ Constant pressure, $C_p = \frac{7}{2}R$

$$Q_{BC} = nC_p \Delta T = nC_p (T_c - T_B)$$

$$T_B = \frac{P_B V_B}{nR} = \frac{1}{nR} V_B P_A \left(\frac{V_A}{V_B}\right)^\gamma$$

$$T_C = \frac{P_c V_c}{nR} = \frac{P_B V_c}{nR} = \frac{V_c}{nR} P_A \left(\frac{V_A}{V_B}\right)^\gamma$$

$$T_C - T_B = P_A \left(\frac{V_A}{V_B}\right)^\gamma \frac{1}{nR} (V_c - V_B)$$

$$Q_{BC} = \frac{C_p}{R} P_A \left(\frac{V_A}{V_B}\right)^\gamma (V_c - V_B) = \frac{7}{2} P_A \left(\frac{V_A}{V_B}\right)^{7/5} (V_c - V_B)$$

$D \Rightarrow A$ Constant volume; $C_V = \frac{5}{2}R$

$$Q_{DA} = nC_V \Delta T = nC_V (T_A - T_D)$$

$$T_D = \frac{P_D V_D}{nR} = \frac{P_D V_A}{nR} = \frac{V_A}{nR} P_D$$

from earlier adiabatic process $C \Rightarrow D$,

$$P_D V_D^\gamma = P_c V_c^\gamma$$

$$P_D V_A^\gamma = P_B V_C^\gamma \Rightarrow P_D = P_B \frac{V_c^\gamma}{V_A^\gamma} = P_A \left(\frac{V_A}{V_B}\right)^\gamma \left(\frac{V_c}{V_A}\right)^\gamma = P_A \left(\frac{V_c}{V_B}\right)^\gamma$$

$$T_D = \frac{V_A}{nR} P_D = \frac{V_A}{nR} P_A \left(\frac{V_c}{V_B}\right)^\gamma$$

$$T_A = \frac{V_A P_A}{nR}$$

$$T_A - T_D = \frac{V_A P_A}{nR} - \frac{P_A V_A}{nR}\left(\frac{V_c}{V_B}\right)^\gamma = \frac{V_A P_A}{nR}\left(1 - \left(\frac{V_c}{V_B}\right)^\gamma\right)$$

$$Q_{DA} = nC_V \frac{V_A P_A}{nR}\left[1 - \left(\frac{V_c}{V_B}\right)^\gamma\right] = \frac{C_V}{R} V_A P_A \left(1 - \left(\frac{V_c}{V_B}\right)^\gamma\right) = \frac{5}{2} V_A P_A \left(1 - \left(\frac{V_c}{V_B}\right)^{7/5}\right)$$

d) $\epsilon = \dfrac{W_{net}}{Q_{in}}$

Q_{in} heat flow into gas (not net)

Q_{out} heat flow out of gas (not net)

$$Q_{total} = Q_{in} - Q_{out}$$

As for the whole cycle $\Delta E_{gas} = 0$, by the first law,

$$0 = \Delta E_{int} = Q - W_{net} = (Q_{in} - Q_{out}) - W_{net}$$

$$W_{net} = Q_{in} - Q_{out}$$

From our earlier work, $Q_{in} = Q_{BC}$

$Q_{out} = Q_{DA}$,

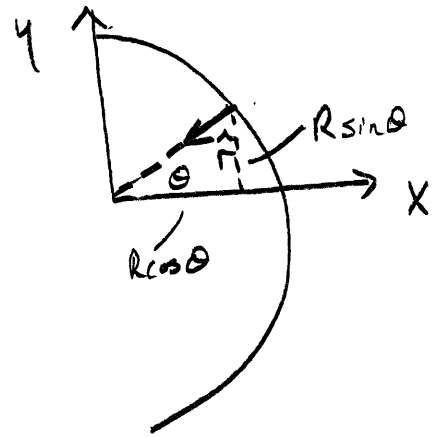
So we have

$$\epsilon = \frac{W_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{Q_{DA}}{Q_{BC}} = 1 - \frac{\frac{C_V}{R} V_A P_A \left(1 - \left(\frac{V_c}{V_B}\right)^\gamma\right)}{\frac{C_P}{R} P_A \left(\frac{V_A}{V_B}\right)^\gamma (V_c - V_B)}$$

$$\epsilon = 1 - \frac{5}{7} \frac{\left(1 - \left(\frac{V_c}{V_B}\right)^{7/5}\right)}{\left(\frac{V_c}{V_B} - 1\right)}\left(\frac{V_B}{V_A}\right)^{2/5} \Leftarrow \epsilon = 1 - \frac{1}{\gamma} \frac{\left(1 - \left(\frac{V_c}{V_B}\right)^\gamma\right)}{\left(V_c/V_B - 1\right)}\left(\frac{V_B}{V_A}\right)^{\gamma - 1}$$

4. a)



$$\vec{r} = -R\cos\theta\,\hat{i} - R\sin\theta\,\hat{j}$$

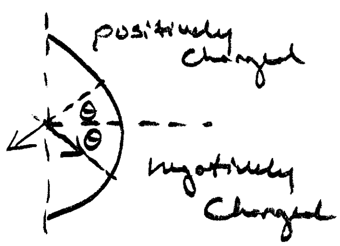
$$\hat{r} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$$

b)

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{R^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda_0 \sin\theta\, R\, d\theta}{R^2}\left(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$$

$$d\vec{E} = \frac{-\lambda_0}{4\pi\epsilon_0 R}\left[\sin\theta\cos\theta\,\hat{i} + \sin^2\theta\,\hat{j}\right]d\theta$$

c)



horizontal components cancel; net direction is along $-\hat{j}$

d) ignoring horizontal component, which will cancel out,

$$\vec{E} = \frac{-\lambda_0}{4\pi\epsilon_0 R}\hat{j}\int_{-\pi/2}^{\pi/2}\sin^2\theta\, d\theta$$

$$= \frac{-\lambda_0}{4\pi\epsilon_0 R}\hat{j}\int_{-\pi/2}^{\pi/2}\frac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$= \frac{-\lambda_0}{8\pi\epsilon_0 R}\hat{j}\left[\theta - \frac{1}{2}\sin 2\theta\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{-\lambda_0}{8\pi\epsilon_0 R}\hat{j}\left[\pi\right] = \frac{-\lambda_0}{8\epsilon_0 R}\hat{j}$$