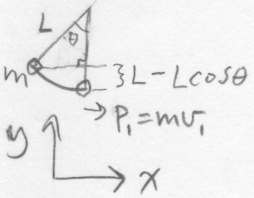
Mike DeWeese Fall 2017 lecture sec. 2 Phys 7A final exam

1. a.) cons. of E: $\frac{1}{2} m v_1^2 = mg\Delta h = mg(L - L\cos\theta)$

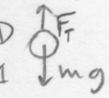


$$1 - \cos\theta = \frac{v_1^2}{2gL}$$

$$\boxed{\theta = \cos^{-1}\left(1 - \frac{p_1^2}{2gLm^2}\right)}$$

b.) N2L$_y$ ball: $F_T - mg = ma_y = m\left(\frac{v_1^2}{L}\right) = \frac{p_1^2}{mL}$

FBD ball1



$$\boxed{F_T = mg + \frac{p_1^2}{mL}}$$

c.) $\theta \ll 1$: harmonic oscillator: $\omega = \sqrt{g/L}$ $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{g/L}$

time to swing down $= \frac{T}{4} = \frac{1}{4f} = \boxed{\frac{\pi}{2}\sqrt{L/g}}$

d.) elastic collision: cons. of KE & cons. of p_x $\Rightarrow$ $v_{5f} = v_{1i} = \boxed{p_1/m}$ ($\vec{F}_T$ acts vertically, so $\vec{F}_T \cdot \vec{v} = 0 \Rightarrow$ no power from tension)

in fact, since we are told that only the fifth ball is moving after the collision, cons. of momentum is enough to get $v_{5f} = v_{1i}$, since mass of each ball is the same

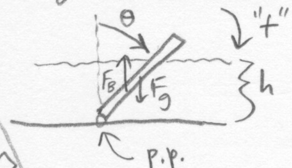
e.) partially inelastic collision:

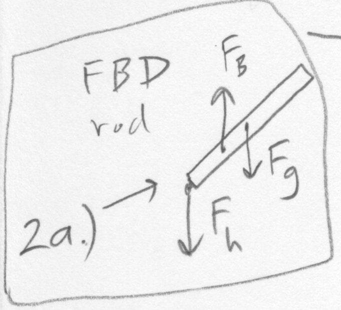
1

cons. of p_x: $\cancel{m}v_{1i} = \cancel{m}v_{4f} + \cancel{m}v_{5f} \Rightarrow v_{5f} = v_{1i} - v_{4f} = \frac{3}{4}v_{1i} = \frac{3}{4}p_1/m$

2 b.) static equilibrium while vertical:

for $\theta > 0$:





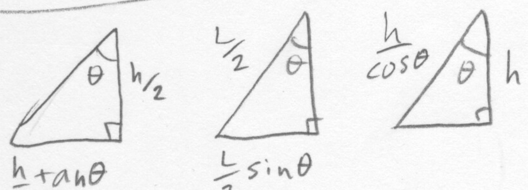
$$N2L_\alpha: \cancel{\tau_h}^{\,0} + \tau_B + \tau_g = I_{end}\cancel{\alpha}^{\,0 \text{ static}}$$

hinge = p.p.

$$-\frac{h}{2}\tan(\theta) F_B + \frac{L}{2}\sin(\theta)\rho_R L A g = 0$$

$$① \Rightarrow \frac{h}{\cancel{2}}\tan(\theta)\frac{hA}{\cos\theta}\cancel{g}\rho_w = \frac{L}{\cancel{2}}\sin(\theta)\rho_R L \cancel{Ag}$$

$$\cos^2\theta = \frac{h^2}{L^2}\frac{\rho_w}{\rho_R} \quad ③$$



$$F_B = \frac{hA}{\cos\theta} g \rho_w \quad ①$$

vertical $\Rightarrow \theta = 0 \Rightarrow \frac{h^2}{L^2}\frac{\rho_w}{\rho_R} = 1$

so $\boxed{\rho_{R,max} = \frac{h^2}{L^2}\rho_w}$ ②

check ballance of forces is satisfied:

$$N2L_y \quad F_B - F_g - F_h = m\cancel{a_y}^{\,0 \text{ static}}$$

$\theta = 0$

static eq: $F_{h,y} \leq 0$

so: $F_B \geq F_g$

$$\frac{hA}{\cancel{\cos\theta}}\cancel{g}\rho_w \geq LA\cancel{g}\rho_R$$

1

$\rho_R \leq \frac{h}{L}\rho_w$, which is always satisfied if ② is true. ✓

c.) ③: $\boxed{\theta = \cos^{-1}\left(\frac{h}{L}\sqrt{\rho_w/\rho_R}\right)}$

d.) N2Ly: rod

$$-F_h + F_B - F_g = m\cancel{a_y}^{\,0 \text{ static}}$$

$$F_h = F_B - F_g = \frac{hA}{\cos(\theta)} g\rho_w - \rho_R L A g$$

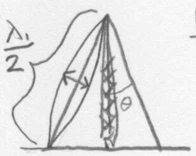
$$F_h = \boxed{Ag\left(\frac{\rho_w h}{\cos\theta} - \rho_R L\right)}$$ downward

another way to write it:

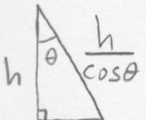
$$③: F_h = Ag\left(\frac{\rho_w h}{\frac{h}{L}\sqrt{\rho_w/\rho_R}} - \rho_R L\right)$$

$$= AgL\rho_R\left(\sqrt{\rho_w/\rho_R} - 1\right)$$

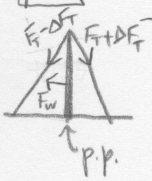
3a.) standing wave in cable w/ lowest resonant frequency f_1



$$f_1 = \frac{v}{\lambda_1} = \boxed{\frac{\sqrt{F_T/\mu}}{2h}\cos\theta}$$



b.)



$$N2L_\alpha: -\cancel{h}(F_T - \Delta F_T)\sin\theta + \cancel{h}(F_T + \Delta F_T)\sin\theta - F_w\frac{h}{2} = I\alpha \to 0 \text{ static}$$

$$2\sin\theta\,\Delta F_T = \frac{F_w}{2} \qquad \boxed{\Delta F_T = \frac{F_w}{4\sin\theta}}$$

c.) $$f_{beat} = |f_1 - f_2| = \boxed{\frac{\cos\theta}{2h}\left(\sqrt{\frac{F_T+\Delta F_T}{\mu}} - \sqrt{\frac{F_T-\Delta F_T}{\mu}}\right)}$$

d.) observer is moving w/r to ground but still w/r to the air, which is the medium transmitting the sound waves.

Doppler shift w/ moving source, moving away

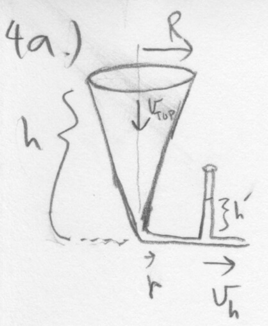
$$f_{obs} = f_B\frac{1}{1+\frac{u}{v}} \qquad \lambda_{obs} = \frac{v}{f_{obs}} = \frac{v(1+u/v)}{f_B} = \boxed{\frac{v+u}{f_B}}$$

e.) new observer moving w/r to air: approaching 1st bird

$$f'_{obs} = f_{obs}\left(1 + \frac{\text{speed of obs' w/r to air}}{v}\right)$$

$$= \boxed{f_B\frac{1}{1+u/v}\left(1 + \frac{u'-u}{v}\right)}$$

4a.)



continuous flow Through vertical & horizontal pipes (separate flow in stand pipe)

$$\text{B.E.} \quad \left(P+\rho g h+\tfrac{1}{2}\rho v_{top}^2\right)_{top} = \left(P+\cancel{\rho g 0}^{\,0}+\tfrac{1}{2}\rho v_{horiz.}^2\right)_{\substack{horiz.\\ pipe}} \quad -①$$

$$\text{C.E.} \quad A_{top} v_{top} = A_h v_h \Rightarrow v_h = \frac{R^2}{r^2} v_{top} \quad -②$$

$$② \text{ in } ① \Rightarrow \quad \rho g h + \tfrac{1}{2}\rho v_{top}^2 = \tfrac{1}{2}\rho \frac{R^4}{r^4} v_{top}^2 \quad -③$$

$$\boxed{R = r\sqrt[4]{\frac{2gh}{v_{top}^2}+1}}$$

$$\text{since } R \gg r \Rightarrow ③: \; gh + \cancel{\tfrac{1}{2} v_{top}^2}^{\approx 0} = \tfrac{1}{2}\rho \frac{R^4}{r^4} v_{top}^2$$

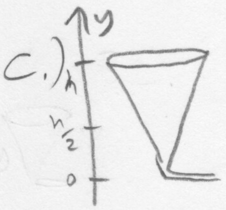
$$R \approx r\sqrt[4]{\frac{2gh}{v_{top}^2}}$$

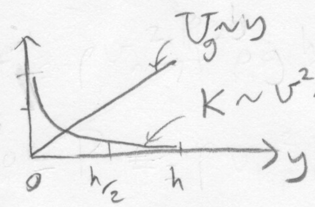
b.) C.E. $A_m v_m = A_t v_{top}$

$$\left(\frac{R+r}{2}\right)^2 v_m = R^2 v_{top}$$

$$R \gg r \Rightarrow \frac{R^2}{4} v_m = R^2 v_{top} \Rightarrow \boxed{v_m = 4 v_{top}}$$

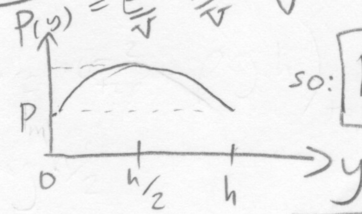
c.)





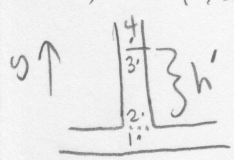
SO

$$P(y) = \underbrace{\frac{E}{V}}_{const.} - \frac{K}{V} - \frac{U_g}{V} \quad \text{(where } V \text{ is volume)}$$



so: $\boxed{P_m > P}$

d.) $\underset{1\to 2}{N2Ly}: P_1 = P_2 \quad \underset{2\to 3}{BE}: P_3 + \rho g h' = P_2 \Rightarrow h' = \frac{P_2 - P_3}{\rho g} = \boxed{\frac{P - 1\text{atm}}{\rho g}}$

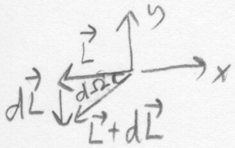


$\underset{3\to 4}{N2L}\; P_3 = P_4 = 1\text{atm}$

e.) C.E. is unchanged by viscosity: ②: $v_h = \boxed{\frac{R^2}{r^2} v_{top}}$

5a.) $N2L_\alpha$: $\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$ sphere $I_{cm} = \frac{2}{5}MR^2$ symmetrical about axis of r. $L = I\omega$

$\omega_2 \ll \omega_1 \Rightarrow \vec{L} = -I_{cm}\omega_1 \hat{i}$, $\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt} = L \underbrace{\dfrac{d\Omega}{dt}}_{\omega_2}(-\hat{j}) = \boxed{-\frac{2}{5}MR^2\omega_1\omega_2\hat{j}}$

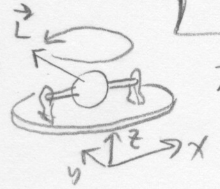


$dL = L\sin(d\Omega) \underset{d\Omega \ll 1}{\approx} L\,d\Omega$

negative y direction

b.) $\vec{L} = I\vec{\omega} = \boxed{\frac{2}{5}MR^2\left(-\omega_1\hat{i} + \omega_2\hat{k}\right)}$

c.)



$\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$ = same as in a) since the x & y components of $\vec{L}$ are the same functions of time as in a) and L_z is not changing.

$$\boxed{|\vec{\tau}_{net}| = \frac{2}{5}MR^2\omega_1\omega_2}$$

d.) half of this torque is achieved by the force acting on one end of axle: $\tau_{\frac{1}{2}\text{end}} = \frac{l}{2}F = \frac{1}{2}\frac{2}{5}MR^2\omega_1\omega_2$

$$\boxed{F = \frac{2}{5}\frac{MR^2\omega_1\omega_2}{l}}$$

e.)

density of original sphere $= \rho = \dfrac{M}{\frac{4}{3}\pi R^3}$

"||" axis thrm: $I = \underbrace{\left(\frac{2}{5}MR^2 + MR^2\right)}_{\text{I for solid sphere about end}} - \underbrace{\left(\frac{2}{5}mr^2 + m(R-d)^2\right)}_{\text{I for solid sphere about axis d from center}}$

$m = \rho\frac{4}{3}\pi r^3 = M\dfrac{r^3}{R^3}$

$$\boxed{I = \left(\frac{2}{5}MR^2 + MR^2\right) - \left(\frac{2}{5}M\frac{r^5}{R^3} + M\frac{r^3(R-d)^2}{R^3}\right)}$$

6a.) $h = \frac{1}{2}H \Rightarrow$ bullet hits c.m. of rack: 1-D problem since no friction or other forces that give torque about pole's c.m.

$$|\vec{J}| = J_x = \Delta P_{x,\,Rack} = v_{Rf} M_R - \cancel{v_{Ri}}^{0} M_R$$

$\longrightarrow x$

totally inelastic collision:

$$m_B v_B = (\cancel{m_B}^{0} + M_R) v_{fR}$$

$$\boxed{|\vec{J}| = m_B v_B} \quad \text{using } m_B \ll M_R$$

b.) $h = \frac{1}{2}H \quad W_{\text{on Rack}} = \Delta K_R = K_{Rf} = \frac{1}{2} M_R v_{fR}^2 = \frac{1}{2} M_R \frac{m_B^2}{M_R^2} v_B^2 = \boxed{\frac{1}{2} M_B v_B^2 \frac{m_B}{M_R}}$

c.) $|\vec{L}| = |\vec{r} \times \vec{p}| = \boxed{\left(\frac{H}{2} - h\right) m_B v_B}$ out of page by R.H.R.

d.)

$v_B \longrightarrow$

$$\text{C.O.}\vec{L}: \left(\frac{H}{2} - \cancel{h}^{0}\right) m_B v_B = I_{\text{Rack cm}}\, \omega_{R,f} \quad (1)$$

(where we have assumed $m_B \ll M_R$ so c.m. of bullet + Rack = cm Rack and $L_{Bf} \ll L_{Rf}$)

$$I_{\text{Rack cm}} = 2\int_0^{H/2} \frac{M_R}{H} dx\, x^2 = \frac{2}{3}\frac{H^3 M_R}{8H} = \frac{1}{12} M_R H^2 \quad (2)$$

(2) in (1) $\Rightarrow$ $\frac{H}{2} m_B v_B = \frac{1}{12} M_R H^2 \omega_{R,f}$

$$\boxed{\omega_{R,f} = \frac{6}{H}\frac{m_B}{M_R} v_B}$$

e.) time for Rack to rotate by $\frac{\pi}{2}$: $t_{rotate} = \frac{\pi/2}{\omega_R} = \frac{\pi H M_R}{12 m_B v_B}$

time for Rack's cm to move by $\frac{H}{2} - D$: $t_{move} = \frac{\frac{H}{2} - D}{v_{cm,R}} = \frac{\frac{H}{2} - D}{v_B} \frac{M_R}{m_B}$

C.O.P_x $\quad m_B v_B = (M_R + \cancel{m_B}^{0}) v_{R,f,cm}$

Rack hits Dr. Maybe: $t_{rot} \le t_{move}$

$$\frac{\pi H \cancel{M_R}}{12 \cancel{m_B} \cancel{v_B}} \le \left(\frac{H}{2} - D\right) \frac{1}{\cancel{v_B}} \frac{\cancel{M_R}}{\cancel{m_B}}$$

$$D \le \frac{H}{2} - \frac{\pi H}{12}$$

$$\boxed{D \le \frac{H}{2}\left(1 - \frac{\pi}{6}\right)}$$