University of California at Berkeley
Department of Physics
Physics 7A, Lecture Section 2, Fall 2017

Michael DeWeese

Final Exam
11:30 am, December 11, 2017

You will be given 180 minutes to work this exam. No books are allowed, but you may use one double-sided, handwritten formula sheet no larger than an 8 ½” by 11” sheet of paper. No electronics that can send or receive wirelessly are allowed, but you may use a calculator if you want (won’t help much…). Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, be careful with signs, and take particular care to explain what you are doing. Please express your answers using the symbols provided in the problem descriptions or define any new symbols you use, tell us why you’re writing any new equations, and clearly label any drawings that you make. Write your answers in a blue book (or green book), and do not use any extra scratch paper. Please $\boxed{\text{BOX}}$ your answers and use a pencil or a blue or black pen. Good luck!

1) (25 points) Newton's Cradle

A "Newton's Cradle" is a toy consisting of 5 steel balls suspended by strings from a metal frame as shown in the diagram (usually, each ball is suspended from two strings, but for this problem assume there is only 1 string per ball). Each of the balls has diameter R and mass m, and the strings are each of length L. The 1st ball (*i.e.*, the one on the far left) is pulled back and released from rest so that the string supporting it is under tension throughout the ball's motion; the other 4 balls are all holding still during this time. You may assume $R << L$.

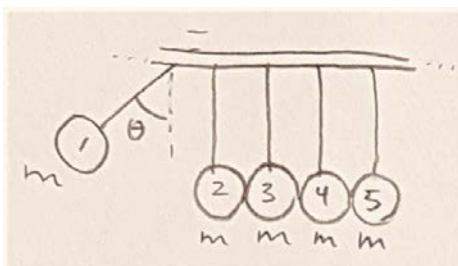
a) If the magnitude of the momentum of the 1st ball is p_1 immediately before it strikes the 2nd ball, then what was the initial angle θ between the string and the vertical as viewed in the diagram? Express your answer in terms of any subset of p_1, m, R, L, and any relevant physical constants.

b) What is the tension in the string supporting the 1st ball immediately before it strikes the 2nd ball? Express your answer in terms of any subset of p_1, m, R, L, and any relevant physical constants.

c) How long does it take for the 1st ball to swing from rest until it hits the 2nd ball? Express your answer in terms of any subset of p_1, m, R, L, and any relevant physical constants. You may assume that $\theta << 1$.

d) After the collision, the first 4 balls are stationary but what is the speed of the 5th ball at the far right? As always, show your work or justify your answer.

e) Now suppose that the experiment is repeated, but this time an insect climbs in between the two balls at the far right so that when the 1st ball collides into the 2nd ball, some of the initial kinetic energy of the 1st ball goes into squashing the insect. If the two balls on the far right are the only balls moving after that collision, and if the 4th ball is moving at a speed $v_{4f} = \frac{1}{4}\, p_1/m$ immediately after the collision, then what is the **speed** of the 5th (last) ball immediately after the collision? You may assume the insect has negligible mass.



2) (20 points) Buoyant Force

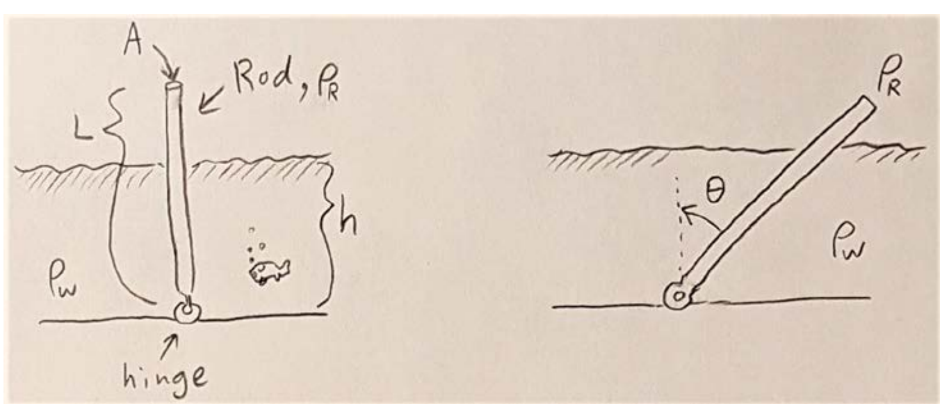
A long thin plastic rod of length L and cross-sectional area A is attached at one end to a hinge at the bottom of a tank of water, as shown in the left diagram. The depth of the water is h, which is less than L. The hinge is frictionless so that the rod is free to rotate.

a) For the case in which the rod's density is low enough so that it is oriented at an angle $\theta > 0$ as shown in the diagram, draw an "extended" free body diagram of the rod indicating where every external force is acting on the rod.

b) What is the maximum possible value for the density of the rod $\rho_{R,max}$ such that the rod will be oriented vertically in stable equilibrium? Express your answer in terms of h, L, and the density of water ρ_W.

c) Now consider the case in which the rod's density ρ_R satisfies $\rho_W > \rho_R > \rho_{R,max}$ so that the rod still floats, but at an angle θ with respect to the vertical, as shown in the second diagram. What is θ in this case? Express your answer in terms of h, L, ρ_R, and ρ_W.

d) In this case, what is the **direction** and **magnitude** of the force from the hinge on the rod?



3) (25 points) Radio Tower

The top of a radio tower of height h is held in place by two cables attached to the ground on either side of the tower, each cable forming an angle θ with the vertical tower, as shown in the diagram. Each of the cables has linear mass density μ (in units of kg/m) and tension F_T.

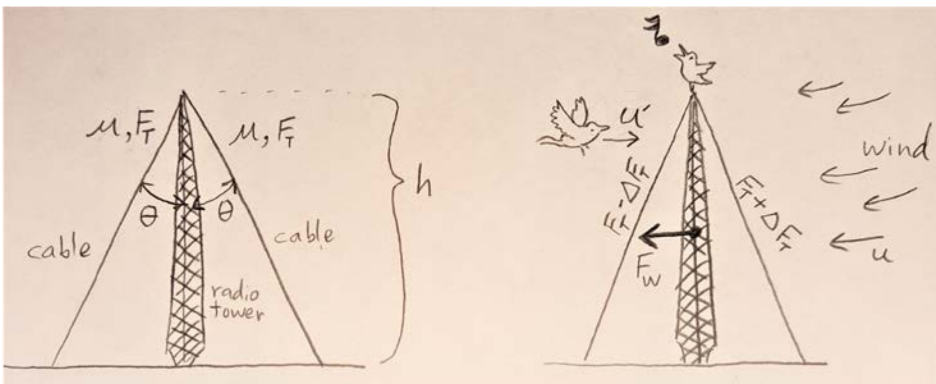
a) What is the lowest resonant frequency f_1 (*i.e.*, the fundamental frequency) for either of the cables? Express your answer in terms of h, θ, μ, and F_T.

b) Now the wind starts to blow, exerting a **leftward** force F_W on the **middle** of the tower so that the tension of the left cable is decreased while the tension in the right cable is increased. By how much ΔF_T does the tension in the right cable increase, assuming that the left cable decreases by the same amount? (Hint: treat the bottom of the tower as a hinge.) Express your answer in terms of h, θ, and F_W.

c) What is the resulting **beat frequency** heard by an observer standing near the tower listening to the notes produced by both cables at the same time? You may express your answer in terms of F_T, ΔF_T, h, θ, and μ.

d) If the wind speed is u, and the speed of sound is v, then what is the **wavelength** of the sound emitted by a singing bird on top of the tower as measured by an observer comoving with the air that is downstream (*i.e.*, to the left) of the tower? You may express your answer in terms of u, v, and frequency f_B the bird's note as heard by the bird itself.

e) If a second bird is flying **towards the tower from the left** with speed u' **with respect to the ground**, then what frequency does that bird hear when it listens to the first bird? Express your answer in terms of u, u', v, and f_B.



4) (25 points) Steady State Flow in a Conical Pipe

Water is flowing downward through a vertical pipe that is conical in shape, as shown in the diagram. The pipe has a height h and it has a circular cross section everywhere. It is widest at the top, with a radius of R, and it tapers linearly to a small radius r at the bottom, where the pipe bends to form a horizontal section with uniform radius r. At every point along this pipe, water is flowing at a steady rate. There is also a stand pipe, which is open to the atmosphere at the top, sticking up out of the horizontal section of the pipe. For the first 4 parts of the problem, you may neglect viscosity.

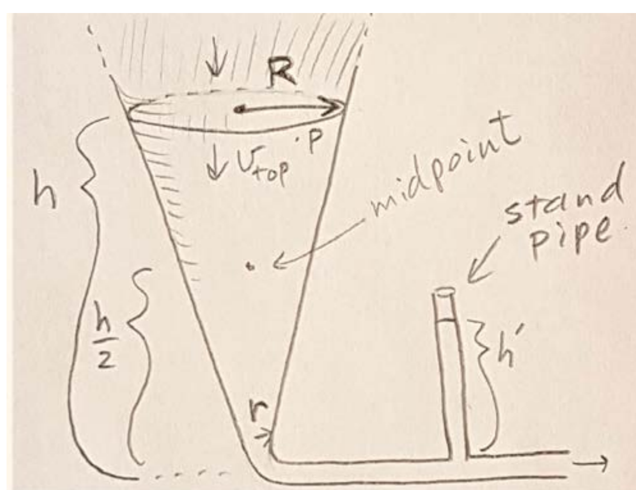
a) If the pressure in the top part of the pipe P is **the same** as the pressure in the horizontal section of the pipe, then what is R expressed in terms of r, h, the speed v_{top} of water entering the top part of the vertical pipe, and any relevant physical constants? You may assume $R >> r$ throughout this problem.

b) What is the speed of the water passing through the midpoint of the vertical section of pipe? Express your answer in terms of v_{top}.

c) Is the pressure in the midpoint of the vertical section of pipe **larger, smaller, or the same** as P, the pressure at the top and bottom of the pipe? Show your work or justify your answer.

d) What is the **height h'** of the water column in the standpipe? Express your answer in terms of any subset of R, r, h, v_{top}, P, and any relevant physical constants.

e) Consider the same physical system, but now viscosity results in a loss of 10% of the water's energy as it travels from the top of the vertical pipe to a point in the horizontal section of the pipe just below the standpipe. What is the **speed** of the water in the horizontal section of the pipe right below the standpipe? Express your answer in terms of any subset of R, r, h, v_{top}, and any relevant physical constants.



5) (25 points) Spinning sphere

The axle passing through the center of a solid sphere of uniform density is mounted on supports that rest on a rotating turntable as shown in the diagram. The sphere, with radius R and mass M, has angular velocity ω_1 about its axle, and the turntable and sphere have angular velocity ω_2 about a vertical axis. Note arrows showing these motions in the figure. Take the z axis vertically upward and the direction of the axle at the moment shown to be the x axis pointing to the right. Take the direction perpendicular to that of the axle at the moment shown to be the y axis pointing into the page. Recall that a uniform density sphere has a moment of inertia $I = 2/5\, MR^2$ as measured about an axis passing through the center of the sphere.

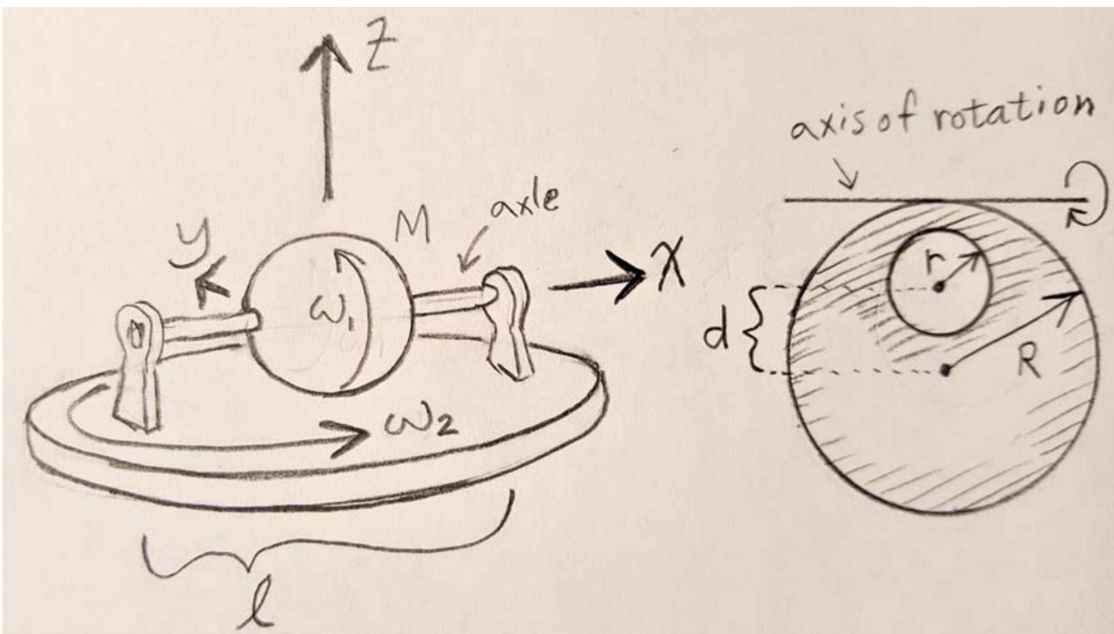
a) If $\omega_2 << \omega_1$, what is the **direction and magnitude** of the torque on the sphere at the moment shown in the diagram needed to maintain this motion? Express your answer in terms of M, R, ω_1, and ω_2.

b) Now do **not** assume $\omega_2 << \omega_1$. What is the resulting **angular momentum** of the sphere at the moment shown in the diagram? Express your answer as a **vector** in terms of any subset of M, R, ω_1, and ω_2.

c) In this case, what is the magnitude of the torque on the sphere at the moment shown?

d) What is the magnitude of the force exerted by the turntable on each end of the axle in order to maintain this motion? The total length of the axle is ℓ. Express your answer in terms of any subset of M, R, ω_1, ω_2, and ℓ.

e) If a spherical hole of radius r centered a distance d from the center of the original sphere is cut out of the sphere, then what is the moment of inertia of this object about an axis tangent to the surface of the original sphere at the point closest to the hole, as shown in the second diagram? Express your answer in terms of M, R, and d.



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6) (25 Points) Dr. Maybe

A secret agent working for the British government has tracked down a master criminal called "Dr. Maybe". Using a powerful handgun, the agent shoots a bullet horizontally with speed v_B ; the bullet misses Dr. Maybe and hits a coat rack that is initially standing still on the **frictionless** tile floor a horizontal distance D behind Dr. Maybe, as shown in the diagram. Upon impact, the bullet lodges in the coat rack a height h above the ground (*i.e.*, the bullet gets stuck in the coat rack). The bullet has mass m_B and the coat rack has mass M_R; you may assume $M_R >> m_B$, and also that the bullet is moving so fast that its velocity remains horizontal until it hits the coat rack. The coat rack is essentially a thin pole of uniform mass density and height H.

a) If $\boldsymbol{h = \frac{1}{2} H}$, what is the magnitude of the **impulse** to the coat rack resulting from the collision with the bullet? Express your answer in terms of any subset of m_B, M_R, v_B.
b) If $\boldsymbol{h = \frac{1}{2} H}$, what is the **work** done by the bullet on the coat rack during the collision?
c) For $\boldsymbol{0 \leq h \leq \frac{1}{2} L}$, what is the **angular momentum** of the **bullet** before it strikes the coat rack using the coat rack's center of mass as the origin? Indicate whether the angular momentum points into the page or out of the page as viewed in the diagram.
d) If the agent shoots the bullet very close to the ground so that $\boldsymbol{h = 0}$, what is the **angular velocity** of the coat rack after the collision?
e) If $\boldsymbol{h = 0}$, what is the **largest possible value for D** such that the coat rack hits Dr. Maybe on the top of his head? Dr. Maybe's head is ½ H above the ground. You may assume that the bullet's momentum is so large that you can neglect the effect of gravity on the coat rack throughout this process. Express your answer in terms of H.