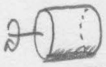
DeWeese spring 2014 7A midterm 2 solutions

1.a.) $\rho_{\substack{\text{front}\\ \text{wheel}}} = \frac{M_{fw}}{V_{fw}} = \boxed{\frac{6M}{\pi r^2 D}}$

b.) $I_{fw} = \int_0^D dz \int_0^r dr' \int_0^{2\pi} d\theta\, r' \rho_{fw}\, r'^2 = D\frac{r^4}{4} 2\pi \rho_{fw} = \frac{r^4}{4} 2\pi \frac{6M\cancel{D}}{\cancel{\pi} r^2 \cancel{D}} = \boxed{3Mr^2}$



$I_{\substack{\text{back}\\ \text{wheel}}} = \int_0^D dz \int_0^{2\pi} d\theta\, R\, \sigma_{bw} R^2 = D\, 2\pi R^3 \sigma_{bw}$

$\sigma_{bw} = \frac{M_{bw}}{D 2\pi R} = \frac{5M}{D 2\pi R}$

$\Bigg\} \; I_{bw} = \frac{\cancel{D 2\pi} R^3 5M}{\cancel{2\pi} R \cancel{D}} = R^2 5M$

$= \left(\frac{3}{2}r\right)^2 5M$

$= \frac{9\cdot 5}{4} Mr^2$

$= \frac{45}{4} Mr^2 > 11 Mr^2$

so $\boxed{I_{bw} > I_{fw}}$

c.) $\frac{KE_{rot.\,fw}}{KE_{rot\,bw}} = \frac{\cancel{\frac{1}{2}} I_{fw}\omega_{fw}^2}{\cancel{\frac{1}{2}} I_{bw}\omega_{bw}^2} = \frac{3\cancel{Mr^2}\,\omega_{fw}^2}{\frac{9\cdot 5}{4}\cancel{Mr^2}\,\omega_{bw}^2} = \frac{4\,\omega_{fw}^2}{3\cdot 5\,\omega_{bw}^2}$ — ①

$v_{bw\,cm} = \omega_{bw} 1.5r$

$v_{fw\,cm} = \omega_{fw} r$

$\Big\}$ rolling w/out slipping

both wheels are connected to same steamroller ⟹ $v_{fw\,cm} = v_{bw\,cm}$ at all times

so $\omega_{bw} 1.5\cancel{r} = \omega_{fw}\cancel{r}$ — ②

② in ① ⟹ $\frac{KE_{rot\,fw}}{KE_{rot\,bw}} = \frac{4}{3\times 5}\frac{(1.5\,\cancel{\omega_{bw}})^2}{\cancel{\omega_{bw}^2}}$

$= \frac{4}{3\times 5}\frac{3^2}{2^2} = \boxed{\frac{3}{5}}$

d.) C.O.E.:

$(KE_{cm} + KE_{rot\,fw} + KE_{rot\,bw} + \cancel{PE_g}^{\,0})_f - (\cancel{KE_{cm}}^{\,0} + \cancel{KE_{rot\,fw}}^{\,0} + \cancel{KE_{rot\,bw}}^{\,0} + M_{tot}gh)_i = \cancel{W_{ext}}^{\,0}$ (static friction does no work)

$\frac{1}{2}M_{tot}v_{cm}^2 + \frac{1}{2}I_{fw}\omega_{fw}^2 + \frac{1}{2}I_{bw}\omega_{bw}^2 = M_{tot}\, g\, \Delta x \sin\gamma$

$12\cancel{M}\, v_{cm}^2 + 3\cancel{Mr^2}\frac{v_{cm}^2}{\cancel{r^2}} + \frac{45}{4}\cancel{Mr^2}\frac{\left(\frac{2}{3}v_{cm}\right)^2}{\cancel{r^2}} = 2\cdot 12\cancel{M} g\, 2\pi r \sin\gamma$

$v_{cm}^2(12+3+5) = 48\pi g \sin(\gamma)\, r$

$v_{cm}^2 = \frac{48}{20}\pi g \sin(\gamma)\, r = 2.4\pi \sin(\gamma)\, g r$

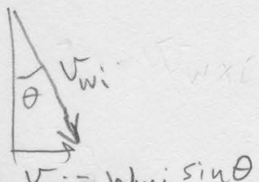
$\boxed{v_{cm\,\text{final}} = \sqrt{2.4\pi \sin(\gamma)\, g\, r}}$

2.a.) $m_w = \boxed{Rt}$

b.) C.O. $\vec{P}$:  $x$: $(m_w v_{wxf} + m_c v_{cxf})_f - (m_w v_{wxi} + m_c \cancel{v_{cxi}}^{\,0})_i = J_x^{ext} \nearrow 0$ no friction

maximally inelastic collision: $v_{wxf} = v_{cxf} = v_{xf}$

one could solve this using C.O.$\vec{P}$ at each moment of time, but since we know that each rain drop has the same initial velocity in our inertial reference frame (the ground) we can just consider all the rain falling into the cart as one big mass w/ the same initial velocity:

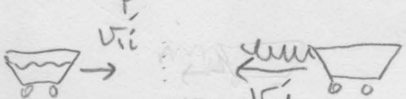



so: $(m_w + m_c)v_{xf} = m_w v_{wi}\sin\theta$

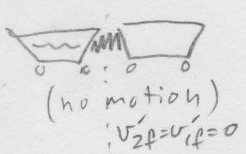
$$v_{xf} = \frac{m_w}{m_w+m_c}\sin\theta\, v_{wi} = \boxed{\frac{Rt}{Rt+M}\sin\theta\, v_{wi}}$$

c.) as seen from center of mass reference frame, nothing is moving when the spring is as compressed as it gets during collision, so at that instant, $KE_{tot} = 0$. and $v_{cm} = 0$

primes denote velocities as measured in center of mass frame




by C.O.E in C.O.M. frame: $KE_{tot\,i} = PE_{spring\ \text{max compression}}$ ①




$$v_{cm} = \frac{v_{1i}M_1 + \cancel{v_{2i}}^{\,0} M_2}{M_1 + M_2} = \frac{\frac{Rt}{\cancel{Rt+M}}\sin\theta\, v_{wi}\cancel{(Rt+M)}}{Rt+M+M} = \boxed{\frac{Rt\sin\theta}{Rt+2M}v_{wi}}$$

d.) $$v'_{1i} = v_{1i} - v_{cm} = \frac{Rt}{Rt+M}\sin\theta\, v_{wi} - \frac{Rt}{Rt+2M}\sin\theta\, v_{wi} = \frac{Rt\sin\theta\, v_{wi}\, M}{(Rt+M)(Rt+2M)}$$

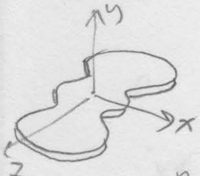
$$v'_{2i} = \cancel{v_{2i}}^{\,0} - v_{cm} = -\frac{Rt}{Rt+2M}\sin\theta\, v_{wi}$$

①: $$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}(M+Rt)\left(\frac{Rt\sin\theta\, M v_{wi}}{(Rt+M)(Rt+2M)}\right)^2 + \frac{1}{2}M\left(\frac{Rt}{Rt+2M}\sin\theta\, v_{wi}\right)^2$$

$$\boxed{\Delta x = \frac{1}{\sqrt{k}}\frac{Rt\sin\theta\, v_{wi}}{Rt+2M}\sqrt{\frac{M^2}{(Rt+M)} + M}}$$

3. a.) perpendicular axis Theorem:

given this choice of coords:



$$I_y = I_x + I_z = 0.3ML^2 + 0.2ML^2 = \boxed{0.5ML^2}$$

b.)  for solid cylinder of radius r, density ρ: $I_{cm_y} = \frac{1}{2}\underbrace{\rho\pi r^2 T}_{m_{cyl}} r^2 = \frac{1}{2}\rho\pi T r^4$

I calculated this for prob. 1 (and it's on my formula sheet)

parallel axis theorem:

$$I = I_{cm} + MD^2 = \frac{1}{2}\pi\rho T r^4 + \pi\rho T r^2 D^2$$
$$= \pi\rho T r^2\left(\frac{1}{2}r^2 + D^2\right)$$

a hole in the original plate can be thought of as decreasing the moment of inertia by an amount equal to the mom. of I. for a solid cylinder at the same location:

$$I_{y\,new} = I_y - 4\,(I_{\text{each offaxis cylinder}})$$
$$= \boxed{\frac{1}{2}ML^2 - 4\pi\rho T r^2\left(\frac{1}{2}r^2 + D^2\right)}$$

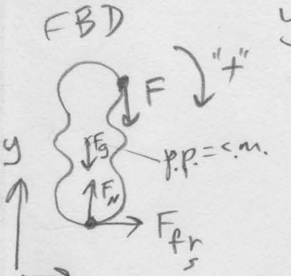
c.)  $|\tau| = \boxed{dF}$ since $r_\perp$ is horizontal, given vertical $\vec{F}$

d.) N2L x: $F_{fr_s} = M' a_{cm_x}$ ①

$M' = M - 4\rho\pi r^2 T$

FBD

y: $-F - M'g + F_N = \cancel{M' a_{cm_y}} \to 0$ ② because $\frac{v^2}{R} = \frac{0^2}{R} = 0$ right as the force is first applied



N2L$_\alpha$: $\cancel{\tau_g}^{\,0} + \cancel{\tau_N}^{\,0} - L F_{fr_s} + dF = I_{y\,new}\,\alpha = I_{y\,new}\frac{a_{cm_x}}{L}$ ③

$a_{cm_x} = L\alpha$

if $\mu_s$ is as small as it can be for rolling w/out slipping:

Then $F_{fr_s} = \mu_{s_{min}} F_N \Rightarrow \mu_{s_{min}} = \frac{F_{fr_s}}{F_N}$ ⑤

① in ③ $\Rightarrow$ $dF - L F_{fr_s} = I_{y\,new}\frac{F_{fr_s}}{M'L} \Rightarrow F_{fr_s}\left(\frac{I_{y\,new}}{M'L} + L\right) = dF$

$$F_{fr_s} = \frac{dF}{\frac{I_{y\,new}}{M'L} + L}$$ ⑥

②: $F_N = F + M'g$ ⑦

⑥ & ⑦ in ⑤ $\Rightarrow$ $\boxed{\mu_{s_{min}} = \frac{dF}{\frac{I_{y\,new}}{M'L} + L}\,\frac{1}{F + M'g}}$

4. a.) $\vec{F}_{net} = \cancel{\vec{F}_1} + \vec{F}_2 + \cancel{\vec{F}_3}$ $\quad (\vec{F}_1 = -\vec{F}_3)$

$= \vec{F}_2 = \boxed{G \dfrac{m_R M_S}{D^2} \hat{i}}$

b.) N3L: $\vec{F}_{\substack{\text{Rocket} \\ \text{on} \\ \text{3 stars}}} = -\vec{F}_{\substack{\text{3 stars} \\ \text{on} \\ \text{Rocket}}} = \boxed{-\dfrac{G m_R M_S}{D^2} \hat{i}}$

for a point particle $M_1$ located a distance $r$ from point particle $M_2$:

c.) $PE_g = -G \dfrac{M_1 M_2}{r}$ this goes to zero as $r \to \infty$ ✓

$PE_{g \text{ of rocket}} = \sum_{i=1}^{3} PE_{g_i} = \boxed{-3 G \dfrac{M_s M_R}{D}}$

d.) C.O.E. $KE_{Ri} = \cancel{PE_{Rf}}^{\,0} - PE_{Ri}$

$\frac{1}{2} \cancel{m_R} v_{Ri}^2 = +3G \dfrac{M_s \cancel{M_R}}{D}$

$\boxed{v_{Ri} = \sqrt{\dfrac{6 G M_s}{D}}}$

5. a.) C.O.$\vec{P}$. $\Rightarrow \vec{P}_{cm_f} = \vec{P}_{cm_i} \Rightarrow v_{cm_f} = \dfrac{Mv_0 + 0 + 0}{M + 2M + 3M} = \boxed{\dfrac{v_0}{6}}$

b.) C.O.$\vec{P}$.:

$$\underbrace{(v_{1fx}, v_{1fy})}_{\vec{v}_{1f}}\cancel{M} + (0.2v_0, -0.1v_0)2\cancel{M} + (0.1v_0, 0.1v_0)3\cancel{M} = (v_0, 0)\cancel{M}$$

$$\vec{v}_{1f} = (v_0 - 0.2v_0 \cdot 2 - 0.1v_0 \cdot 3,\ 0 + 0.1v_0 \cdot 2 - 0.1v_0 \cdot 3)$$

$$= (-0.4 - 0.3,\ 0.2 - 0.3)v_0$$

$$= \boxed{(+0.3,\ -0.1)v_0}$$

c.) elastic $\Rightarrow KE_{tot_f} = KE_{tot_i}$

$$KE_{tot_i} = \frac{1}{2}Mv_0^2$$

$$KE_{tot_f} = \frac{1}{2}M(0.3^2 + 0.1^2)v_0^2 + \frac{1}{2}2M(0.2^2 + 0.1^2)v_0^2 + \frac{1}{2}3M(0.1^2 + 0.1^2)$$

$$= \frac{1}{2}Mv_0^2(.09 + .01 + 2(.04 + .01) + 3(.01 + .01))$$

$$= \frac{1}{2}Mv_0^2(.09 + .01 + .08 + .02 + .03 + .03)$$

$$= \frac{1}{2}Mv_0^2(0.26) < KE_{tot_i}$$

so kinetic energy was lost

$\boxed{\text{not elastic}}$

d.) impulse

$$\vec{J}_{1 on \{2 \& 3\}} = \Delta\vec{P}_{(2\&3)} = -\Delta\vec{P}_1 = \vec{P}_{1i} - \vec{P}_{1f}$$

$$= (v_0, 0)M - (+0.3, -0.1)v_0 M$$

$$= \boxed{Mv_0(.7,\ 0.1)}$$