University of California at Berkeley
Department of Physics
Physics 7A, Spring 2016

Midterm 2
April 5, 2016

You will be given 120 minutes to work this exam. No books are allowed, but you may use a double-sided, handwritten formula sheet no larger than an 8 ½” by 11” sheet of paper. No calculators or other electronics are allowed (wouldn’t help much anyhow…). Your description of the physics involved in a problem is worth significantly more than the final answer by itself. Show all work, be careful with signs, and take particular care to explain what you are doing. Please express your answers using the symbols provided in the problem descriptions or define any new symbols you use, tell us why you’re writing any new equations, and clearly label any drawings that you make. Write your answers in a blue book (or green book), and do not use any extra scratch paper. Please BOX your answers. Good luck!

1) (20 points) Geosynchronous orbit

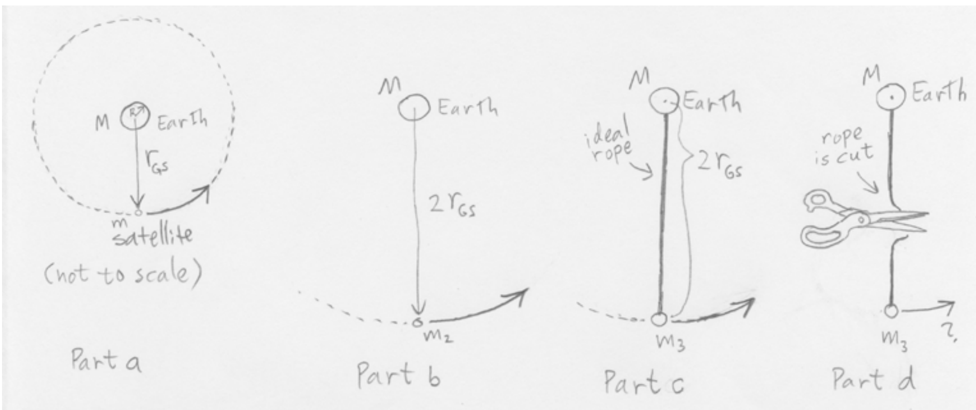
A satellite of mass m rotates around the equator of the Earth at a distance r_{GS} from the center of the Earth, as shown in the first diagram. A geosynchronous orbit is a circular orbit in which the satellite remains directly above the same point on the Earth's surface so that a straight line can always be drawn that connects the center of the Earth, the same point on the Equator, and the satellite. The Earth has mass M, radius R, and period of rotation T (approximately one day).

a) What is the distance r_{GS} from the center of the Earth to the satellite? Express your answer in terms of any combination of m, M, T, and any relevant physical constants.

b) Consider a second object, with mass m_2, orbiting around the Earth in a circular orbit at twice this radius, $r = 2r_{GS}$. What is the orbital period T_2 of this satellite? Express your answer in terms of any combination of m_2, M, T, and any relevant physical constants.

c) Now consider a third object of mass m_3 that is tethered to a point on the equator by an ideal rope of length $L = (2r_{GS} - R)$ so that the object travels in a circular path at a distance $r = 2r_{GS}$ from the center of the Earth and completes one full revolution in time T (1 day). What is the tension F_T in the rope? Express your answer in terms of any combination of m_3, M, T, r_{GS}, and any relevant physical constants.

d) If the rope is cut so that the object is no longer tied to the Earth, then is it travelling fast enough to escape the Earth's gravity so that it will travel arbitrarily far from the Earth? As always, show your work or justify your answer.



2) (30 points) Rolling without slipping

Consider a wheel of radius R and thickness d that is rolling without slipping down a ramp that forms an angle β with respect to the horizontal, as shown in the upper diagram. The wheel has total mass M; it consists of an inner disk of uniform density and mass ½M, surrounded by a very thin hollow cylinder of mass ½M. As always, show your work and/or justify your answer for every part of each problem.

a) What is the moment of inertia of the wheel about its axis of symmetry? Express your answer as a function of M and R.

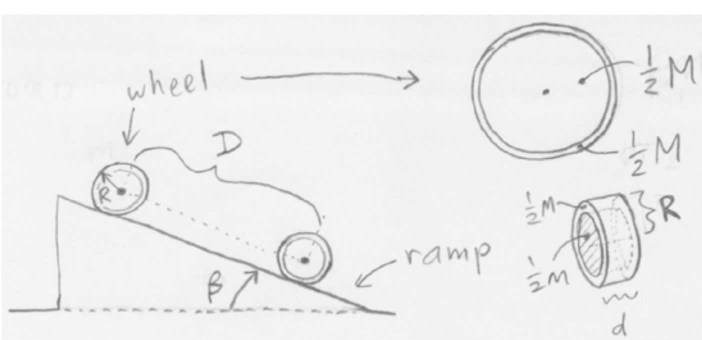
b) How much work is done by the *normal force* on the wheel as it rolls a distance D down the ramp? Express your answer as a function of any combination of D, M, R, β, and any relevant physical constants.

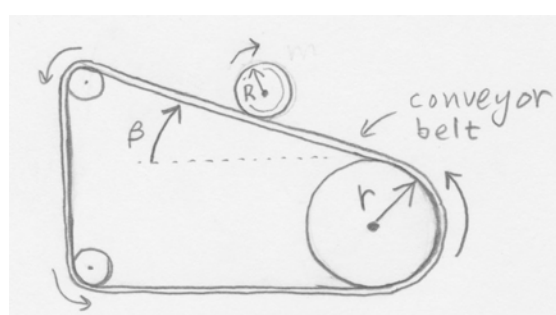
c) What is the magnitude *and direction* of the force of *friction* on the wheel? Express your answer as a function of M, β, and any relevant physical constants.

d) If the wheel starts from rest, what is the angular velocity ω of the wheel after it has rolled a distance D? Express your answer in terms of any combination of D, M, R, β, and any relevant physical constants.

e) Now consider the same wheel as it rolls without slipping on the conveyor belt shown in the lower diagram. If the conveyer belt moves so that the center of mass of the wheel remains in the same place over time, what is the magnitude of the angular acceleration α_{roller} of the large roller at the right of the diagram? The roller turns without slipping on the underside of the belt. Express your answer in terms of M, β, R, r, and any relevant physical constants.

f) How much work is performed on the wheel by the force of friction after a length D of the conveyor belt has passed under the wheel?





3) (25 points) Supremo, the human cannonball

Supremo the carnival performer is shot from a cannon at an initial speed of v_o and at an angle of θ from the horizontal, as shown in the diagram. While he is still ascending, he attempts to grab a large bouncy rubber ball placed on a pedestal. Supremo's mass M is the same as the mass of the ball.

a) If it takes $t_{hit} = v_o\sin(\theta)/(2g)$ after Supremo is shot from the cannon for him to reach the ball, then what are the *horizontal and vertical* components of Supremo's velocity *immediately before* he reaches the ball? Express your answer to this and every later part of this problem in terms of any combination of M, v_o, θ, and g, where $g = +10\text{m/s}^2$.

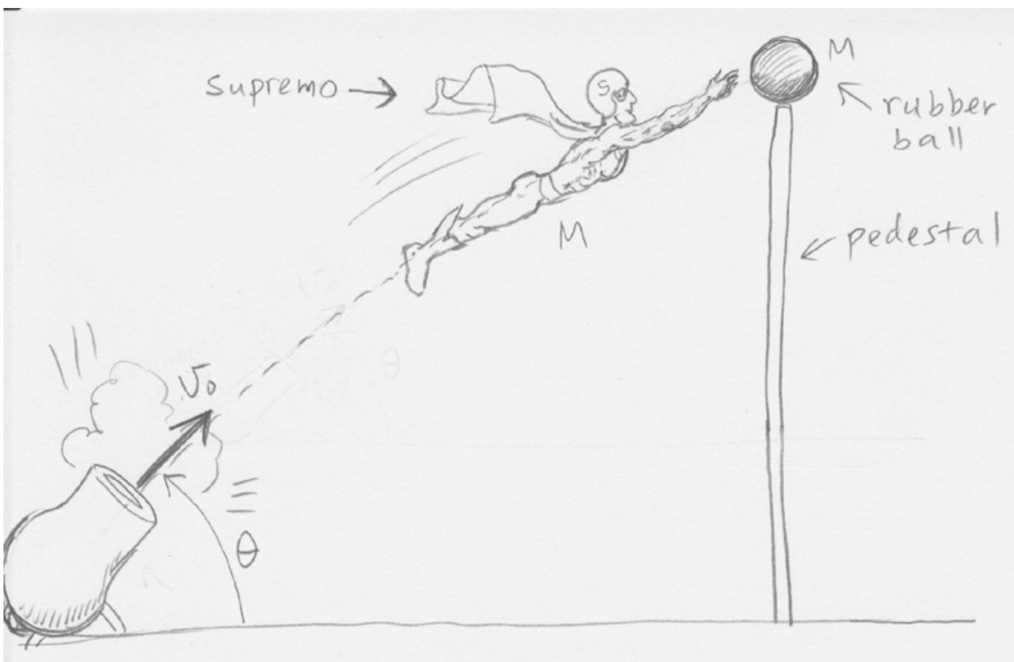
b) On his first attempt, the ball bounces off of Supremo's helmet so that the ball's velocity immediately after the collision points in the same direction as Supremo's velocity immediately before the collision. What is the speed of the *ball* immediately after the impact?

c) Undaunted, Supremo is shot from the cannon a second time. On this second attempt, he gets his hands on the ball, but he can't hold on and the ball moves *horizontally* (to the right as viewed in the diagram) immediately after the collision so that the ball has no vertical component to its velocity. Immediately *after* the collision, if the horizontal component of Supremo's velocity is exactly *one third* of the ball's speed after the collision, then what is the ball's speed immediately after the collision?

d) In this case, what is the *direction and magnitude* of the impulse on the ball during the collision?

e) On Supremo's third attempt, he successfully grabs the ball and holds on to it for the duration of his flight. What is Supremo's speed immediately after the collision?

Cannon →



4) (25 points) Water for thrust

Water shoots out of a powerful water gun at a rate of R, which is in units of kg/s. The water leaves the gun at speed v, with respect to the gun.

a) A boy, who is standing still, aims the water gun horizontally so that water shoots out continuously and splashes against the (vertical) side of a small cart of mass M_C. The water bounces off the cart elastically. What is the acceleration of the cart immediately after the boy starts to spray the cart? The cart starts from rest and it rides on frictionless, massless wheels.

b) Once the cart has sped up to speed V_C, what is the magnitude of the acceleration of the cart due to the water? ($V_C < v$).

c) Now the boy refills his water gun and climbs on board the stationary cart. If the boy's mass is M_B and the water in his gun has mass M_W, then what is the acceleration of the cart with the boy riding on it right at the moment when he begins shooting water horizontally over the side of the cart? The boy is standing still with respect to the cart, and you may assume that the mass of the plastic gun is negligible when it is empty.

d) What is the acceleration of the cart when the gun is almost empty but still shooting if the cart is moving at speed V_C?

e) What is the final speed of the cart after the gun has run out of water?