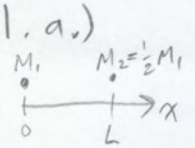
1. a.)

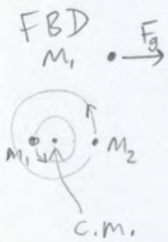


$$R_1 = X_{c.m.} = \frac{M_1 \cancel{x_1}^{\,0} + M_2 x_2}{M_1+M_2} = \frac{\frac{1}{2}M_1 L}{M_1 + \frac{1}{2}M_1} = \frac{\frac{1}{2}L}{3/2} = \boxed{\frac{L}{3}} \quad -(1)$$

b.) circular orbit $\Rightarrow$ uniform circular motion, for either star.

N2L x: M_1

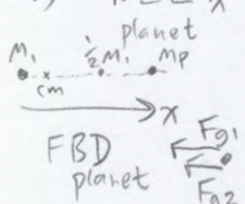
$$G\frac{M_1 M_2}{L^2} = M_1 a_{1x} = M_1 \frac{v_1^2}{r_1}$$



$$v_1 = \sqrt{G\frac{M_2 r_1}{L^2}} \underset{(1)}{=} \sqrt{G\frac{\frac{1}{2}M_1}{3L}} = \sqrt{\frac{GM_1}{6L}}$$

$$T = \frac{2\pi r_1}{v_1} = \frac{2\pi L/3}{\sqrt{\frac{GM_1}{6L}}} = \boxed{2\pi\sqrt{\frac{2L^3}{3GM_1}}}$$

c.) N2L x:



$$-G\frac{M_1 m_p}{(R_1+R_p)^2} - G\frac{M_2 m_p}{(R_p-R_2)^2} = m_p a_{px} \quad -(2)$$

$$R_2 = L - R_1 \underset{(1)}{=} \frac{2L}{3} \quad -(3)$$

uniform circular motion: $a_{px} = -\frac{v_p^2}{R_p} = -\frac{\left(\frac{2\pi R_p}{T}\right)^2}{R_p} = -\frac{\cancel{4\pi^2} R_p}{\cancel{4\pi^2}\frac{2L^3}{3GM_1}} = -\frac{3GM_1R_p}{2L^3}$ (4)

$$(2) \,\&\, (3) \,\&\, (4) \Rightarrow -\cancel{G}\frac{M_1 \cancel{m_p}}{(R_1+R_p)^2} - \cancel{G}\frac{M_2 \cancel{m_p}}{(R_p-L+R_1)^2} = -\frac{\cancel{m_p}\, 3\cancel{G}M_1R_p}{2L^3}$$

$$\frac{\cancel{M_1}}{(R_1+R_p)^2} + \frac{\frac{1}{2}\cancel{M_1}}{(R_p-L+R_1)^2} = \frac{3\cancel{M_1}R_p}{2L^3}$$

$$(1) \Rightarrow \boxed{\frac{2}{(L/3+R_p)^2} + \frac{1}{(R_p-2L/3)^2} = \frac{3R_p}{L^3}}$$

d.) $K_p = \frac{1}{2}m_p v_2^2 = \frac{1}{2}m_p\left(\frac{2\pi R_p}{T}\right)^2 = \frac{1}{2}m_p\frac{\cancel{4\pi^2}R_p^2}{\cancel{4\pi^2}\frac{2L^3}{3GM_1}} = \boxed{\frac{3Gm_pM_1R_p^2}{4L^3}}$

e.) cons. of E.: $K_{pi} + U_{gpi} = \cancel{K_{pf}}^{\,0} + \cancel{U_{gpf}}^{\,0}$

$$K_{pi} = -U_{gpi} = \boxed{G\frac{m_pM_1}{R_p+L/3} + G\frac{m_p\frac{1}{2}M_1}{R_p-\frac{2}{3}L}}$$

this is the smallest value for the planets Kinetic Energy required to leave the solar system.

2a.) cons. of E.: set $U_g = 0$ for cm $= r$ above ground

$$(\cancel{U_{gf}}^{\,0} + K_{cm,f} + K_{rot,f})_f - (U_{gi} + \cancel{K_{cm,i}}^{\,0} + \cancel{K_{rot,i}}^{\,0})_i = \cancel{W_{ext}}^{\,0}$$

(rolling w/o slipping)

rolling w/o slip:

$v_{cm} = r\omega$ — (1)

$$\tfrac{1}{2}Mv_{cm,f}^2 + \tfrac{1}{2}I_{cm}\omega_f^2 - Mg(h-r) = 0 \quad (3)$$

solid cylinder:

$$I_{cm} = \int_0^r \underbrace{2\pi r\,dr\,\sigma}_{dm} r^2$$

$$I_{cm} = \frac{2\pi\sigma r^4}{4} = \tfrac{1}{2}Mr^2 \quad (2), \qquad \sigma = \frac{M}{\pi r^2}$$

(1) & (2) in (3): $\tfrac{1}{2}\cancel{M}v_{cm,f}^2 + \tfrac{1}{2}\tfrac{1}{2}\cancel{M}v_{cm,f}^2 = \cancel{M}g(h-r)$

$$\boxed{v_{cm,f} = \sqrt{\tfrac{4}{3}g(h-r)}} \quad (4)$$

b.) C.O.E

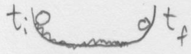
i: at rest at left

f: max height on right

$$(U_{gf} + \cancel{K_{cm,f}}^{\,0} + K_{rot,f})_f - (U_{gi} + \cancel{K_{cm,i}}^{\,0} + \cancel{K_{rot,i}}^{\,0}) = \cancel{W_{ext}}^{\,0}$$

roll w/o slip or no $F_{fr,k}$ at every point

$$\cancel{M}gh_f + \tfrac{1}{2}\tfrac{1}{2}\cancel{M}v_{cm,bottom}^2 - \cancel{M}gh = 0 \quad (5)$$



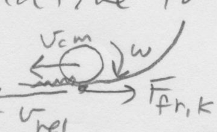
t_i t_f

(4) in (5) $\Rightarrow$ $h_f = h - \frac{1}{4\cancel{g}}\frac{4}{3}\cancel{g}(h-r) = h - \tfrac{1}{3}h + \tfrac{1}{3}r = \boxed{\tfrac{2}{3}h + \tfrac{1}{3}r}$

c.) C.O.E. from upper right down to flat section & the fact that there is no torque on wheel about cm. since there is no friction as it slides back down: ω is still $\frac{v_{cm}}{r} \underset{(4)}{=} \frac{1}{r}\sqrt{\tfrac{4}{3}g(h-r)}$ clockwise

so velocity of bottom of wheel relative to ground is

$$v_{rel.} = 2v_{cm} = 2\sqrt{\tfrac{4}{3}g(h-r)}$$



instant. Power $= P = \vec{v}_{rel} \cdot \vec{F}_{fr,k} = -2\sqrt{\tfrac{4}{3}g(h-r)}\,F_N\mu_k = \boxed{-4\mu_k Mg^{3/2}\sqrt{\frac{h-r}{3}}}$

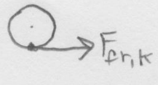
$N2L_y$ wheel: $-Mg = F_N$

d.) $N2L_\alpha$ wheel: $\tau_{net,cm} = I_{cm}\alpha \Rightarrow \alpha = \frac{\tau_{net,cm}}{I_{cm}} = \frac{-F_{fr,k}r}{I_{cm}} = \frac{-\mu_k \cancel{M}g\cancel{r}}{\tfrac{1}{2}\cancel{M}r^{\cancel{2}}} = \boxed{\frac{-2\mu_k g}{r}}$ (6)

$\curvearrowright$ "+" p.p. = c.m.

(<0 ⇒ counterclockwise)

FBD$_\alpha$ wheel



e.) (time to stop moving to the left assuming $v_{cm} \to 0$ befor $\omega \to 0$) $= t_{cm\,stop}$

$N2L_x$: $F_{fr,k,x} = Ma_{cm,x} \Rightarrow a_{cm,x} = \mu_k g$ const. a_x: $t_{cm,stop} = \frac{\Delta v_{cm,x}}{a_{cm,x}} = \frac{\sqrt{\tfrac{4}{3}g(h-r)}}{\mu_k g}$

(time to stop spinning assuming $\omega \to 0$ before $v_{cm} \to 0$) $= t_{\omega\,stop}$

const α & (6) $\Rightarrow$ $t_{\omega\,stop} = \frac{\Delta\omega}{\alpha} = \frac{\sqrt{\tfrac{4}{3}g(h-r)}/\cancel{r}}{2\mu_k g/\cancel{r}} = \tfrac{1}{2}t_{cm\,stop} \Rightarrow$ wheel does $\boxed{\text{NOT}}$ reverse direction another way to solve 2e.):

$$v_{\substack{\text{bottom}\\ \text{of wheel}\\ x}}(t) = v_{cm,x}(t) - r\omega(t) = v_{cm,x,0} + a_{cm,x}t - r(\omega_0 + \alpha t)$$

wheel stops sliding when $v_{bottom}(t^*) = 0$

so ⑦: $(a_{cm,x} - r\alpha)t^* = r\omega_0 - v_{cm,x,0}$

$$\left[\mu_k g - \cancel{r}\left(-\frac{2\mu_k g}{\cancel{r}}\right)\right]t^* = \frac{\cancel{r}\sqrt{\frac{4}{3}g(h-r)}}{\cancel{r}} - \left(-\sqrt{\frac{4}{3}g(h-r)}\right)$$

$$3\mu_k g\, t^* = 2\sqrt{\frac{4}{3}g(h-r)}$$

$$t^* = \frac{2}{3\mu_k g}\sqrt{\frac{4}{3}g(h-r)} = \frac{2}{3\mu_k}\sqrt{\frac{4}{3}\frac{(h-r)}{g}}$$

which way is cm of wheel moving at this time?

$$v_{cm,x}(t^*) = v_{cm,x,0} + a_{cm,x}t^*$$

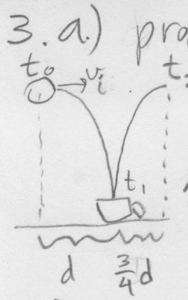
$$= -\sqrt{\frac{4}{3}g(h-r)} + \cancel{\mu_k} g \frac{2}{3\cancel{\mu_k}}\sqrt{\frac{4}{3}\frac{(h-r)}{g}}$$

$$= \left(-1 + \frac{2}{3}\right)\sqrt{\frac{4}{3}g(h-r)}$$

$$= -\frac{1}{3}\sqrt{\frac{4}{3}g(h-r)} < 0 \Rightarrow$$

c.m. moving to left when wheel stops sliding so it does <u>NOT</u> reverse direction due to backspin

3. a) projectile motion: $\vec{a}_x = 0$ for all times except during bounce.



velocity at t_0 & t_2 is purely horizontal. During bounce, the ball experiences a torque about c.m., so its ω changes, but the vertical component of $\vec{v}_{cm}$ has same magnitude before & after.

$$v(t_2) = \frac{v_i \frac{3}{4} d}{d} = \boxed{\frac{3v_i}{4}}$$

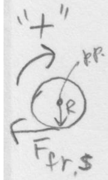
b.) C.O.P$_x$: ball & truck

$$\left(m_b v_{bf,x} + m_t v_{tf,x}\right) - \left(m_b v_{bi,x} + m_t \cancel{v_{ti,x}}^{\,0}\right) = \cancel{J_{b+t,x}}^{\,0}$$

$$v_{tf,x} = \frac{m_b}{m_t}\left(v_{bi,x} - v_{bf,x}\right) = \frac{m_b}{m_t} v_i \left(1 - \frac{\frac{3}{4}\cancel{d}}{\cancel{d}}\right)$$

$$J_{truck,x} = \Delta p_x = m_t\left(v_{tf,x} - 0\right) = m_b v_i \left(1 - \tfrac{3}{4}\right) = \boxed{\frac{1}{4} m_b v_i} \quad \text{— (1)}$$

c.)
$$\bar{F}_{\substack{fr,s,x \\ \text{on ball}}} = \frac{J_{ball,x}}{\Delta t} \underbrace{=}_{\substack{J_{ext,x} = 0 \\ \text{ball \& truck}}} -\frac{J_{truck,x}}{\Delta t} = -\frac{m_b v_i}{4\Delta t}$$



$$\bar{\tau}_{cm} = +R\bar{F}_{fr,s} = \boxed{\frac{m_b v_i R}{4\Delta t}}$$ clockwise

d.) N2Lα ball:
$$\tau_{\substack{net \\ cm}} = I_{cm}\alpha \Rightarrow \bar{\alpha} = \frac{\bar{\tau}_{cm}}{I_{cm}}$$

$$\omega_f = \cancel{\omega_i}^{\,0} + \bar{\alpha}\Delta t = \frac{\bar{\tau}_{cm}}{I_{cm}}\Delta t = \frac{m_b v_i R \cancel{\Delta t}}{4\cancel{\Delta t} I_{cm}} = \boxed{\frac{m_b v_i R}{4 I_{cm}}} \quad \text{— (3)}$$

e.) elastic collision: C.O.E. : ball & truck $t_0 \to t_2$

$U_{gb2} = U_{gb0}$

$$\left(\cancel{U_{gb2}} + K_{cm,b,2} + K_{rot,b,2} + K_{t,2}\right)_{t_2} - \left(\cancel{U_{gb0}} + K_{\substack{cm \\ b0}} + \cancel{K_{\substack{rot \\ b0}}}^{\,0} + \cancel{K_{t0}}^{\,0}\right)_{t_0} = \cancel{W_{ext}}^{\,0}$$

(The truck did do work on the ball, but $\Delta KE = 0$ for ball bouncing elastically off of truck on ground since $M_{earth} \approx \infty$)

$$\frac{1}{2}M_b\left(\frac{3v_i}{4}\right)^2 + \frac{1}{2}I_{cm}\omega_2^2 + \frac{1}{2}M_t v_{t2}^2 - \frac{1}{2}M_b v_i^2 = 0 \quad \text{— (2)}$$

(1) $\Rightarrow$ $v_{t,2} = \frac{P_{t,2}}{m_t} = \frac{m_b v_i}{4 m_t}$ in (2) $\Rightarrow$
$$\cancel{\tfrac{1}{2}}M_b \frac{9v_i^2}{16} + \cancel{\tfrac{1}{2}}I_{cm}\omega_2^2 + \cancel{\tfrac{1}{2}}M_b\frac{M_b v_i^2}{M_t 16} - \cancel{\tfrac{1}{2}}M_b v_i^2 = 0$$

$$m_b v_i^2\left(\frac{9}{16} + \frac{M_b}{M_t}\frac{1}{16} - \frac{16}{16}\right) = -I_{cm}\omega_2^2$$

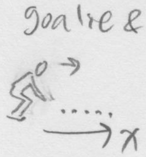
$$\omega_2^2 = \frac{M_b v_i^2}{I_{cm}}\left(\frac{16 - 9 - \frac{M_b}{M_t}}{16}\right) = \frac{v_i^2}{16}\frac{M_b}{I_{cm}}\left(7 - \frac{M_b}{M_t}\right) \quad \text{— (4)}$$

(3) & (4) $\Rightarrow$
$$\left(\frac{m_b v_i R}{4 I_m}\right)^2 = \frac{v_i^2 M_b}{16\, I_{cm}}\left(7 - \frac{M_b}{M_t}\right)$$

$$m_b R^2 = I_{cm}\left(7 - \frac{M_b}{M_t}\right) \Rightarrow \boxed{I_{cm} = \frac{m_b R^2}{7 - M_b/M_t}}$$

4.a.) totally inelastic collisions between pucks & goalie.

C.O.P$_x$ goalie & pucks: $(Nm_p + M_G)v_f = M_G v_G \Rightarrow v_f = \frac{M_G}{Nm_p+M_G} v_G$



$$|\bar{a}_{G,x}| = \left|\frac{\Delta v_{G,x}}{\Delta t}\right| = \left|\left(\frac{M_G}{Nm_p+M_G}-1\right)\frac{v_G}{\Delta t}\right| = \boxed{\frac{-Nm_p}{Nm_p+M_G}\,\frac{v_G r}{N}}$$

b.) $$J_G = \Delta p_G = M_G \Delta v_G = \left|M_G\left(\frac{M_G}{Nm_p+M_G}-1\right)v_G\right| = \boxed{M_G v_G\left(\frac{-Nm_p}{Nm_p+M_G}\right)}$$

c.) C.O.P$_x$ $\quad Nm_p v = (Nm_p+M_G)v_{Gf} \Rightarrow v_{Gf} = \frac{Nm_p v}{Nm_p+M_G}$

$v_{Gi}=0 \Rightarrow \bar{a}_G = \frac{\Delta v_G}{\Delta t} = \boxed{\frac{Nm_p v r}{(Nm_p+M_G)N}}$

d.) variable mass: $M\frac{dv_x}{dt} = \cancelto{0}{F_{ext,x}} + v_{x\,rel}\frac{dM}{dt}$ — (1)

$v_{x\,rel}$ is velocity of pucks w/r to machine $= v_p$ (const.)

(1): $\int_0^{v_{xf}} dv_x = v_p \int_{M_M}^{M_M+Nm_p}\frac{dM}{M}$

so $v_{M,x,f} = v_p \ln(M)\Big|_{M_M}^{M_M+Nm_p} = \boxed{v_p \ln\left(\frac{M_M+Nm_p}{M_M}\right)}$ — (2)

e.) C.O.P$_x$ Goalie & Machine & pucks: $(P_{G,xf}+P_{pxf}+P_{Mxf})_f - (\cancelto{0}{P_{tot\,i}}) = \cancelto{0}{J_x}$

(2) $\Rightarrow P_{Mxf} = M_M v_p \ln\left(\frac{M_M+Nm_p}{M_M}\right)$

so $v_{Gf} = \frac{|P_{Mxf}|}{M_G+Nm_p} = \boxed{\frac{M_M v_p}{M_G+Nm_p}\ln\left(\frac{M_M+Nm_p}{M_M}\right)}$