University of California at Berkeley
Department of Physics
Physics 7A, Fall 2017

Midterm 2
October 31, 2017

You will be given 120 minutes to work this exam. No books are allowed, but you may use a double-sided, handwritten formula sheet no larger than an 8 ½" by 11" sheet of paper. No calculators or other electronics are allowed (wouldn't help much anyhow…). Your description of the physics involved in a problem is worth significantly more than the final answer by itself. Show all work, be careful with signs, and take particular care to explain what you are doing. Please express your answers using the symbols provided in the problem descriptions or define any new symbols you use, tell us why you're writing any new equations, and clearly label any drawings that you make. Write your answers in a blue book (or green book), and do not use any extra scratch paper. Please $\boxed{\text{BOX}}$ your answers. Good luck!

1) (25 points) Two stars and a planet

Consider a pair of stars that form a binary solar system. One star has mass M_1 and the second has mass ½ M_1. They are separated from one another by a distance L, and they are each in circular orbits about the center of mass of the two stars.

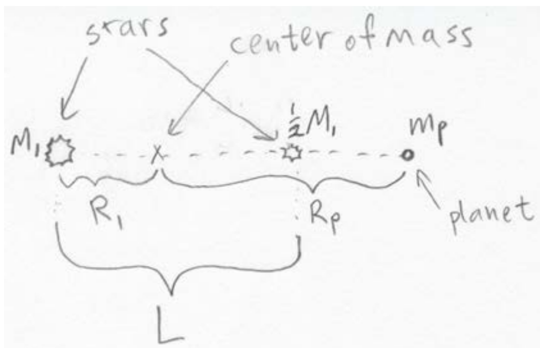
a) What is the distance R_1 from the first star to the center of mass of the two stars? As always, show your work and/or justify your answer.

b) What is the orbital period T of either of these two stars? Express your answer in terms of any combination of M_1, L, and any relevant physical constants. If you could not find R_1 in part a), you may use the symbol R_1 in your answers for the rest of this problem.

c) Now consider a planet located along the line through the two stars, and beyond the smaller of the two so that its distance R_p from the center of mass of the two stars is greater than the second star's orbital radius. If this planet travels in a circular orbit, and if its orbital period is the **same** as that of either of the two stars, then derive an equation that must be satisfied by R_p (This equation doesn't allow for a closed-form solution for R_p, so do not try to isolate this variable. This type of special location for a small co-rotating third body is called a "Lagrange point."). You may assume that $m_p \ll M_1$.

d) What is the translational (*i.e.*, linear) kinetic energy of the planet? Express your answer in terms of any combination of R_p, m_p, M_1, L, and any relevant physical constants.

e) What would the planet's kinetic energy have to be for it to be able to leave this binary solar system from its current location?



2) (25 points) Rolling wheel

Consider a wheel of radius r and mass M that is rolling along a surface consisting of round segments at the left and right, each with radius R, and a flat horizontal section connecting them, as shown in the diagram. The wheel is a solid cylinder of uniform density. The surface on the left and along the flat section at the bottom exert frictional forces on the wheel, but the curved section on the right is frictionless. The wheel starts from rest at the left side of the surface such that its center is at a height h above the flat section. The wheel rolls without slipping all the way to the right side of the flat section. Do ***not*** assume that $r \ll R$.

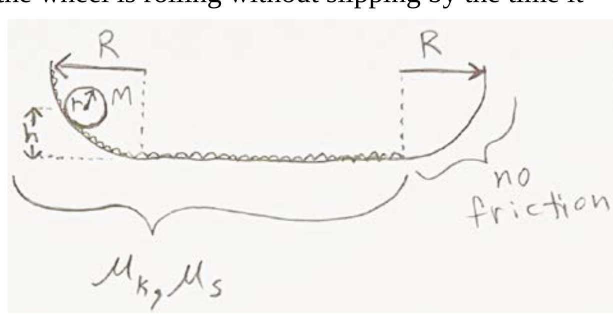
a) What is the speed of the center of mass of the wheel once it reaches the flat section at the bottom? Express your answer as a function of M, h, r, R, and any relevant physical constants.

b) How high does the center of mass of the wheel get when it goes up the frictionless slope at the right? Express your answer as a function of any combination of M, h, r, R, and any relevant physical constants.

c) What is the power delivered to the wheel from kinetic friction immediately after the wheel starts to slide from the right to the left of the horizontal section after sliding down the slope on the right? Express your answer as a function of the coefficient of kinetic friction μ_k, r, h, M, and any relevant physical constants; and take care with signs.

d) What is the angular acceleration of the wheel as it slides from the right to the left along the horizontal section after coming back down the slope on the right? Indicate whether it is **clockwise** or **counterclockwise** as viewed in the diagram. Express your answer as a function of μ_k, r, and any relevant physical constants.

e) As the wheel slides along the horizontal section, will the backspin of the wheel cause it to reverse direction? Assume that the width D of the horizontal section is large enough so that the wheel is rolling without slipping by the time it reaches one end or the other. As always, show your work and/or justify your answer.



3) (25 points) Bouncy ball and a toy truck

A boy throws a rubber ball of radius R and mass M_B from a height h above the ground so that it initially has a horizontal velocity of magnitude v_i, as shown in the diagram. The ball then bounces elastically off of the top of an initially stationary toy truck resting on the wooden floor. The truck's mass is M_T, and it is located a horizontal distance d from the point where the ball was thrown. The ball reaches its highest point after the bounce at a horizontal distance ¾ d from the truck. Assume the truck can roll along the floor without friction, but note that there are static frictional forces between the ball and the top of the truck.

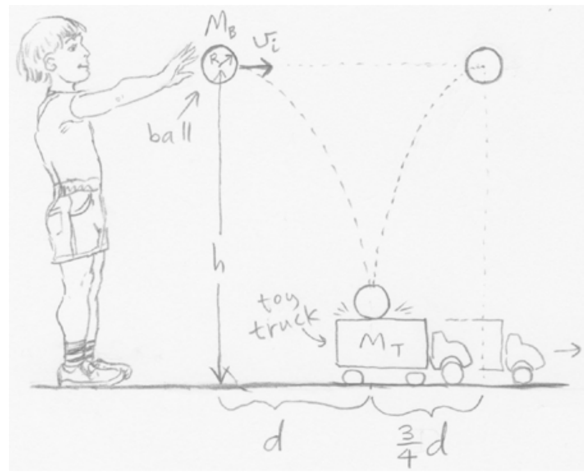
a) What is the speed of the ball's center of mass once it bounces back up to its initial height h above the floor?

b) What is the magnitude of the impulse to the truck during the time the ball bounced? (Hint: the impulse is in the horizontal direction).

c) If Δt is the brief duration of the bounce, then what was the average torque on the ball about its center of mass during the bounce? Indicate whether this was **clockwise** or **counter-clockwise**. You may assume that the ball's radius remains equal to its full value of R during the bounce. Express your answer in terms of M_B, v_i, R, and Δt.

d) If the ball's moment of inertia about its center of mass is I_{cm}, then what is the angular velocity of the ball after the bounce? Assume that the ball was not spinning before it bounced. Express your answer in terms of M_B, v_i, R, and I_{cm}.

e) What is I_{cm}, the moment of inertia of the ball about its center of mass, expressed in terms of M_B, M_T, and R? (Hint: the ball may not have uniform mass density, so you cannot find this by direct calculation using the definition for the moment of inertia for a known mass distribution; consider a second way of computing the angular frequency of the ball after the bounce.)



4) (25 points) Hockey practice

During a Cal hockey team practice session, the goalie of mass M_G skates on the ice while he picks up hockey pucks at a rate of r pucks per second. The pucks are all initially sitting on the ice at rest, and each puck has mass m_p. The goalie initially has a speed of v_G (in m/s) and he travels in a straight line and experiences no friction with the ice.

a) What is the magnitude of the average acceleration of the goalie during the time it takes him to pick up N pucks? Express your answer in terms of M_G, m_p, v_G, r, and N.

b) What was the magnitude of the impulse experienced by the goalie during the time that the picked up these N pucks?

c) Now the goalie comes to a stop, puts down the pucks he collected, and catches new pucks fired at him at speed v at a rate of r pucks/s by a puck shooting machine. As before, the goalie is on ice skates and can slide without friction. What is the average acceleration of the goalie during the time he catches a total of N pucks? Assume he holds on to all pucks he catches during this process.

d) Now the puck shooting machine comes loose so that it can slide without friction on the ice. Starting from rest, the machine fires off another N pucks at a speed v_P with respect to the **machine**. The mass of the machine is M_M, which does **not** include the mass of the N pucks. What is the final speed of the machine after all the pucks have been fired? You may approximate the ejection of pucks as a continuous flow of matter from the machine.

e) If the goalie was initially holding still and not holding any pucks when the machine came loose allowing it to slide, then what is his final speed after he catches N pucks shot at him by the sliding machine? Again assume that the goalie can slide without friction and that he holds on to all of the pucks he catches.