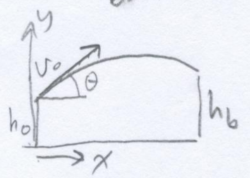
DeWeese MT1 Physics 7A spring 2014 Lec sec. 2 & 3

1. a.) FBD_{ball}: $\downarrow mg$ $\uparrow y$ N2L_{ball} y: $-\cancel{m}g = \cancel{m}a_y$ $\boxed{a_y = -g}$ $g = +10\,m/s^2$

so magnitude of $\vec{a}$ is g

b.) 

projectile motion: const. $\vec{a}$:

x: $x = v_0 \cos(\theta) t \Rightarrow d_0 = v_0 \cos(\theta)\, t_f \Rightarrow t_f = \dfrac{d}{v_0 \cos\theta}$ — ①

y: $y = h_0 + v_0 \sin(\theta) t + \frac{1}{2} a_y t^2$

$$① \Rightarrow h_b = h_0 + \frac{\cancel{v_0}\sin\theta}{\cancel{v_0}\cos\theta} d_0 - \frac{1}{2} g \frac{d_0^2}{v_0^2 \cos^2\theta}$$

$$-h_b + h_0 + d_0 \tan\theta = + \frac{g d^2}{2\cos^2\theta} \frac{1}{v_0^2}$$

$$v_0 = \sqrt{\frac{g d_0^2}{(h_0 - h_b + d_0 \tan\theta) 2\cos^2\theta}} = \boxed{\frac{d_0}{\cos\theta}\sqrt{\frac{g}{2(h_0 - h_b + d_0 \tan\theta)}}}$$

c.) smallest value of speed occurs when $v_y = 0$

$v_x = \text{const} = v_0 \cos\theta$ so $v_{min} = \boxed{v_0 \cos\theta}$

d.) $x_s = \cancel{x_{s0}}^{\,0} + \cancel{v_{0sx}}^{\,0} t + \frac{1}{2} a_{sx} t_f^2$

$$d_s = \frac{1}{2} a_{sx} \frac{d^2}{v_0^2 \cos^2\theta} \Rightarrow \boxed{a_{sx} = \frac{2 d_s v_0^2 \cos^2\theta}{d_0^2}}$$

2. a.) $[A] = [v] = \boxed{m/s}$

$[B] = \left[\frac{v}{t^2}\right] = \frac{[v]}{[t]^2} = \frac{m/s}{s^2} = \boxed{m/s^3}$

b.) $a = \frac{dv}{dt} = \cancel{\frac{dA}{dt}}^{\,0} + \frac{d}{dt} B t^2 = \boxed{2Bt}$ (not constant a ...)

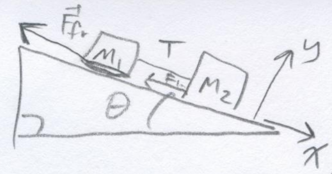
c.) $\int_0^x dx = \int_0^t dt (A + Bt^2)$

$$\boxed{x = At + \frac{1}{3} B t^3}$$

d.) $A = 80\,m/s$, $B = 6\,m/s^3$, $t = 2s$

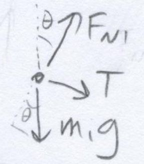
$$x(t=2s) = 80\,m/s \cdot 2s + \frac{1}{3} \cdot 6\,m/s^3 \cdot (2s)^3$$
$$= 160\,m + 16\,m$$
$$= \boxed{176\,m}$$

3. a.) sliding down ramp $\Rightarrow$ $\vec{F}_{kfr1}$ is $\boxed{\text{up ramp}}$:



kinetic friction: $F_{kfr1} = \mu_{1k} F_{N1}$

FBD m_1:

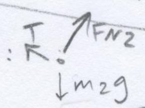


N2L$_{m_1}$ y: $F_{N1} - m_1 g\cos\theta = m a_y$

$F_{N1} = m_1 g\cos\theta$

$$\boxed{F_{kfr1} = \mu_{1k} m_1 g\cos\theta}$$

FBD m_2:



b.) $T > 0 \Rightarrow a_{1x} = a_{2x}$ — ①

N2L$_{m_1}$ x: $-F_{frk1} + T + m_1 g\sin\theta = m_1 a_{1x} \Rightarrow a_{1x} = \dfrac{-F_{frk1} + T + m_1 g\sin\theta}{m_1}$ — ②

N2L$_{m_2}$ x: $-F_{kfr2} - T + m_2 g\sin\theta = m_2 a_{2x} \underset{T>0 \Rightarrow a_{1x}=a_{2x}}{=} m_2 a_{1x}$ — ③

② & ③ $\Rightarrow$ $-F_{kfr2} - T + \cancel{m_2 g\sin\theta} = \dfrac{m_2}{m_1}(\cancel{m_1 g\sin\theta} - F_{frk1} + T)$ — ④

upper limit for μ_{k2} occurs right as $T \to 0$

T=0 in ④: $+\mu_{2k} m_2 \cancel{g\cos\theta} < +\dfrac{m_2}{\cancel{m_1}} \mu_{1k} \cancel{m_1} \cancel{g\cos\theta}$

$$\boxed{\mu_{2k} < \mu_{1k}}$$

c.) $\mu_{2k} = \frac{1}{2}\mu_{1k}$

② $\Rightarrow T = -m_1 g\sin\theta + m_1 a_{1x} + F_{kfr1}$ in ③ $\Rightarrow -F_{kfr2} + m_1 g\sin\theta - m_1 a_{1x} - F_{kfr1} + m_2 g\sin\theta = m_2 a_{1x}$

$$a_{1x}(m_1 + m_2) = (m_1 + m_2) g\sin\theta - F_{kfr1} - F_{kfr2}$$

$$a_{1x} = g\sin\theta - \frac{F_{kfr1} + F_{kfr2}}{m_1 + m_2}$$

$$a_{1x} = g\sin\theta - \frac{(\mu_{1k} m_1 + \mu_{2k} m_2) g\cos\theta}{m_1 + m_2} \quad ⑤$$

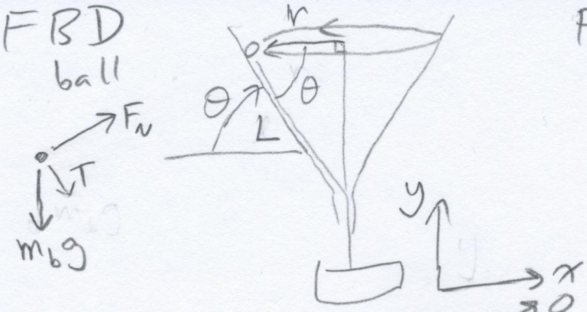
$m_2 = 2m_1$, $\mu_{k2} = \frac{1}{2}\mu_{k1}$ $\Rightarrow$ $a_{1x} = g\sin\theta - \dfrac{(1 + \frac{2}{2})\mu_{1k} \cancel{m_1} g\cos\theta}{3\cancel{m_1}} = \boxed{g\sin\theta - \frac{2}{3}\mu_{1k} g\cos\theta}$

d.) ② $T = F_{frk1} - m_1 g\sin\theta + m_1 a_{1x}$

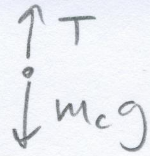
⑤ $\Rightarrow T = \mu_{1k} m_1 g\cos\theta - m_1 \cancel{g\sin\theta} + m_1(\cancel{g\sin\theta} - \frac{2}{3}\mu_{1k} g\cos\theta)$

$$\boxed{T = \frac{1}{3}\mu_{1k} m_1 g\cos\theta}$$

④ a.) FBD ball



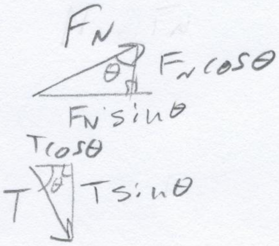
FBD cork



b.) N2L cork y: $T - m_c g = m_c \cancel{a_y}^{0} \Rightarrow \boxed{T = m_c g}$

c.) uniform circular motion

$$a = \frac{v^2}{r} = \boxed{\frac{v^2}{L\cos\theta}}$$



d.) N2L ball x: $F_N\sin\theta + T\cos\theta = \frac{m_b v^2}{L\cos\theta}$ — (1)

y: $F_N\cos\theta - m_b g - T\sin\theta = \cancel{m_b a_y}^{0}$ — (2)

(2): $F_N = \frac{m_b g + T\sin\theta}{\cos\theta}$ in (1) $\Rightarrow (m_b g + T\sin\theta)\frac{\sin\theta}{\cos\theta} + T\cos\theta = \frac{m_b v^2}{L\cos\theta}$

$$T\left(\frac{\sin^2\theta}{\cos\theta} + \cos\theta\right) = \frac{m_b v^2}{L\cos\theta} - m_b g\frac{\sin\theta}{\cos\theta}$$

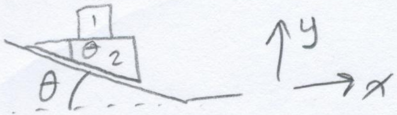
$$T(\underbrace{\sin^2\theta + \cos^2\theta}_{1}) = \frac{m_b v^2}{L} - m_b g\sin\theta$$

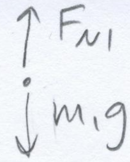
$$T = m_b\left(v^2/L - g\sin\theta\right)$$

$T = m_c g \Rightarrow \quad m_c g L = m_b(v^2 - gL\sin\theta$

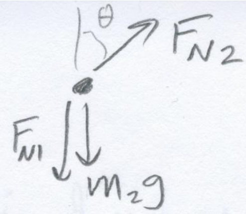
$$\boxed{m_b = \frac{m_c\, gL}{v^2 - gL\sin\theta}}$$

(5) a.) FBD$_{m_1}$:

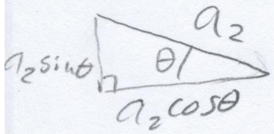




FBD$_{m_2}$:



b.) $\vec{a}_2 = (a_2\cos\theta, -a_2\sin\theta) = (a_{2x}, a_{2y})$



$$\frac{a_{2y}}{a_{2x}} = -\tan\theta \Rightarrow \boxed{a_{2y} = -\tan\theta\, a_{2x}} \quad (1)$$

c.) N2L$_{m_1}$ y: $F_{N1} - m_1 g = m_1 a_{1y}$ (2)

$$\text{N2L}_{m_2}\begin{cases} y: -F_{N1} - m_2 g + F_{N2}\cos\theta = m_2 a_{2y} \overset{a_{1y}=a_{2y}}{=} m_2 a_{1y} \quad (3) \\ x: F_{N2}\sin\theta = m_2 a_{2x} \underset{(1)}{=} -\dfrac{m_2 a_{2y}}{\tan\theta} \underset{a_{1y}=a_{2y}}{=} -\dfrac{m_2 a_{1y}}{\tan\theta} \quad (4) \end{cases}$$

(2) in (3) $\Rightarrow -m_1 g - m_1 a_{1y} - m_2 g + F_{N2}\cos\theta = m_2 a_{1y}$ (5)

(4) in (5) $\Rightarrow -m_1 g - m_1 a_{1y} - m_2 g - \dfrac{m_2 a_{1y}\cos\theta}{\tan\theta \sin\theta} = m_2 a_{1y}$

$$a_{1y}\left(-m_1 - m_2\frac{\cos^2\theta}{\sin^2\theta} - m_2\right) = m_1 g + m_2 g$$

$$\boxed{a_{1y} = -g\frac{m_1 + m_2}{m_1 + m_2\left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right)}} = -g\frac{(m_1 + m_2)\sin^2\theta}{m_1\sin^2\theta + m_2}$$

so The magnitude is $|a_{1y}|$ ✓

d.) no forces act in the horizontal direction on m_1

so by N2L: x: $0 = m_1 a_{1x} \Rightarrow \boxed{a_{1x} = 0}$