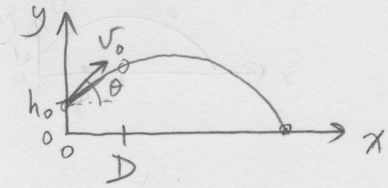
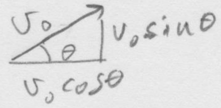
Mike DeWeese 7A Midterm 1 Fall 2017

Prob. 1a) constant $\vec{a}$: $g = +10\,\text{m/s}^2 = -a_y$





$$x = \cancel{x_0}^{\,0} + v_{0x} t_n + \tfrac{1}{2} \cancel{a_x}^{\,0} t_n^2$$

$$D = v_0 \cos\theta \, t_n \Rightarrow \boxed{t_n = \frac{D}{v_0 \cos\theta}} \quad (1)$$

b.) const. $\vec{a}$ projectile motion: $\boxed{|\vec{a}| = g} = 10\,\text{m/s}^2$ at <u>every</u> point along trajectory

c.) v_x is constant. $v_y = 0$ at highest point of trajectory

so $v_{min} = |v_x| = \boxed{v_0 \cos\theta}$

d.) $y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$

$0 = h_0 + v_0 \sin(\theta)\, t_g + \frac{1}{2}(-g) t_g^2$ $\quad t_g \equiv$ time to hit ground

quadratic formula: $$t_g = \frac{-v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta - 4(-g/2) h_0}}{2(-g/2)}$$

$$\boxed{t_g = \frac{v_0}{g} \sin\theta + \sqrt{\left(\frac{v_0}{g}\sin\theta\right)^2 + \frac{2h_0}{g}}}$$

must be "+" since we know $t_h > 0$

e.) find min v_0 s.t. $y(x=D) \geq h_n$: $\longrightarrow$ $v_0' = \text{min.}\ v_0$

$y_n = y_0 + v_{0y} t_n + \frac{1}{2} a_y t_n^2$

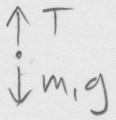
(1) $\Rightarrow$ $$h_n = h_0 + \frac{\cancel{v_0'} \sin(\theta)\, D}{\cancel{v_0'} \cos(\theta)} - \frac{1}{2} g \frac{D^2}{v_0'^2 \cos^2\theta}$$

$$h_n - h_0 - D\tan\theta = -\frac{\frac{1}{2} g D^2}{\cos^2\theta} \frac{1}{v_0'^2}$$

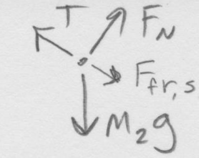
$$v_0'^2 = \frac{g D^2}{2\cos^2\theta} \frac{1}{D\tan\theta - (h_n - h_0)}$$

so: $$\boxed{v_0 \geq \sqrt{\frac{g D^2}{2\cos^2\theta}\left(\frac{1}{D\tan\theta - (h_n - h_0)}\right)}}$$

prob 2. a) F.B.D. m_1



F.B.D. M_2



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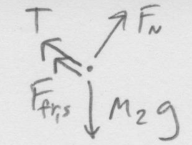

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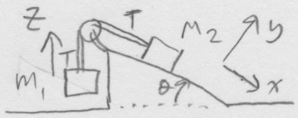
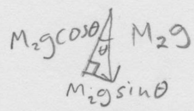
b.)





static: N2L M_2 $\begin{cases} x: -T + M_2 g\sin\theta + F_{fr,s} + \cancel{F_{Nx}}^{\,0} = m_2 \cancel{a_x}^{\,0} \text{ static} \quad (1) \end{cases}$

we want the maximum M_1, so T is as big as it can be, which requires $F_{fr,s}$ to point down the incline.

$y: \cancel{F_{fr,s,y}}^{\,0} + \cancel{F_{T,y}}^{\,0} + F_N - m_2 g\cos\theta = m_2 \cancel{a_y}^{\,0}$ (static)

$$\boxed{F_N = m_2 g\cos\theta} \quad (2)$$

c.) N2L M_1: z: $T - M_1 g = m_1 \cancel{a_{1z}}^{\,0}$ static

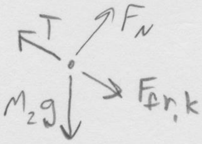
$T = M_1 g$ (3)

$F_{fr,s} \leq \mu_s F_N$ so $F_{fr,s,max} = \mu_s F_N \underset{(2)}{=} \mu_s M_2 g\cos\theta$ (4)

(3) & (4) in (1) $\Rightarrow -M_{1,max}\cancel{g} + M_2\cancel{g}\sin\theta + \mu_s M_2 \cancel{g}\cos\theta = 0$

$$\boxed{M_{1,max} = M_2(\sin\theta + \mu_s\cos\theta)}$$

d.) sliding up ramp: F.B.D. M_2



N2L: M_2 $\begin{cases} x: -T + M_2 g\sin\theta + F_{fr,k} + \cancel{F_{Nx}}^{\,0} = m_2 a_{2x} \\ -T + M_2 g\sin\theta + \mu_k F_N = m_2 a_{2x} \quad (5) \\ y: \quad F_N = M_2 g\cos\theta \quad (6) \end{cases}$

N2L: m_1 z: $T - M_1 g = M_1 a_{1z}$ (7)

blocks tied together with ideal rope $\Rightarrow$ $a_{1z} = + a_{2x}$ (8)

(6), (7), & (8) in (5): $-M_1 a_{1z} - M_1 g + M_2 g\sin\theta + \mu_k M_2 g\cos\theta = m_2 a_{1z}$

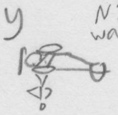
$a_{1z}(m_1 + m_2) = g(-M_1 + M_2\sin\theta + \mu_K M_2\cos\theta)$

$$\boxed{a_{1z} = g\,\frac{-M_1 + M_2\sin\theta + \mu_k M_2\cos\theta}{M_1 + M_2}} \quad (9)$$

e.) (9) in (7) $\Rightarrow T = M_1\left(g + g\,\frac{-M_1 + M_2\sin\theta + \mu_k M_2\cos\theta}{m_1 + M_2}\right)$

$$\boxed{T = gM_1 M_2\left(\frac{1 + \sin\theta + \mu_K\cos\theta}{M_1 + M_2}\right)}$$

3.a.) uniform circular motion $a_y = \frac{v^2}{R}$ — ①



$\uparrow y$ N2L y: walter $-F_{plane} - Mg = Ma_y \underset{①}{=} -\frac{Mv^2}{R}$

$$F_{plane} = \frac{Mv^2}{R} - Mg = \boxed{M\left(\frac{v^2}{R} - g\right)}$$

b.)

$a = \frac{v^2}{R} \rightarrow$

$\uparrow y \rightarrow x$

N2L walter $\begin{cases} x: & F_{p,x} = ma_x = \frac{Mv^2}{R} \quad (F_{wr,x} = 0) \\ y: & F_{p,y} + F_{wr,y} - Mg = M\cancel{a_y}^{\,0} \end{cases}$

so $\vec{F}_{wr} + \vec{F}_p = \left(\frac{Mv^2}{R}, Mg\right) \Rightarrow |\vec{F}_{wr} + \vec{F}_p| = \sqrt{\frac{M^2v^4}{R^2} + M^2g^2}$

$$= \boxed{M\sqrt{\frac{v^4}{R^2} + g^2}}$$

c.) $[A] = \frac{m}{s}$ $[B] = \frac{m}{s^4}$ $[C] = \frac{m}{s}$ $[D] = \frac{m}{s^2}$

d.) at highest point, $v_h = 0$: $\left.\frac{dh}{dt}\right|_{t'} = C - 2Dt' = 0$ t' = time to reach highest point

$h \uparrow \rightarrow x$

$t' = \frac{C}{2D}$ — ②

$$\boxed{v(t') = A + B\frac{C^3}{8D^3}}$$

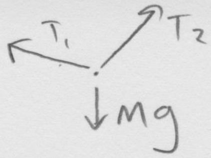
e.) at highest point, horizontal component of $\vec{a}$ points along direction of motion, so: $a_x = \frac{dv}{dt}$

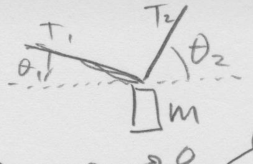
$$\left.\frac{dv}{dt}\right|_{t'} = 3Bt^2 \underset{②}{=} \boxed{3B\frac{C^2}{4D^2}} = 3\,0.1\,\frac{200^2}{4\,10^2}\,\frac{m}{s^2}$$

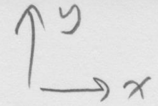
$$= 3 \times 10^{-1+4-2}\ m/s^2$$

$$= \boxed{30\ m/s^2}$$

4.a.) FBD slug







N2L slug $x: -T_1\cos\theta_1 + T_2\cos\theta_2 + \cancel{F_{gx}}^{\to 0} = \cancel{ma_x}^{\to 0}$ stationary — ①

$y: T_1\sin\theta_1 + T_2\sin\theta_2 - mg = \cancel{ma_y}^{\to 0}$ stationary — ②

① $: T_2 = T_1 \dfrac{\cos\theta_1}{\cos\theta_2}$ in ② $\Rightarrow T_1\sin\theta_1 + T_1\dfrac{\cos\theta_1}{\cos\theta_2}\sin\theta_2 = mg$

$$\boxed{T_1 = \frac{mg}{\sin\theta_1 + \cos\theta_1\tan\theta_2}}$$

b.) $\theta_1 = \theta_2 = \theta$ in ① $\Rightarrow -T_1\cancel{\cos\theta} + T_2\cancel{\cos\theta} = 0$

$$\boxed{T_1 = T_2}$$ — ③

c.) N2L slug $y: T_1\sin\theta + T_2\sin\theta - mg = ma_y = ma$ ← upward accel.

① still holds because $a_x = 0$ $\Rightarrow$ ③ $\Rightarrow 2T\sin\theta = mg + ma$

$$T_1 = T = \boxed{\frac{m(a+g)}{2\sin\theta}}$$

d.) N2L slug $\begin{cases} x: -T_1\cos\theta + T_2\cos\theta = ma_x = ma' & \text{— ④} \leftarrow \text{accel. to the right} \\ y: T_1\sin\theta + T_2\sin\theta = mg \Rightarrow T_2 = \dfrac{mg - T_1\sin\theta}{\sin\theta} & \text{— ⑤} \end{cases}$

⑤ in ④ $\Rightarrow -T_1\cos\theta + \dfrac{mg - T_1\sin\theta}{\sin\theta}\cos\theta = ma'$

$$T_1(\cos\theta + \cos\theta) = \frac{mg\cos\theta}{\sin\theta} - ma'$$

$$T_1 = \left(\frac{mg\cos\theta}{\sin\theta} - ma'\right)\frac{1}{2\cos\theta}$$

$$\boxed{T_1 = \frac{m}{2}\left(\frac{g}{\sin\theta} - \frac{a'}{\cos\theta}\right)}$$

e.) max a' occurs when $T_1 \to 0$, since we can't have a "−" tension. What would happen if we tried to exceed this a' value is that the slug would lift up, and not have purely horizontal acceleration.

$$T_1 = 0 \Rightarrow \frac{g}{\sin\theta} = \frac{a'_{max}}{\cos\theta} \Rightarrow \boxed{a'_{max} = g\frac{\cos\theta}{\sin\theta}}$$