Chemical Engineering 150A
Midterm Exam
Tuesday, February 20, 2018
6:10 pm – 7:00 pm

The exam is 100 points total.

Name: ______________________________ (in Uppercase)

Student ID: __________________________

You are allowed one 8.5”×11” sheet of paper with your notes on both sides and a calculator for this exam.

The exam should have 11 pages (front and back) including the cover page.

Instructions:

1) Please write your answers in the box if provided.
2) Do your calculations in the space provided for the corresponding part. Any work done outside of specified area will not be graded.
3) Please sign below saying that you agree to the UC Berkeley honor code.
4) The exam contains one problem with sub-parts.
5) Use the blank white full pages behind the question pages as scratch sheets.

Honor Code:

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

Signature: _

1.a	1.b	1.c	1.d	1.e	1.f	1.g	1.h	Total

Problem 1. (100 points)

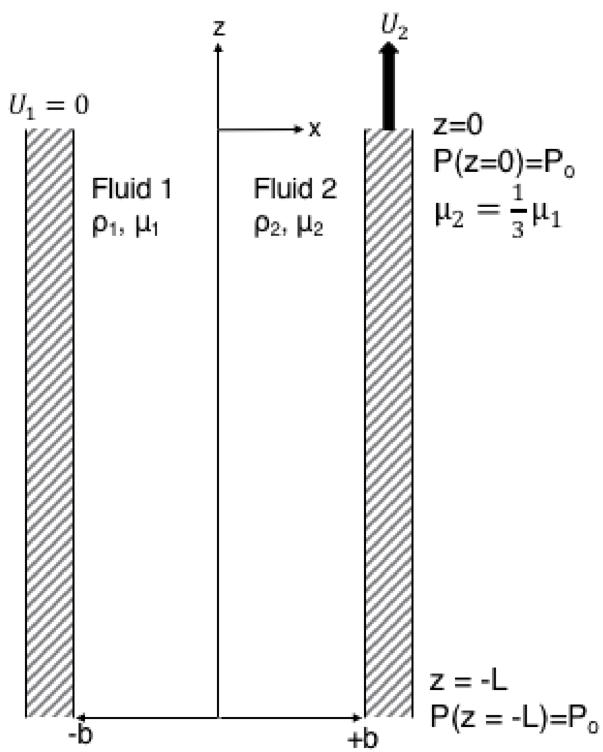
Suppose we have two immiscible, Newtonian fluids with the same densities but different viscosities between two parallel, vertical plates separated by a width 2**b** and height **L**. The first plate is **stationary** and the second plate has a constant velocity **U_2**. Do not forget gravity in the z-direction!

1. Assume that there is no pressure gradient in the z-direction.
2. The viscosity of fluid 2 is 1/3rd of viscosity of fluid 1, i.e., $\mu_2 = \frac{1}{3}\mu_1$

2. Assume that the system is at steady state and that the velocity of the fluid has the following form:

$$\underline{v} = v_z(x)\underline{e_z}$$

A schematic of this setup is given below along with a coordinate system.



a. Is the flow incompressible or not? Prove it. (10 points)

$\nabla \cdot \underline{v} = 0$ or $\rho\nabla \cdot \underline{v} = 0$ or $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$ **+4**

For each dv_x/dy and $dv_y/dy = 0$ **+1**

$\frac{\partial v_z}{\partial z} = 0$ **+2**

Correct conclusion **+3**

b. The fluid flowing between the plates can be described by the following constitutive relationships between shear stress and velocity gradients, where μ is the coefficient of viscosity.

Please circle the components that are non-zero. (10 points)

$$\tau_{xx} = 2\mu \frac{\partial v_x}{\partial x} \qquad \tau_{xy} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right] \qquad \tau_{xz} = \mu\left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]$$

$$\tau_{yx} = \mu\left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right] \qquad \tau_{yy} = 2\mu \frac{\partial v_y}{\partial y} \qquad \tau_{yz} = \mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$$

$$\tau_{zx} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right] \qquad \tau_{zy} = \mu\left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right] \qquad \tau_{zz} = 2\mu \frac{\partial v_z}{\partial z}$$

For each correct τ_{xz} *and* τ_{zx} circled **+5**

For each incorrect τ_{ij} **-5 (max -10)**

c. Give the Cauchy momentum balance *only* in the *x*-direction and simplify it. What can you conclude from the x-direction for the pressure? (10 points)

Correct Cauchy balance **+3**

$$\frac{\partial}{\partial t}\rho\underline{v} + \nabla\cdot(\rho\underline{v}\underline{v}) = -\nabla P + \nabla\cdot\underline{\underline{\tau}} + \rho g$$

Or

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial}{\partial x}(\rho v_x v_x) + \frac{\partial}{\partial y}(\rho v_x v_y) + \frac{\partial}{\partial z}(\rho v_x v_z) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) + \rho g_x$$

For LHS = 0 **+2**

$$\cancel{\frac{\partial(\rho v_x)}{\partial t}} + \cancel{\frac{\partial}{\partial x}(\rho v_x v_x)} + \cancel{\frac{\partial}{\partial y}(\rho v_x v_y)} + \cancel{\frac{\partial}{\partial z}(\rho v_x v_z)} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) + \rho g_x$$

For canceling three τ and pg terms **+3**

$$0 = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) + \rho g_x$$

For correct final expression, $\frac{\partial P}{\partial x} = 0$ **+1**

Correct conclusion about P (P is constant with respect to x) **+1**

Partial credit or deductions:

For canceling a non-zero term **-1**

For not fully simplifying **-1**

For canceling only three of four zero terms on RHS **+2 (of 3)**

d. Give the Cauchy momentum balance *only* in the *z*-direction and simplify it using the constitutive relationships from part b. Write the final ordinary differential equation in the box for the velocity. (20 points)

Correct Cauchy momentum balance **+4**

$$\frac{\partial}{\partial t}\rho \underline{v} + \nabla \cdot (\rho \underline{v}\underline{v}) = -\nabla \mathrm{P} + \nabla \cdot \underline{\underline{\tau}} + \rho g$$

Or

$$\frac{\partial(\rho v_z)}{\partial t} + \frac{\partial}{\partial x}(\rho v_z v_x) + \frac{\partial}{\partial y}(\rho v_z v_y) + \frac{\partial}{\partial z}(\rho v_z v_z) = -\frac{\partial P}{\partial z} + \frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + \frac{\partial}{\partial z}(\tau_{zz}) + \rho g_z$$

For cancelling each term on LHS **+2 (total of 8 pts)**

$$\frac{\partial(\rho v_z)}{\partial t} + \frac{\partial}{\partial x}(\rho v_z v_x) + \frac{\partial}{\partial y}(\rho v_z v_y) + \frac{\partial}{\partial z}(\rho v_z v_z) = -\frac{\partial P}{\partial z} + \frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + \frac{\partial}{\partial z}(\tau_{zz}) + \rho g_z$$

For cancelling $\frac{\partial}{\partial y}(\tau_{yz})$ and $\frac{\partial}{\partial z}(\tau_{zz})$ **+3 (total of 6 pts)**

$$0 = -\frac{\partial P}{\partial z} + \frac{\partial}{\partial x}(\tau_{xz}) + \frac{\partial}{\partial y}(\tau_{yz}) + \frac{\partial}{\partial z}(\tau_{zz}) + \rho g_z$$

For correct answer ($0 = \mu \frac{d^2 v_z}{dx^2} - \rho g$) **+2**

Partial credit or deductions:

For $0 = \mu \frac{d^2 v_z}{dx^2} + \rho g$ (not fully simplified) **-1**

1 incorrect term in Cauchy momentum balance **+2 (of 4)**

e. Solve the ordinary differential equation derived in part (d) for the velocity profile for fluid 1 and fluid 2 with viscosities μ_1 and μ_2. Do not solve for the constants of integration yet, which means you can leave the constants of integration as they are. Write the answers in the box. (15 points)

NOTE: If you cannot solve the flow profile, set up the problem appropriately.

$$\mu_i \frac{\partial^2 v_{z,i}}{\partial x^2} = \rho g$$

$$\frac{\partial^2 v_{z,i}}{\partial x^2} = \frac{\rho g}{\mu_i}$$

$$\frac{dv_z}{dx} = \frac{\rho g}{\mu_i} x + c_i$$

$$v_{z,i}(x) = \frac{\rho g}{2\mu_i} x^2 + c_i x + c_j$$

$$v_{z,1} = \frac{\rho g_z}{2\mu_1} x^2 + C_1 x + C_2$$
$$v_{z,2} = \frac{\rho g_z}{2\mu_2} x^2 + C_3 x + C_4$$

For $\frac{dv_z}{dx} = \frac{\rho g}{\mu_i} x + c_i$ **+3**

For $v_z(x) = \frac{\rho g}{2\mu_i} x^2 + c_i x + c_{i+1}$ **+3**

Having two velocity profiles written (one per fluid) **+1**

$v_{z,1}(x) = \frac{\rho g}{2\mu_1} x^2 + c_1 x + c_2$ **+2**

$v_{z,2}(x) = \frac{\rho g}{2\mu_2} x^2 + c_3 x + c_4$ **+2**

Having 4 unique integration constants **+1 (total of 4 pts)**

Deductions:

Wrong sign on g **-1**

Missing μ **-1**

Missing factor of 2 in $\frac{\rho g}{2\mu_i}$ **-1**

f. Give appropriate boundary conditions for the flow. Write the answers in the box. (15 points)

1. x = -b, $v_{z,1} = 0$ **+2.5**

2. x = +b, $v_{z,2} = U_2$ **+2.5**

+2.5 **+5**

3. x = 0, $v_{z,1} = v_{z,2}$

+5 **+5**

4. x = 0, [illegible] $dv_{z,1}$ [illegible] = [illegible] $dv_{z,2}$ [illegible] +5

g. Now use the boundary conditions and solve the problem for the velocity profiles including constants of integration. (15 points)

Use no slip boundary condition at interface (BC 3): x=0, $v_{z,1}=v_{z,2}$

$$C_2 = C_4$$

Use constant shear stress at interface boundary condition (BC 4)

@ x=0, $\mu_1 \frac{dv_{z,1}}{dx}|_{x=0} = \mu_2 \frac{dv_{z,2}}{dx}|_{x=0}$

$$\mu_1 \frac{dv_{z,1}}{dx}|_{x=0} = \mu\left(\frac{\rho g}{\mu_1}(0) + C_1\right) = \mu_1 C_1$$

$$\mu_2 = \frac{1}{3}\mu_1$$

$$\mu_2 \frac{dv_{z,2}}{dx}|_{x=0} = \mu_2\left(\frac{\rho g}{\mu_2}(0) + C_2\right) = \frac{1}{3}\mu_1 C_3$$

$$3C_1 = C_3$$

Use no slip boundary condition at walls (BC 2): x=-b, $v_{z,1}= 0$

$$v_{z,1} = 0 = \frac{\rho g}{2\mu_1}b^2 - C_1 b + C_2$$

BC 3: x=b, $v_{z,2}=U_2$

$$v_{z,2} = U_2 = \frac{\rho g}{2\mu_2}b^2 + C_3 b + C_4$$

Sub in relationship for C_1 and C_2

$$v_{z,2} = U_2 = \frac{\rho g}{2\mu_2}b^2 + 3C_1 b + C_2$$

Solve for C_1 and C_2 (2 equations, 2 unknowns)

$$C_1 = \frac{U_2}{4b} - \frac{\rho g}{4\mu_1}b$$

$$C_2 = \frac{U_2}{4} - \frac{3\rho g}{4\mu_1}b^2$$

$$v_{z,1}(x) = \frac{\rho g}{2\mu_1}x^2 + \left(\frac{U_2}{4b} - \frac{\rho g}{4\mu_1}b\right)x + \frac{U_2}{4} - \frac{3\rho g}{4\mu_1}b^2$$

$$v_{z,2}(x) = \frac{3\rho g}{2\mu_1}x^2 + 3\left(\frac{U_2}{4b} - \frac{\rho g}{4\mu_1}b\right)x + \frac{U_2}{4} - \frac{3\rho g}{4\mu_1}b^2$$

$3C_1 = C_3$ **+3**

$C_2 = C_4$ **+3**

Applying no slip at +/-b **+1 (+2 total)**

Applying $v_{z,1}|_{x=0} = v_{z,2}|_{x=0}$ **+1**

Applying $\tau_{xz,1}\big|_{x=0} = \tau_{xz,2}\big|_{x=0}$ **+1**

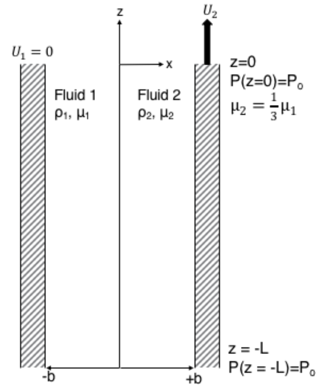
Solving for $C_1 = \frac{U_2}{4b} - \frac{\rho g}{4\mu_1} b$ **+1**

Solving for $C_2 = \frac{U_2}{4} - \frac{3\rho g}{4\mu_1} b^2$ **+1**

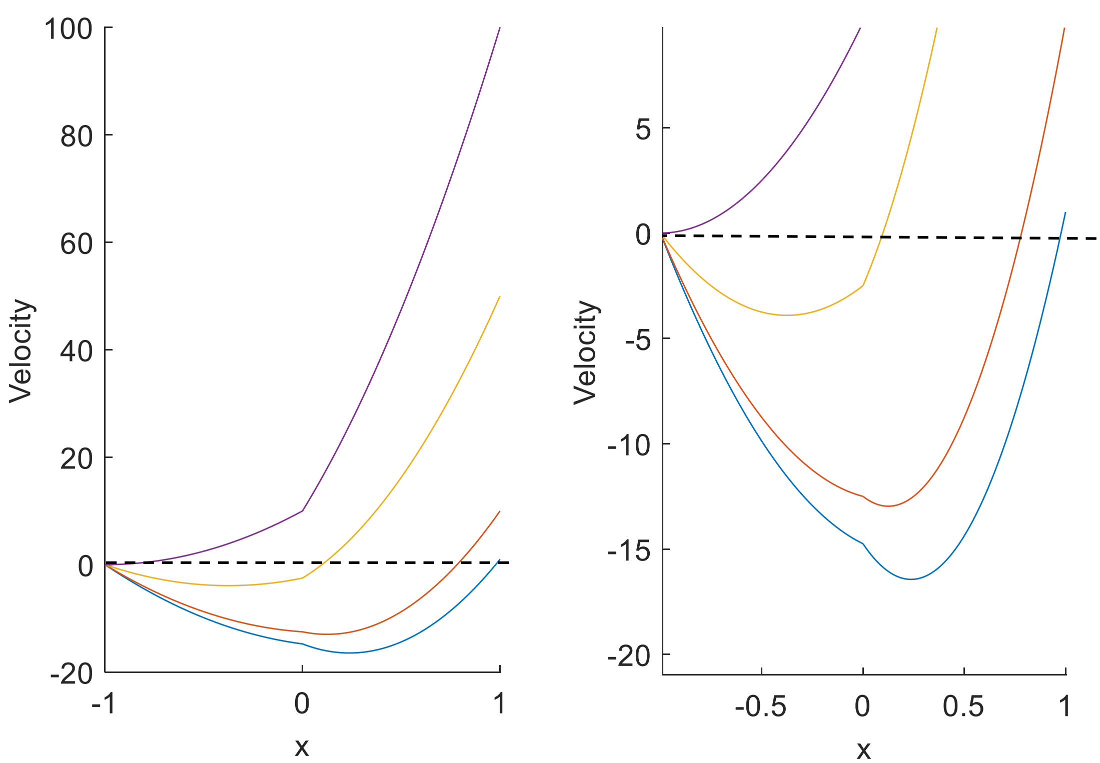
Correct $v_{z,1}(x) = \frac{\rho g}{2\mu_1} x^2 + \left(\frac{U_2}{4b} - \frac{\rho g}{4\mu_1} b\right) x + \frac{U_2}{4} - \frac{3\rho g}{4\mu_1} b^2$ **+1.5**

Correct $v_{z,2}(x) = \frac{3\rho g}{2\mu_1} x^2 + 3\left(\frac{U_2}{4b} - \frac{\rho g}{4\mu_1} b\right) x + \frac{U_2}{4} - \frac{3\rho g}{4\mu_1} b^2$ **+1.5**

h. Sketch the flow profile in the figure provided. If you are not certain about the profile, draw based on your intuition and provide explanation for what you drew (5 points)



Velocity profiles plotted from $-b \geq x \geq b$, where $b = 1$, $\frac{\rho g}{\mu_1} = 1$, and for $U_2 = [\mathbf{1}, \mathbf{10}, \mathbf{50}, \mathbf{100}]$



Continuous velocity at interface	**+0.5**
Derivatives same sign with slope change at interface	**+2**
Faster velocity for fluid 2	**+0.5**
Concave up parabolic slopes	**+1**
Reasonable explanation for fluid profile	**+1**