Instructions:

- This is a multiple choice exam with 25 questions. You have 80 minutes to complete it.
- **All of your answers must be recorded on the score sheet**. The score sheet is the final page in this exam packet.
- **Write your name on this page, and on the top of your score sheet**. You may separate the score sheet from the rest of the exam for convenience.
- **Grading scheme:** There are 75 possible points on this exam.
  - Correct answers are worth +3 points each.
  - Unanswered questions are worth 0 points each.
  - Incorrect answers are worth -1 point each.

  No work/explanantion is needed for any of your answers, and no partial credit will be given. **Mark at most one answer per question**; multiple marks will be considered incorrect.
- You may use one double-sided sheet of notes. No calculators are allowed (or needed).

Your Name:

Your Student ID:

Name of Student on Your Left:

Name of Student on Your Right:

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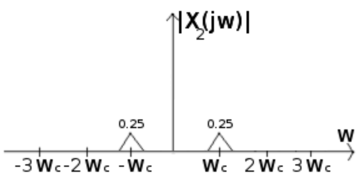
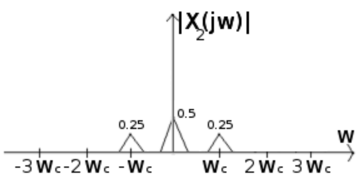
1. $x[n]$ is a periodic signal with period N and fourier series coefficients a_k. You are told that $a_0 = 1$. Consider, $y[n] = 2x[n] + 5$. Evidently, $y[n]$ is also periodic with period N; denote its Fourier coefficients by b_k. What do we know about b_0?

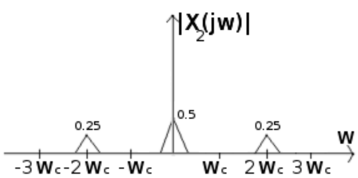
   a) $b_0 = 1$
   b) $b_0 = 2$
   c) $b_0 = 7$
   d) We do not have enough information to get b_0.

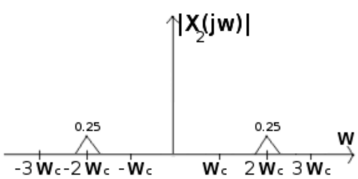
   (c)

2. Suppose you are given a signal $x(t)$, with spectrum that looks like a triangle of height 1. Define $x_1(t) = x(t)\cos(\omega_c t)$ and $x_2(t) = x_1(t)\cos(\omega_c t)$. (We modulate twice). What plot best illustrates the spectrum of $x_2(t)$?

a) 

b) 

c) 

d) 

   (c)

3. Which of the following systems are LTI?

   (i) $y(t) = x(2t)$
   (ii) $y(t) = \int_{-\infty}^{t} x(\tau)e^{t-\tau}d\tau$
   (iii) $y(t) = \int_{0}^{t} \tau x(t-\tau)d\tau$

   a) (i)

b) (ii)
c) (i) and (ii)
d) (i) and (ii) and (iii)

(b)

4. Consider the signal $x(t) = \sum_{k=-\infty}^{+\infty} \cos(k\pi) e^{jk\omega_0 t}$. What is the fundamental frequency in Hz?

a) None, $x(t)$ is not periodic
b) $\frac{\omega_0}{2}$
c) $\frac{\omega_0}{2\pi}$
d) $\frac{1}{2}$

(c)

5. Which of the following statements is correct?

a) If $X(e^{j\omega}) = X(e^{j(\omega-1)})$, then $x[n] = 0$ for $|n| > 0$
b) If $X(e^{j\omega}) = X(e^{j(\omega-\pi)})$, then $x[n] = 0$ for $|n| > 0$
c) If $X(e^{j\omega}) = X(e^{j\omega/2})$, then $x[n] = 0$ for $|n| > 0$
d) None of the above

(a)

6. Suppose we have a continuous time LTI system with frequency response

$$H(j\omega) = \begin{cases} 2 & |\omega| \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

Which of the functions below is an eigenfunction of this system?

(i) $e^{j\frac{\pi}{2}t}$
(ii) $\cos(2\pi t)$
(iii) $1 + e^{j2\pi t}$

a) (i)
b) (i) and (ii)
c) (i) and (ii) and (iii)
d) None of the above

(b)

7. For $\omega > 0$, is possible to build an LTI system that transforms $x(t) = \cos(2\omega t)$ to $y(t) = \cos(4\omega t) + \cos(2\omega t)$?

a) Yes
b) No

c) Not enough information given to say either ‘Yes’ or ‘No’.
d) It depends on the particular value of ω.

(b)

8. A real periodic signal with fundamental frequency f_0 can be written as:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k \cos(2\pi k f_0 t) + b_k \sin(2\pi k f_0 t)$$

for some numbers $a_k, b_k \in \mathbb{R}$.

a) True
b) False

(a)

9. What is Fourier transform of signal $x(t) = e^{-a|t|}$, where $a > 0$?

a) $\left|\frac{1}{a+j\omega}\right|$
b) $\frac{1}{(a+j\omega)^2}$
c) $\frac{1}{a^2+\omega^2}$
d) $\frac{2a}{a^2+\omega^2}$

(d)

10. What is $\delta(t-1) * \delta(t)$?

a) $\delta(t)$
b) $\delta(t-1)$
c) $\delta(t+1)$
d) $\delta(1)$

(b)

11. Let $x(t) = \cos(\omega_1 t) + \sin(\omega_2 t)$. What is the value of a_3, the 3rd coefficient in the Fourier series representation of $x(t)$?

a) 0
b) 1
c) j
d) The question doesn’t make sense, or there is not enough information to determine a_3.

(d)

12. Which of the following systems are causal?

(i) $y(t) = \int_{-\infty}^{t} x(\tau+1)e^{t-\tau} d\tau$

(ii) $y[n+2] + y[n] = x[n+1] + x[n]$

(iii) The LTI system with frequency response $H(j\omega) = \frac{\sin(2\omega)}{\omega}$

a) (i)

b) (ii)

c) (iii)

d) None of the above

(b)

13. Consider the continuous-time LTI system with frequency response

$$H(j\omega) = \frac{1 - j\omega}{1 + j\omega}$$

Determine the output of the system when the input is $x(t) = \cos(t)$

a) $-j\cos(t)$

b) $\sin(t)$

c) $-\sin(t)$

d) None of the above

(b)

14. Suppose we have a signal $x[n] = u[n]$, and an LTI system $H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$, what is the output $y[n]$ if we pass $x[n]$ into the system?

a) $a^n u[n]$

b) $\frac{1-a^n}{1-a}$

c) $\frac{1-a^{n+1}}{1-a}$

d) $\frac{1-a^{n+1}}{1-a} u[n]$

(d)

15. Which of the following systems are linear?

(i) $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} - 5y(t) = 2\frac{dx(t)}{dt} + x(t)$

(ii) $\frac{d^2y(t)}{dt^2} + 2\sin(\pi t)y(t) - \int_{-\infty}^{t} y(\tau)d\tau = x(t)$

(iii) $2\frac{dy(t)}{dt} + y(t) = x^2(t)$

a) (i)

b) (ii)

c) (i) and (ii)

d) (ii) and (iii)

(c)

16. What is an equivalent expression for the quantity $e^{-at}\delta(t-1)$?

   a) 1
   b) e^{-a}
   c) $e^{-a}\delta(t-1)$
   d) $e^{-a(t-1)}\delta(t)$

   (c)

17. The frequency response of an LTI system is given by

$$H(j\omega) = \begin{cases} j & \text{if } \omega \geq 0 \\ -j & \text{if } \omega < 0. \end{cases}$$

   If $x(t) = \sin(\omega t)$ is input into the system, the system output will be:

   a) $y(t) = -\cos(\omega t)$.
   b) $y(t) = \cos(\omega t)$.
   c) Not enough information to determine $y(t)$.
   d) None of the above.

   (b)

18. Suppose you convolve the two vectors $x = [1, 1, 1]$ and $y = [1, -1]$ in Python. What is the result?

   a) $[1, 0, 0, -1]$
   b) $[1, 0, -1]$
   c) $[-1, 0, 1]$
   d) $[-1, 0, 0, 1]$

   (a)

19. Suppose we have a system with frequency response

$$H(j\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

   What filter type best describes this system?

   a) low-pass
   b) high-pass
   c) band-pass
   d) all-pass

   (b)

20. Consider a system with input-output relation as follows: $y(t) = (x(t))^2$. Which of the following statements is accurate?

a) The system is LTI.

b) The system is linear but not time-invariant.

c) The system is time-invariant but not linear.

d) The system is neither linear nor time-invariant.

(c)

21. Frequency responses of four different systems are given below. Which system is BIBO stable?

a) $H(j\omega) = \delta(\omega - \omega_0)$

b) $H(j\omega) = \frac{2\sin(2\omega)}{\omega}$

c) $H(j\omega) = \frac{1}{-1+j\omega}$

d) $H(e^{j\omega}) = \sum_{n=0}^{\infty} e^{j\omega n}$

(b)

22. When presented with input $x(t) = \cos(\omega t)$, a stable LTI system outputs $y(t) = A(\omega)\cos(\omega t + \theta(\omega))$, where both $A(\omega)$ and $\theta(\omega)$ are real-valued functions of ω. Is this enough information to determine the impulse response of the system?

a) Yes.

b) No.

(a)

23. Consider the system defined by: $y[n] = 0.5y[n-1] + x[n]$. What is the frequency response $H(e^{j\omega})$:

a) $H(e^{j\omega}) = 1 - 0.5e^{-j\omega}$

b) $H(e^{j\omega}) = \frac{1}{1-0.5e^{-j\omega}}$

c) $H(e^{j\omega}) = \frac{1}{1-0.5e^{j\omega}}$

d) $H(e^{j\omega}) = 1 - 0.5e^{j\omega}$

(b)

24. Suppose a linear system takes as input $x(t) = e^{j\omega t}$ and outputs $y(t) = 2e^{-j\omega t}$ for all $\omega \in \mathbb{R}$. In words, describe the system operation.

a) The system scales the input by a factor of 2.

b) The system scales the input by a factor of -2.

c) The system scales the input by a factor of 2, and reflects it across the vertical axis.

d) The system scales the input by a factor of -2, and reflects it across the vertical axis.

(c)

25. Suppose $x[n] = 0$ for $n < 10$ and $h[n] = 0$ for $n > 5$. Which of the following statements are accurate regarding the function $y[n] = x[n] * h[n]$?

i) $y[n] = 0$ for all n.

ii) $y[n] = 0$ for $n < 10$

iii) $y[n] = 0$ for $n > 5$

a) (i)

b) (ii) and (iii)

c) (ii)

d) None of the above.

(d)

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Name: A

Clearly mark your answers below next to the appropriate problem number

1. (a) (b) (c) (d)
2. (a) (b) (c) (d)
3. (a) (b) (c) (d)
4. (a) (b) (c) (d)
5. (a) (b) (c) (d)
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