University of California, Berkeley, Department of Physics

Physics 7B,

Midterm 2, Spring 2017

- Calculators or other electronic devices are not permitted.
- Put a box around your final answer and cross out any work you wish the grader to disregard.
- Try to be neat and organized.

Problems are weighted as indicated. Remember to look over your work. Good Luck!

Problem 1	_____/18
Problem 2	_____/18
Problem 3	_____/24
Problem 4	_____/22
Problem 5	_____/18
Total	_____/100

Problem 1. [18 points] Short questions

a) **[8 points]** Find the electric potential at point P, a distance z along the perpindicular bisector of a loop of charge Q and radius R.

b) **[8 points]** A radially directed electric field is given by $\vec{E} = E_0 \frac{exp(-\kappa r)}{r^2}\hat{r}$ where E_0 *and* κ are constants. Find the charge within the radius $1/\kappa$.

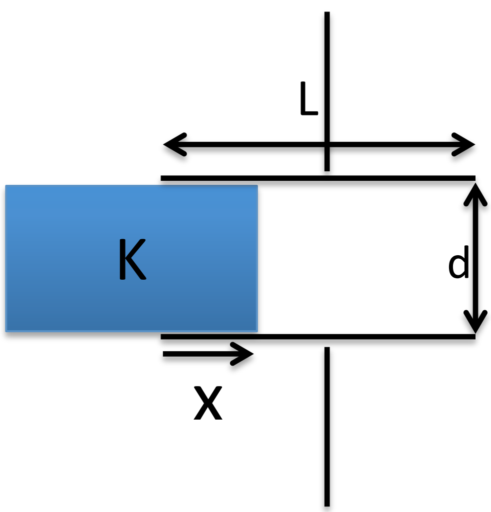
c) **[2 points]** Challenge. Find the charge density that creates the field in (b) as a function of radius.

Problem 2. [18 points] Capacitor with dielectric

A square capacitor of side L ($A=L^2$) and plate separation d is partially filled with a dielectric with dielectric constant K, which is inserted a distance $x<L$ into the capacitor

Express your answer to the questions below in terms of the given parameters.

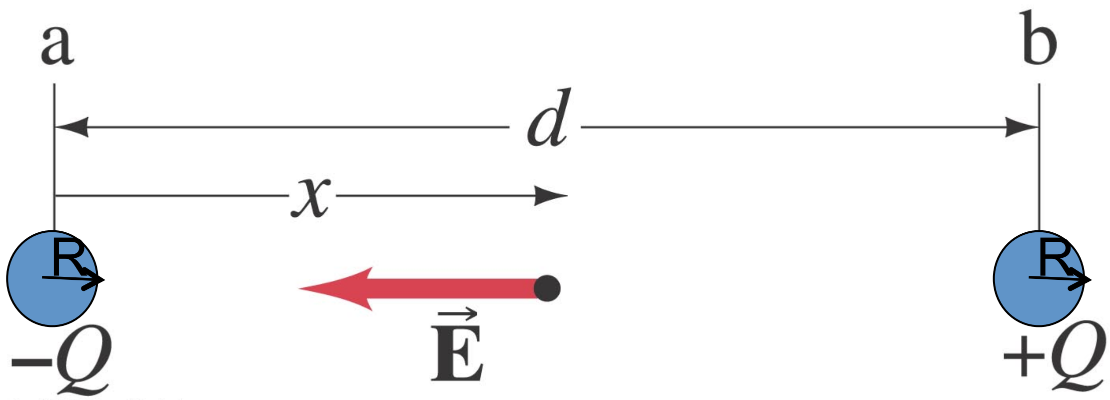
(a) **[4 points]** What is the capacitance of the capacitor?
(b) **[4 points]** The capacitor is charged to a voltage V. What is the stored energy in the capacitor?
(c) **[5 points]** Suppose the voltage source is disconnected. How do the charge, voltage and stored energy change if the dielectric is removed?
(d) **[5 points]** Suppose instead the voltage source *remains connected* while the dielectric is removed. How do the charge, voltage and stored energy change if the dielectric is removed?



Problem 3. [24 points] Two finite size long rods

Estimate the capacitance per unit length of two very long straight parallel conducting wires, each of nonzero radius R, carrying uniform charges +Q and –Q, and separated by a distance d which is large compared to R (d >> R). The separation is so large that the charge distribution on each wire can be approximated as a uniform surface charge density.

(a) **[8 points]** Find the electric field at a point x along the line joining the wires.
(b) [**8 points]** Find the potential at the point x.
(c) **[8 points]** Using (b) or any other method find the capacitance per unit length of the two wires.



Problem 4. [22 points]

A cylindrical wire resistor with resistivity ρ has a radius that doubles between Z=0 and Z=L, as in the figure. The radius of the cone is given by $R=R0\ (1+z/L)$. Find

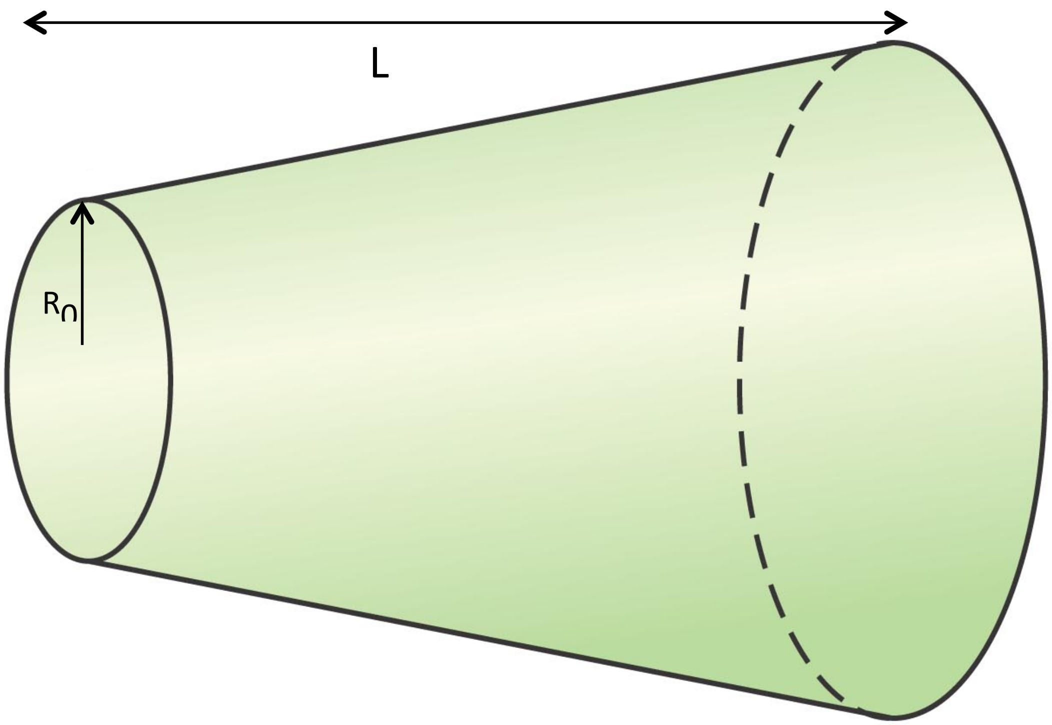
(a) **[12 points]** The total resistance between z=0 and z=L

In what follows assume a current I flows in the resistor.

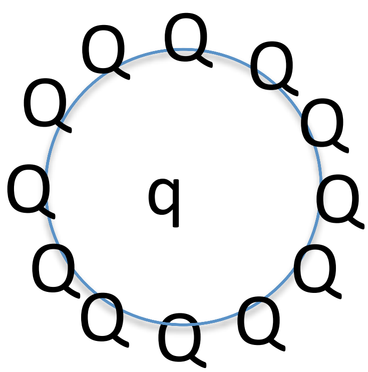
(b) **[4 points]** Find the power dissipation through the resistor.

(c) **[3 points]** Find the power dissipation per unit length in the resistor. Does it depend on z?

(d) **[3 points]** Challenge: Find the power loss per unit volume in the resistor. Express your answer in terms of the current density J



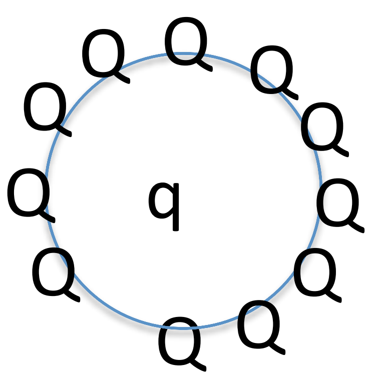
Problem 5. [18 points] Twelve equal charges Q>0 are arrayed at equal intervals (hour marks) on a circle of radius R as shown in the diagram. A charge q>0 is placed in the center.



(a) **[5 points]** What is the force on the charge q?

(b) **[5 points**] How much work had to be done to bring this charge in from infinity?

(c) **[8 points]** The charge Q located 7 o'clock is removed. What is the force on the charge q now (magnitude and direction)?



Physics 7B Spring 2017 - Midterm 2 Formula Sheet

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r} = \frac{kQ_1Q_2}{r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} = \int \frac{kdQ}{r^2}\hat{r}$$

$$\rho = \frac{\Delta Q}{\Delta V}$$

$$\sigma = \frac{\Delta Q}{\Delta A}$$

$$\lambda = \frac{\Delta Q}{\Delta l}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{dQ}{4\pi\epsilon_0 r} = \int \frac{kdQ}{r}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$C = \kappa C_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2}|\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$C = \epsilon_0 \frac{A}{d} \quad (1)$$

$$R = \rho \frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nq\vec{v_d} = \frac{\vec{E}}{\rho}$$

$$R_{eq} = R_1 + R_2 \text{ (In series)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (In parallel)}$$

$$\sum_{\text{junction}} I = 0$$

$$\sum_{\text{loop}} V = 0$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ \vec{\nabla} \cdot \vec{V} &= \frac{1}{r}\frac{\partial (rV_r)}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} \end{aligned}$$

(Cylindrical Coordinates)

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ \vec{\nabla} \cdot \vec{V} &= \frac{\partial (r^2 V_r)}{r^2 \partial r} + \frac{1}{r\sin\theta}\left(\frac{\partial (V_\theta \sin\theta)}{\partial \theta} + \frac{\partial V_\phi}{\partial \phi}\right) \end{aligned}$$

(Spherical Coordinates)

$$\epsilon_0 = 8.9 \times 10^{-12}\text{Fm}^{-1}$$