Physics 110a Fall 2004 Prof. Orenstein

Quiz #2: To help with grading, please either show your work or explain (briefly) your reasoning.

Problem 1: 20 points

Consider a sphere of total charge Q, suspended so that its center is z_0 above a conducting plane. Find an expression for the electric field at the surface of the conductor in the limit that the distance from the sphere tends to infinity.

Problem 2: 40 points

The potential outside of a spherical shell of charge (radius R) varies along the positive z-axis as:

$$V_{out}(z) = V_0 \frac{R^3}{z^3}$$

assuming that the origin of coordinates is the center of the sphere.

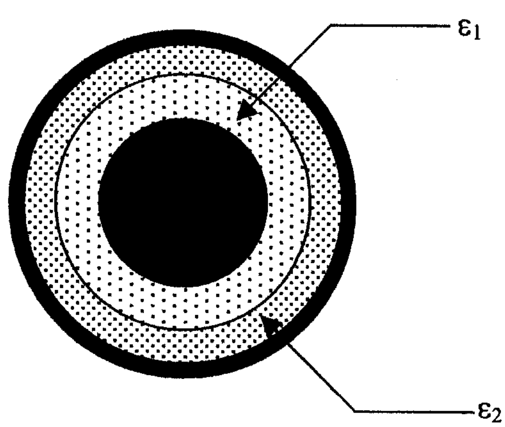
(a) Find the potential everywhere outside of the shell. **[25 points]**

(b) What is the direction of the electric field in the z=0 plane? (It may help to express $\cos\theta$ as z/r) **[15 points]**

Problem 3: 40 points

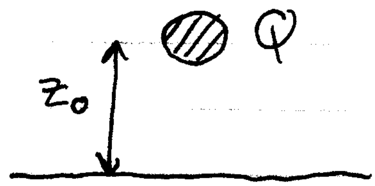
The region between two concentric spherical shell conductors is filled with two kinds of dielectric material, as shown in the diagram below. The inner conductor has uniform surface charge σ and its radius is a. The inner dielectric has permittivity ε_1 and the outer has permitivity ε_2. The interface lies halfway between the conductors.

Find the bound charge at the interface.



Solutions to Quiz #2

1.



$$\vec{p} = 2Q z_0 \hat{z}$$

$$\vec{E}_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$$

$$\boxed{\vec{E}_{dip}(\vec{r}) \to \frac{-1}{4\pi\epsilon_0 r^3}\cdot 2Q z_0 \hat{z}}$$

2. a. In general $V_{out}(r,\theta) = \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$

on +z-axis, $r = z$ $\theta = 0$

$V_{out}(z) = \sum_\ell \frac{B_\ell}{z^{\ell+1}}$. There fore we have only $\ell = 2$ term

and $B_2 = V_0 R^3$. So $\boxed{V_{out}(r,\theta) = \frac{V_0 R^3}{r^3}\,\frac{1}{2}(3\cos^2\theta - 1)}$

b. Use substitution $z = r\cos\theta$

$$V_{out} = \frac{V_0 R^3}{r^3}\,\frac{1}{2}\left(\frac{3z^2}{r^2} - 1\right) \quad \text{or}$$

$$V_{out} = \frac{3}{2}\frac{V_0 R^3 z^2}{r^5} - \frac{1}{2}\frac{V_0 R^3}{r^3}$$

The E-field is the gradient of V_{out} $(\times -1)$. The second term can give only a radial field. To see what the first term gives calculate $E_z = -\frac{\partial V}{\partial z}$.

$$\frac{\partial V}{\partial z} \propto \frac{\partial}{\partial z}\frac{z^2}{r^5} = \frac{r^5 \cdot 2z - z^2 \cdot 5r^4}{r^{10}}.$$ This

clearly vanishes for $z=0$. Therfore no z-component, and the field in the plane is purely radial. For V_0 positive $\vec{E}$ points away from sphere.

3. $\nabla \cdot D = \rho_f \rightarrow D = \dfrac{4\pi a^2 \sigma}{4\pi r^2}$

At interface $D(b) = \dfrac{\cancel{4\pi} a^2 \sigma}{b^2}$

Inside shell $E_1 = \dfrac{D(b)}{\epsilon_1}$ Outside $E_2 = \dfrac{D(b)}{\epsilon_2}$

$$P_1 = D - \epsilon_0 E_1 = D - \frac{\epsilon_0 D}{\epsilon_1} = D\left(1 - \frac{\epsilon_0}{\epsilon_1}\right)$$

$$P_2 = D\left(1 - \frac{\epsilon_0}{\epsilon_2}\right)$$

$$\sigma_b = P_2 - P_1 = D\epsilon_0\left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2}\right) = \frac{\cancel{4\pi} a^2 \sigma_f}{b^2}\epsilon_0\left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2}\right)$$