University of California, Berkeley
Department of Mechanical Engineering
ME 104, Fall 2016

Midterm Exam 1 (5 October 2016)

1. Choosing cylindrical coordinates in a Newtonian space, we may write the position vector $\mathbf{r}$ of a particle B as

$$\mathbf{r} = \chi(B,t) = R\,\mathbf{e}_R + z\,\mathbf{k}, \tag{1}$$

where

$$\mathbf{e}_R = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}, \quad \mathbf{e}_\theta = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}. \tag{2}$$

(a) Calculate $\dot{\mathbf{e}}_R$ and $\dot{\mathbf{e}}_\theta$ and obtain the expression for the velocity of B.

(b) Consider a rotating right-angled rigid rod OAC (Fig.1), where the position vector of C is

$$\mathbf{r}_C = R_0\,\mathbf{e}_R + h\,\mathbf{k} \tag{3}$$

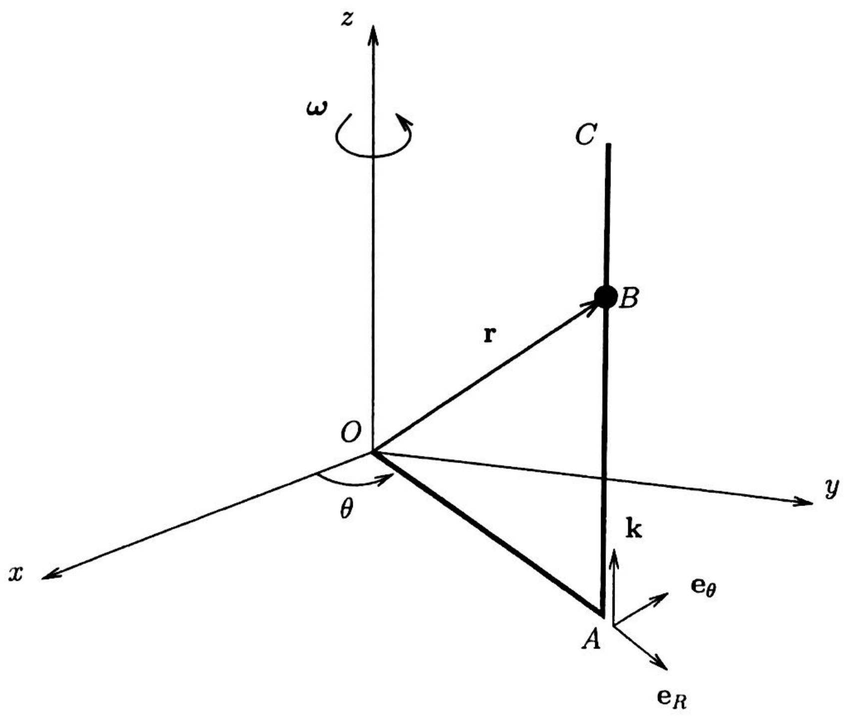

Figure 1: Rotating right-angled rigid rod OAC and slider B

with $OA = R_0$ (and $AC = h > 0$). Let the angular velocity of OAC be constant ($\boldsymbol{\omega} = \dot{\theta}\,\mathbf{k}, \dot{\theta} = \text{const.}$). Suppose that a slider B of mass m is driven up the vertical leg AC at constant vertical velocity. Neglect friction. Recall the general expression for acceleration in cylindrical coordinates, namely (you do not need to prove this)

$$\mathbf{a} = \left(\ddot{R} - R\dot{\theta}^2\right)\mathbf{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\mathbf{e}_\theta + \ddot{z}\,\mathbf{k}. \tag{4}$$

Calculate the components of the force $\mathbf{N}$ that the rod ~~exerts on the slider~~. Also, calculate the driving ~~force~~.

(c) Show that the velocity and acceleration of the point C may be expressed as

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C \tag{5}$$

and

$$\mathbf{a}_C = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_C)\,. \tag{6}$$

2. Consider a particle B which is moving along a space curve $\mathcal{C}$ with positive curvature κ $(=1/\rho)$ everywhere. Let s be the arclength of $\mathcal{C}$ and let $\mathbf{e}_t$ be the unit tangent vector to $\mathcal{C}$. The principal unit normal vector $\mathbf{e}_n$ is defined by

$$\mathbf{e}_n = \frac{1}{\kappa}\frac{d\mathbf{e}_t}{ds}, \tag{7}$$

and the unit binormal vector is $\mathbf{e}_b = \mathbf{e}_t \times \mathbf{e}_n$.

(a) Show that

$$\mathbf{e}_n \cdot \mathbf{e}_t = 0. \tag{8}$$

(b) Prove that velocity and acceleration of B are given by

$$\mathbf{v} = \dot{s}\,\mathbf{e}_t \tag{9}$$

and

$$\mathbf{a} = \ddot{s}\,\mathbf{e}_t + \kappa\dot{s}^2\,\mathbf{e}_n. \tag{10}$$

(c) Deduce that

$$\kappa = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3}. \tag{11}$$

(d) If the slider B in Problem 1 has a vertical speed of 1 m/s, and $R_0 = 200$ mm, and ω is 30 revolutions per minute, calculate the radius of curvature ρ of the path of the slider in space.

(e) Suppose that an airplane of mass m is flying counterclockwise in a vertical loop at an airshow. Indicate the directions of $\mathbf{e}_t$ and $\mathbf{e}_n$ at a few points on the loop. Draw the free-body diagram of the airplane at the bottom point A of the loop (denote the lift, thrust, and drag by $L > 0$, $T > 0$, $D > 0$, respectively). If the speed of the airplane in the vicinity of the point A is constant at 750 km/h and the radius of curvature of the loop at A is 1800 m, find the relationship between T and D, and calculate the lift.