

Final Exam Answers

1. Elements of environmental engineering

- (a) Water density is 1000 g/L and the molecular mass is 18 g/mol, so $[H_2O] = 1000/18 = \boxed{56\text{ M}}$
- (b) In air, ppb refers to *mole* fraction (or volume fraction); in water, ppb refers to *mass* fraction.
- (c) $J_d [=] \text{M L}^{-2} \text{T}^{-1}$; $D [=] \text{L}^2 \text{T}^{-1}$. Here, M = mass, L = length, T = time.
- (d) **Viscosity**. Both Stokes' law and Darcy's law describe processes governed by frictional interactions between particles and a fluid for low Re, where viscosity makes the dominant contribution to drag.
- (e) Carbonic acid, H_2CO_3 .
- (f) X represents the active biomass (cell) concentration, which is measured as volatile suspended solids. S represents the "food" or substrate concentration, which is measured as dissolved BOD_5 .
- (g) Sedimentation and disinfection.
- (h) To monitor the outflow of a rapid sand filter as one of the parameters to detect when backwashing is required.
- (i) All particles that collide with fibers adhere (to a good approximation). For the smallest particles, Brownian motion controls the collision probability and increases in effectiveness with decreasing size. For the largest particles, inertial impaction and interception control the collision probability and increase in effectiveness with increasing size. With both processes acting in concert, there is a minimum efficiency in the middle where neither is strong.
- (j) The characteristic time for a chemical reaction is the initial stock divided by the initial reaction rate. For a zeroth order reaction, we would have $\boxed{\tau \sim A_0/k}$.
- (k) Cell washout occurs in the activated sludge process when the mean cell residence time is too short and the microbes do not have sufficient time to reproduce to maintain a stable elevated concentration.
- (l) Any of the following are acceptable answers: pathogenic microorganisms, nutrients (specifically N and P), metals.
- (m) Good ozone is in the stratosphere and absorbs UV radiation from the sun. Bad ozone is in the troposphere and can damage living tissue through oxidative processes. Ozone is bad if you can come into contact with it.
- (n) CO binds to the hemoglobin in the blood and thereby interferes with the effective transmission and delivery of oxygen to muscles (including the heart) and the brain.
- (o) Biochemical oxygen demand, specifically biodegradable organic molecules.
- (p) They are subjected to anaerobic digestion, which makes them more biochemically stable, produces methane that can be harvested and used as a fuel, and facilitates separation from water.

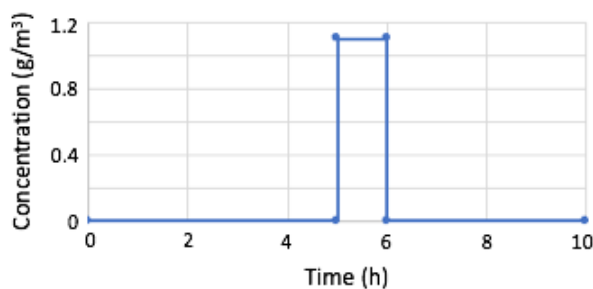
2. Environmental engineering exercises

- (a) The volumes add, so the total volume of water is 10 L. The masses of Na^+ also add; the total mass of sodium ion is $1\text{ L} \times 9\text{ mg/L} + 9\text{ L} \times 1\text{ mg/L} = 18\text{ mg}$. So, the sodium ion concentration of the mixture is $18/10 = \boxed{1.8\text{ mg/L}}$.
- (b) Approach: Solve the governing differential equation for $[A]$ and then use stoichiometry to evaluate $[B]$ at $t = 2\text{ h}$. For first-order decay, $d[A]/dt = -k[A]$, so $[A](t) = [A](0) \times \exp(-kt)$. In this case, $[A](2\text{ h}) = 10\text{ mM} \times \exp(-1) = 3.7\text{ mM}$. The stoichiometric argument is that the

- sum of [A] and [B] is conserved through the reaction. So $[A](0) + [B](0) = [A](2 \text{ h}) + [B](2 \text{ h})$. Solve for the one unknown: $[B](2 \text{ h}) = 15 - 3.7 \text{ mM}$; $[B](2 \text{ h}) = 11.3 \text{ mM}$.
- (c) The important detail here is that a solution made up of a strong acid in water must have a conjugate base at the same concentration as $[H^+]$. Let's call the concentration of that conjugate base $[A^-]$. In the first sample, it's concentration must be $[A^-] = 10^{-3} \text{ M}$. When we mix the two parcels, the conjugate base becomes diluted and, for the specific parameters of the problem, the dilution is to 40% of the initial concentration, or $0.4 \times 10^{-3} \text{ M}$. By electroneutrality, $[H^+]$ must be present at the same level in the mix. Hence, $\text{pH} = 3.4$.
- (d) The two principles are (i) electroneutrality and (ii) that the amount of C in the water in its three forms sum to the amount of Na^+ . The equations: (i) $[H^+] + [\text{Na}^+] = [\text{OH}^-] + [\text{HCO}_3^-] + 2[\text{CO}_3^{2-}]$; (ii) $[\text{Na}^+] = [\text{H}_2\text{CO}_3^*] + [\text{HCO}_3^-] + [\text{CO}_3^{2-}]$.
- (e) Here, we use the Einstein relationship: $x \sim (2 D t)^{1/2}$ where x represents the characteristic distance traveled in time t , by a diffusive process with diffusivity D . We also note that the distance X must satisfy the condition that the time of advective transport through X balances the time of diffusive transport across W . Putting the pieces together, we have $W \sim (2 \varepsilon T)^{1/2}$ and $T \sim X/U$, so $W \sim (2 \varepsilon X/U)^{1/2}$ or $X \sim (U W^2)/(2\varepsilon)$. (The factor 2 in the denominator may be omitted.)
- (f) Combustion stoichiometry: $\text{C}_4\text{H}_{10} + \alpha (\text{O}_2 + 3.78 \text{ N}_2) \rightarrow 4 \text{ CO}_2 + 5 \text{ H}_2\text{O} + 3.78 \alpha \text{ N}_2$. Solve for α by balancing O: $2\alpha = 8 + 5 \Rightarrow \alpha = 6.5$. Mass of air = $6.5 \times (32 + 3.78 \times 28) = 896 \text{ g air per mole of butane}$. The mass of butane = $4 \times 12 + 10 = 58 \text{ g/mol}$. The air:fuel mass ratio is thus $896/58 = 15.4$

3. CMFR and PFR for ENV

- (a) Key ideas for ideal PFR: (a) shape of inlet concentration profile is reproduced at outlet, but delayed by time $\theta = V/Q$; (b) transformation can be modeled by "batch reactor on a conveyor belt" idea. Hydraulic detention time is $\theta = V/Q = 5 \text{ h}$. Transformation is by means of first-order decay, so $dC/d\theta_x = -kC \Rightarrow C(\theta) = C(0) \exp(-1) = 0.37 C(0) = 1.1 \text{ g/m}^3$ when it is not zero. The outlet concentration that satisfies this description given the input data is plotted below.



- (b) Assume steady state here (not necessary to do time-dependent analysis). Solve for the concentration of [A] by writing the material balance equation and evaluating in steady-state: $[A] = A_0/[1+(k\theta)]$ where (as usual) $\theta = V/Q$. Then, apply the stoichiometric condition that $B_0 - [B] = A_0 - [A]$ (change in B is the same as change in A in the reactor). Substitute and solve for $[B] = B_0 - [k\theta/(1+k\theta)] A_0$.
- (c) Key idea here is that the initial amount of either element in the solid must balance the total removal by flow out. We'll focus on calcium in the analysis. Initial quantity = $2 \text{ g}/78.1 \text{ g/mol} = 25.6 \text{ millimoles}$. At equilibrium, $[\text{Ca}^{2+}][\text{F}^-]^2 = 3 \times 10^{-11} \text{ M}^3$. By stoichiometry (or

electroneutrality), $[F^-] = 2[Ca^{2+}]$. Therefore, $4[Ca^{2+}]^3 = 3 \times 10^{-11} M^3$, so $[Ca^{2+}] = 0.196 \text{ mM}$. Time for dissolution, t , satisfies $Qt [Ca^{2+}] = 25.6 \text{ millimoles}$. Flow rate, Q , is 2 L/h , so $t = 25.6/(2 \times 0.196) = \boxed{65 \text{ h}}$.

4. Water treatment technologies

- (a) For a sedimentation basin, the overflow rate, Q/A_s , equals the critical settling velocity, v_c . Here, $Q/A_s = U \times H \times W / (L \times W) = U \times H/L = 600 \times 4/40 = 60 \text{ m/d}$. For each particle size considered separately, the removal efficiency is $\eta = v_s/v_c$, where v_s is the settling velocity. For the $20 \mu\text{m}$ particles, $\eta = 28/60 = 47\%$. For the $10 \mu\text{m}$ particles, $\eta = 7.1/60 = 12\%$. The concentration of each particle size escaping the control unit is the inlet concentration multiplied by $(1-\eta)$. To get the total outlet concentration, apply this relationship to each size separately and add: $C = 4 \text{ g/m}^3 \times (1-0.47) + 1 \text{ g/m}^3 \times (1-0.12) = \boxed{3.0 \text{ g/m}^3}$.
- (b) To transform the starting chlordane concentration of 0.020 mg/L to a controlled level of 0.001 mg/L , we'll need to sorb the difference, 0.019 mg/L , on PAC. The amount sorbed per mass of PAC is obtained by the Freundlich isotherm. Key is that C is the final residual quantity, so $q = 606 (0.001)^{0.51} = 17.9 \text{ mg chlordane sorbed per g PAC}$. The required dose would be $0.019 \text{ mg/L} \div 17.9 \text{ mg/g} = 0.00106 \text{ g/L}$ or $\boxed{1.06 \text{ g/m}^3}$.

5. Air in the aircraft passenger cabin

- (a) $MW_{\text{air}} = 29 \text{ g/mol} = 0.029 \text{ kg/mol}$. Use ideal gas law. $M = MW_{\text{air}} \times n = MW_{\text{air}} \times PV/(RT) = 0.029 \text{ kg/mol} \times 0.74 \text{ atm} \times 150 \text{ m}^3 / (82 \times 10^{-6} \text{ m}^3 \text{ atm mol}^{-1} \text{ K}^{-1} \times 293 \text{ K}) = \boxed{134 \text{ kg}}$.
- (b) Mass flow rate of ventilation air = $0.25 \text{ kg/min per occupant} \times 135 \text{ occupants} = 33.8 \text{ kg/min}$. Hydraulic detention time must be mass of air (from (a)) divided by mass flow rate of ventilation air = $134 \text{ kg}/33.8 \text{ kg/min} = \boxed{4.0 \text{ min}}$.
- (c) The mass rate of water emission must equal the mass rate of water removal by ventilation. Key issue: what is the mass fraction of water vapor in air for the specified in-cabin conditions: $15\% \text{ RH}$, $P = 0.74 \text{ atm}$, $T = 293 \text{ K}$? Mass concentration of air = $29 \text{ g/mol} \times 0.74 \text{ atm} / (82 \times 10^{-6} \text{ m}^3 \text{ atm mol}^{-1} \text{ K}^{-1} \times 293 \text{ K}) = 893 \text{ g/m}^3$. Mass concentration of H_2O in air = $18 \text{ g/mol} \times 0.15 \times 0.023 \text{ atm} / (82 \times 10^{-6} \text{ m}^3 \text{ atm mol}^{-1} \text{ K}^{-1} \times 293 \text{ K}) = 2.58 \text{ g/m}^3$. The mass ratio of water to air is $2.58/893 = 0.00289$. With ventilation supplied at $0.25 \text{ kg/min per occupant} = 15 \text{ kg/h per occupant}$, the rate of water removal in the ventilation air = $0.00289 \times 15 \text{ kg/h per occupant} \times 1000 \text{ g/kg} = \boxed{43 \text{ g/h}}$ per occupant. That must be the water emission rate from each person.

6. Understanding the fate of contaminants spilled in a river

- (a) Total mass release can be determined from material balance as equal to the mass that flows by advection past Harbin. The time-averaged concentration in river water between hours 212.5 and 262.5 is $0.3 \text{ mg/L} = 0.3 \text{ g/m}^3$. The quantity of water flowing past Harbin during this time is $0.44 \text{ m/s} \times 1250 \text{ m}^2 \times 50 \text{ h} \times 3600 \text{ s/h} = 99 \times 10^6 \text{ m}^3$. So, the quantity of nitrobenzene released is $0.3 \text{ g/m}^3 \times 99 \times 10^6 \text{ m}^3 = 30 \times 10^6 \text{ g} = \boxed{30 \text{ tonnes}}$.
- (b) Consider parcel of water with thickness (in river flow direction) of Δx . It has volume $V = A \Delta x$. The interfacial area exposed to the atmosphere is $W \Delta x$. Write a material balance for this parcel as it is advected downriver. Ignore longitudinal dispersion. In the "batch reactors on a conveyor belt" representation, the only process influencing the abundance of benzene is interfacial mass transfer, described by the two-film model. We can write $d(CV)/d\theta_x = -k_{gl} W$

$\Delta x C$. Divide both sides by V and cancel common terms to obtain: $dC/d\theta_x = - (W/A)/k_{gl} C$. Solve to yield $C(\theta) = C_0 \exp[- (W/A) k_{gl} \theta]$. Evaluate at $\theta = 237.5$ h (transit time from Jilin to Harbin). Result is $C(\theta)/C_0 = 0.62$. Meaning: 62% of the benzene spilled into the river remains in the river when the water reaches Harbin. Conversely, 38% of the benzene has volatilized in transit. So, the answer sought is $f = 0.38$.