Department of Physics, University of California, Berkeley

Final Examination Physics 7B, Lecture 1, Prof. Smoot

8 AM - 11 AM, 12 May 2006

Name:____

SID No:____

Discussion Section:____________________________

Name of TA:___________________________________

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8	

Score:____

$$
\begin{aligned}
\nabla V &= \frac{\partial V}{\partial x}\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} \\
\nabla V &= \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} \\
\nabla V &= \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}
\end{aligned}
$$

Some useful Integrals:

$$
\int \frac{dx}{\sqrt{x^2+a^2}} = log\left(x + \sqrt{x^2+a^2}\right) \quad \text{or} \quad sinh^{-1}\frac{x}{a}
$$

$$
\int \frac{dx}{x^2+a^2} = \frac{1}{a} arctan(\frac{x}{a}) \qquad \int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}
$$

Answer all eight problems. Write clearly and explain your work. Partial credit will be given for incomplete solutions provided your logic is reasonable and clear. Cross out any parts that you don't want to be graded. Enclose your answers with boxes. **Express all numerical answers in SI units**. Answers with no explanation or disconnected comments will not be credited. If you obtain an answer that is questionable, explain why you think it is wrong.

Constants and Conversion factors

Avogadro number, N_A	6.022×10^{23}		
Permittivity of vacuum, ϵ_0	8.85×10^{-12} F·m^{-1}		
Permeability of vacuum, μ_0	$4\pi \times 10^{-7}$ T·m·A^{-1}		
Speed of light in vacuum, c	$1/\sqrt{\mu_0\epsilon_0} = 3 \times 10^8$ m/s		
Electron: Charge, q_e	-1.602×10^{-19} C	Mass, m_e	9.91×10^{-31} kg
Universal gas constant, R	8.315 J·mol^{-1}·K^{-1} = 1.99 cal·mol^{-1}·K^{-1}		
Boltzmann constant, k	1.381×10^{-23} J·K^{-1}		
Stefan-Boltzmann constant, σ	5.67×10^{-8} W·m^{-2}·K^{-4}		
Acceleration due to gravity, g	9.8 m·s^{-2}		
Water: Specific Heat	1 kcal·kg^{-1}·°C^{-1}		
Water: Heat of fusion	80 kcal·kg^{-1}	Heat of vaporization	539 kcal·kg^{-1}

1 atm 1.013×10^5 N·m^{-2} 1 kcal 4.18×10^3 J 1 hp 746 W 1 liter 10^3 cm^3

Equations and formulae:

$$Coulomb's\ law: \mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}} \qquad Lorentz\ Force\ Law: \vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$Electric\ field: d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \qquad Potential: dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$$

$$Potential\ difference: V_{ab} = -\int_a^b \mathbf{E} \cdot d\mathbf{l} \qquad Potential\ energy: U_{ab} = qV_{ab}$$

$$Electric\ field\ and\ potential: \mathbf{E} = -\nabla V$$

$$Electric\ dipole: \mathbf{p} = q\mathbf{d} \qquad Magnetic\ dipole: \vec{\mu} = NI\vec{A}$$

$$Torque\ on\ a\ dipole: \vec{\tau} = \mathbf{p} \times \mathbf{E} \qquad \vec{\tau} = \vec{\mu} \times \vec{B}$$

$$Potential\ energy\ of\ a\ dipole: U = -\mathbf{p} \cdot \mathbf{E} \qquad U = -\vec{\mu} \cdot \vec{B}$$

$$Gauss's\ Law(s) \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enclosed}}{\epsilon_0} \qquad \oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$Biot-Savart\ Law: d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{l} \times \hat{\mathbf{r}}}{r^2} \qquad d\mathbf{F} = I\vec{\ell} \times \mathbf{B}$$

$$Faraday\ Induction\ Law: \oint \mathbf{B} \cdot d\ell = -\frac{d}{dt}\Phi_B \quad where \quad \Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

$$Ampere's\ Law: \oint \mathbf{E} \cdot d\ell = \mu_o\epsilon_0\frac{d}{dt}\Phi_E + \mu_0 I_{enclosed} \quad where \quad \Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$$

$$Energy\ density: u_E = \frac{1}{2}\epsilon_0 E^2 \qquad u_B = \frac{1}{2\mu_0}B^2$$

$$Poynting\ vector: \mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B} \qquad Displacement\ Current: I_d = \epsilon_0\frac{d\Phi_E}{dt}$$

$$
\begin{aligned}
Capacitance:\ C &= \frac{q}{V_{ab}} \quad Energy\ stored\ in\ a\ capacitor:\ U = \frac{1}{2}CV^2 \\
Capacitors\ in\ series:\ \frac{1}{C_{eq}} &= \sum \frac{1}{C_i} \qquad Capacitors\ in\ parallel;\ C_{eq} = \sum C_i \\
Inductance:\ L &= \frac{N\phi_B}{I} \quad Energy\ stored\ in\ an\ Inductor:\ U = \frac{1}{2}LI^2 \\
Inductors\ in\ series:\ L_{eq} &= \sum L_i \qquad Capacitors\ in\ parallel;\ \frac{1}{L_{eq}} = \sum \frac{1}{L_i} \\
Resistors\ in\ series:\ R_{eq} &= \sum R_i \quad Resistors\ in\ parallel:\ \frac{1}{R_{eq}} = \sum \frac{1}{R_i} \\
Current:\ I &= \frac{dq}{dt} \quad Current\ and\ current\ density:\ dI = \mathbf{J} \cdot d\mathbf{A} \\
Resistivity\ and\ resistance:\ R &= \rho\frac{l}{A} \qquad Conductivity:\ \sigma = \frac{1}{\rho} \\
Ohm's\ law:\ V &= IR \qquad Ohm's\ law:\ \mathbf{J} = \sigma\mathbf{E} \quad \mathbf{E} = \rho\mathbf{J} \\
Generalized\ Ohm's\ Law:\ V &= IZ \qquad P = IV \quad \overline{P} = I_{rms}V_{rms}cos\phi = I_{rms}^2 Z\cos\phi \\
RLC\ series\ circuit:\ \omega_{LRC} &= \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad \alpha = \frac{R}{2L} \\
Z_{LRC} &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad \phi = tan^{-1}\left(\frac{R}{2L\omega_{LRC}}\right) \\
Electric\ power\ (DC):\ P &= VI \quad Average\ electric\ power\ (AC):\ \overline{P} = \frac{1}{2}I_0^2 R \\
rms\ current\ (AC):\ I_{rms} &= \frac{1}{\sqrt{2}}I_0 \quad Average\ power\ (AC):\ \overline{P} = V_{rms}I_{rms} = I_{rms}^2 R \\
Kirchhoff\ current\ rule:\ \sum I &= 0\ \ at\ a\ node \\
Kirchhoff\ potential\ rule:\ \sum \varepsilon &= \sum IR\ \ around\ a\ loop \\
E_H &= v_d B
\end{aligned}
$$

$$
\begin{aligned}
\Delta L/L &= \alpha\Delta T \qquad \Delta V/V = \beta\Delta T \\
Q &= mc\Delta T \qquad Q = nC\Delta T,\ \text{where } C = C_V \text{ or } C_P \\
\frac{dN_v}{dv} &= 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} \qquad v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\rho}{P}} \qquad \overline{v} = \sqrt{\frac{8kT}{\pi m}} \\
Adiabatic:\ PV^\gamma &= constant \qquad TV^{\gamma-1} = constant \\
W &= \frac{P_1V_1}{\gamma-1}\left[\left(\frac{V_1}{V_2}\right)^{\gamma-1} - 1\right] \qquad \gamma = \frac{C_P}{C_V} \\
\frac{dQ}{dt} &= \sigma\epsilon AT^4 \qquad \frac{dQ}{dt} = -\kappa A\frac{dT}{dx}
\end{aligned}
$$

1. [41 points] **Short Questions**

(a) [15 points] Circle T or F for True or False

T F (i) An induced EMF is produced by a changing magnetic field.

T F (ii) An induced EMF gives rise to a current whose magnetic field opposes the original change in flux.

T F (iii) An EMF is induced in a conductor moving in a magnetic field.

T F (iv) An electric generator using the Faraday effect is a DC generator. A dynamo is an AC generator.

T F (v) Because of Lenz's law, there is always a back or counter EMF and a counter torque. There are also eddy currents.

T F (vi) A transformer is either a step-up transformer or a step-down transformer.

T F (vii) The general form of Faraday's Law states that a changing magnetic flux produces an electric field around the magnetic flux.

T F (viii) The forces produced by a changing magnetic field are conservative.

T F (ix) Mutual inductance between two circuits is symmetric and equal and due to the linkage of magnetic flux between the two circuits.

T F (x) Self-inductance is the coupling of a circuit to itself as a result of the coupling of its magnetic flux.

T F (xi) There is both energy and pressure stored in a magnetic field.

T F (xii) An LR (inductance and resistance) circuit has a characteristic time of L/R.

T F (xiii) An LC (inductance and capacitance) circuit has a characteristic angular frequency of $\sqrt{1/LC}$

T F (xiv) An LRC circuit has both a characteristic frequency $\sqrt{1/LC}$ and damping time of $2L/R$.

T F (xiv) In AC circuits the AC voltages and AC currents have the same frequency.

T F (xvi) An AC current going through an inductor lags the voltage across the inductor by $90°$.

T F (xvii) The AC charge on a capacitor precedes the voltage across the capacitor by 90°.

T F (xviii) Maxwell's equations gives the relationship between the electric and magnetic fields and their corresponding charges and currents.

T F (xix) Maxwell's equations predicted electromagnetic waves. That is the wave equation can be derived from Maxwell's equations.

T F (xx) Electromagnetic waves carry momentum and produce pressure.

T F (xxi) The linkage of magnetic flux from circuit A to circuit B is the same as the linkage of magnetic flux from circuit B to circuit A.

T F (xxii) All materials are diamagnetic. The diamagnetism is sometimes over ridden by ferromagnetism or paramagnetism.

T F (xxiii) Lenz's law states that two parallel wires carrying current in the same direction will oppose each other and push each other apart.

T F (xxiv) The result that $E = 0$ inside a conductor can be derived from Gauss's Law.

T F (xxv) Electromagnetic waves are transverse waves.

T F (xxvi) In a paramagnetic material, each atom or molecule has a permanent magnetic moment but the dipole moments are randomly oriented by thermal effects. Applying an external magnetic field will orient some of these dipole moments.

T F (xxvii) The magnetic field inside a toroid is uniform.

T F (xxviii) If the electric flux through a closed surface is zero, then the surface is at a fixed potential.

T F (xxix) A good radiation emitter is a good radiation absorber.

T F (xxx) An electromagnetic waves carries energy, momentum and sometimes angular momentum.

T F (xxxi) Optical Tweezers use light to manipulate microscopic objects as small as a single atom. The radiation pressure from a focused laser beam is able to trap small particles. In the biological sciences, these instruments have been used to apply forces in the pN-range and to measure displacements in the nm range of objects ranging in size from 10 nm to over 100 mm

(b) [5 points]

(i) To take an MRI (magnetic resonance image) a patient is put in a 3 Tesla magnetic field with a sinusoidally varying smaller field $B(t) = B_0 cos(\omega t)$. Why must the medical technician who attaches a pulse monitor to the patient have to be careful about what direction and orientation he has for the two sense wires?

(A) For the same reason that the patient can't wear a watch. The wires will be attracted into the magnet.

(B) If the patient moves, then the changing flux can generate a voltage that would shock the patient.

(C) The varying magnetic field will create a voltage if the wires present a net loop. That will ruin the pulse readings and could burn the patient if a high current is induced in the loop.

(D) There could be a large force on the sensor wires because they carry sensing currents. So the medical technician must align them parallel to the magnetic field to avoid these forces.

(E) none of those reasons. It is for the tidiness and convenience of the patient.

(ii) An all electric car uses six 12VDC-batteries rated at 100 Ampere-hours. How much total energy is contained in them?

(A) 7,200 joules

(B) 4.32×10^5 J

(C) 7.2×10^6 J

(D) 2.6×10^7 J

(E) 4.32×10^7 J

(iii) The **Q factor** of a resonance RLC series circuit can be **not** be defined as

(A) Ratio of the Stored energy to the energy loss per radian (cycle/2π). $Q = 2\pi \times$ maximum stored energy / energy loss per cycle.

(B) ratio of the peak voltage across the capacitor or inductor to the peak voltage across the resistor.

(C) $Q = \sqrt{L/C}/R$

(D) $Q = \frac{\omega_0 L}{R.}$. $\quad \omega_0 = 1/\sqrt{LC}$

(E) $Q = R\sqrt{C/L}$.

(F) $Q = \frac{1}{\omega_0 RC}$ $\quad \omega_0 = 1/\sqrt{LC}$

(iv) An RC series filter circuit (A) passes high frequencies to the next stage by the voltage across the resistor.

(B) passes high frequencies to the next stage by the voltage across the capacitor.

(C) filters out everything with a frequency below $\omega = 1/RC$.

(D) filters out everything with a frequency above $\omega = 1/RC$.

(E) is just like an RL filter.

(v) A traveling electromagnetic wave (radio station signal) has a peak electric field value of $E_{max} = 6 \times 10^{-6}$ V/m what is the peak value of the magnetic field? (A) 2×10^{-14} T

(B) 2×10^{-12} T

(C) 2×10^{-10} T

(D) 2×10^{-8} T

(E) 2×10^{-6} T

(c) [5 points] A circuit contains a capacitor and resistor in series is connected to an AC source. The circuit schematic is shown in the accompanying figure.

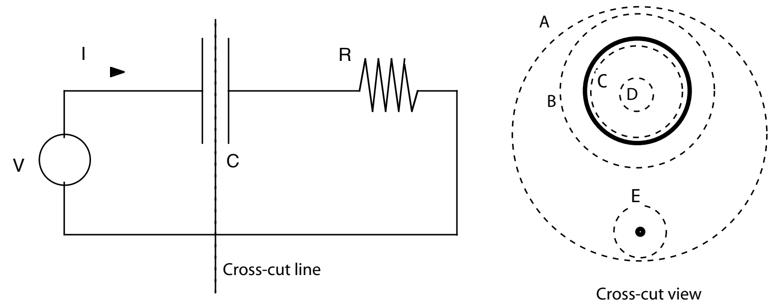

Figure 1: sketch of circuit for problem 1 (c)

The right portion of the figure shows a cross-section through the circuit at the mid-point of the parallel plate capacitor. There are 5 imaginary closed paths (A, B, C, D, E) drawn around portions of the capacitor and the return wire (the wire coming back to the AC source).

(i) Rank the absolute value of the $\int \vec{B} \cdot d\vec{s}$ in order (maximum = 1 to minimum = 5). (Break ties by which has the highest mean absolute or rms magnetic field.)

Path	(i) $\|\int \vec{B} \cdot d\vec{s}\|$	(ii) $\|\int \vec{E} \cdot d\vec{s}\|$
A		
B		
C		
D		
E		

(ii) If the capacitor is replaced by a solenoid - inductor that produces a uniform magnetic field inside the solid circle (between imaginary circles C and B and perpendicular to them), rank the absolute value of $\int E \cdot d\vec{s}$ around the imaginary paths in order (maximum = 1 to minimum =5).

(d) [6 points] Circle correct answer

(i) A Physics 7B student is in a depressed mood and approaches his (her) GSI guru. The student states"I want to change the world, but I feel helpless." The GSI guru turns and pushes a 5 kilogram rock over a ledge. It hits the ground 6 m below and comes to rest. "There," said the GSI guru, "I have changed the world." If the rock, the ground, and the atmosphere are all initially at 300 K, what is the entropy change of the universe?

(A) increased by 2 J/K.
(B) decreased by 2 J/K.
(C) increased by $\sqrt{2}$ J/K.
(D) decreased by $\sqrt{2}$ J/K.
(E) increased by 1 J/K
(F) decreased by 1 J/K
(G) not enough information to determine the change.

Did the GSI guru make the world better? In an engineering or scientific sense. Y N
Did the GSI guru make an irreversible change? Y N

(ii) A large sheet of metal has a hole cut in the middle of it. When the sheet is uniformly heated, the area of the hole will

(A) not change.
(B) always increase
(C) always decrease
(D) increase if the hole is not in the center of the sheet.
(E) decrease only if the hole is in the exact center of the sheet.

(iii) On average, the temperature of the Earth's crust increases 1.0°C for every 30 m of depth. The average thermal conductivity of the Earth's crust is 0.74 J / m · s · K. What is the heat loss of the Earth per second due to conduction from the core? The radius of the Earth is $R = 6378km$.

(A) 100 W.
(B) $10^{13}W$
(C) $10^{26}W$
(D) $10^{4}W$.
(E) $10^{14}W$.

(iv) If the rms voltage in an AC circuit (made of resistors, capacitors, and inductors) is doubled, the peak current is

(A) increased by a factor of 2.
(B) decreased by a factor of 2.
(C) increased by a factor of $\sqrt{2}$.
(D) decreased by a factor of $\sqrt{2}$.
(E) not enough information to determine the change.

(v) Mountaineers say that you cannot hard boil an egg on the top of Mount Rainer. This is true because (A) the air is too cold to boil water.

(B) the air pressure is too low for stoves to burn.
(C) boiling water is not hot enough to hard boil the egg.
(D) the oxygen content of the air is too low..
(E) eggs always break in their backpacks before they get to the top because of small amount of air trapped inside.

(e) [5 points] Circle correct answer

(i) A complicated structure is designed to connect to parts of a well-engineered device. e.g. simple version would be a spherical shell of inner radius r_1 and outer radius r_2 If the thermal conductivity is k and the electrical conductivity is ρ, the temperature difference is ΔT and the voltage difference is ΔV between the two parts, the ratio of the electrical current to the heat current (dQ/dt) through the structure is

(A) $\frac{4\pi k r_1 r_2}{r_2 - r_1}\Delta T$.
(B) $\frac{\rho \Delta V}{k \Delta T}$
(C) $\frac{\rho}{k}\Delta V k \Delta T$
(D) $\frac{T}{T_c}\frac{\rho_c}{\sigma_c}\frac{\Delta V}{\Delta T}$.
(E) not uniquely determined

(ii) If the current in an inductor is doubled, its stored energy will

(A) increase by a factor of 2.
(B) decrease by a factor of 2.
(C) increase by a factor of 4.
(D) increase by a factor of 8.
(E) not changed.

(iii) A positively charge particle is moving northward in a static magnetic field The magnetic force on the particle is toward the northeast. What is the direction of the magnetic field?

(A) Upward
(B) West
(C) South
(D) Downward
(E) This situation cannot exist.
(F) Upward at an angle of 45° to East .

(iv) If the AC frequency driving an inductor is doubled, the inductive reactance of the inductor will

(A) increase by a factor of 2.
(B) not change.
(C) decrease by a factor of 2.
(D) increase by a factor of 4.
(E) decrease by a factor of 4.

(v) An ideal transformer has N_p turns on the primary and N_s turns on its secondary. The power dissipated in a load resistance R connected across the secondary is P_s, when the primary voltage is V_p. The current in the primary is then

(A) P_s/V_p
(B) $\frac{N_p}{N_s}\frac{P_s}{V_p}$
(C) $\frac{N_s}{N_p}\frac{P_s}{V_p}$
(D) $\left(\frac{N_s}{N_p}\right)^2 \frac{P_s}{V_p}$
(E) $\left(\frac{N_s}{N_p}\right)^2 \frac{V_p^2}{R}$

(f) [5 points] Circle correct answer

(i) Compasses point north because

(A) the north star attracts them.
(B) the Earth has an electric charge.
(C) there are electric currents in the iron core of the Earth.
(D) there are magnetic monopoles near the North Pole.
(E) there is a very large bar magnet in the Earth.

(ii) The Earth's magnetic field flips are used for

(A) creating new permanent magnets.
(B) proving the Earth has a solid iron core..
(C) generate useful power.
(D) geologic dating.
(E) bird migration.

(iii) Which is **not** a property of EM waves?

(A) E and B waves have the same velocity $v_E = V_B = c = 1/\sqrt{\epsilon_0 \mu_0}$.
(B) E and B waves are in phase.
(C) EM waves are transverse with E and B perpendicular to the direction of wave motion
(D) E and B magnitudes are in the ratio $E/B = c$.
(E) EM waves can self-propagate. They require no medium for propagation.
(F) EM waves carry both energy and momentum.
(G) The electric field of a wave decreases as one over the square of the distance from the source.

(iv) Magnetism comes from

(A) magnetic monopoles.
(B) moving quanta of light.
(C) quantization of charge.
(D) moving electric charge.
(E) magnesia and lodestones transferred to soft iron.

(v) AC is used instead of DC power because

(A) it is safer.
(B) it carries more power.
(C) it makes the use of transformers straight forward.
(D) it is higher voltage
(E) Westinghouse had deeper-pocket backers than Edison.

2. [25 points] **Resistor and Voltage Source Network**

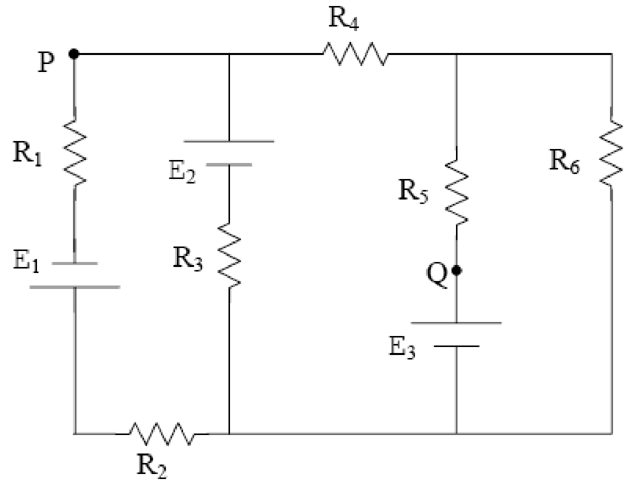

Figure 2: Resistor and Voltage Source Network.

(a) [5 points] Write down the appropriate equations to determine the four currents: (1) I_1 through points P, (2) I_2 through resistor R_3, (3) I_3 though point Q and (4) I_4 through resistor R_6.

(b) [5 points] Assume that the voltages $E_1 = 12$ V , sources E_2 and E_3 are replaced with capacitors, and that all the resistors have the same value $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 =$ 100 Ohms. Solve for the current I_1 and the voltage on each of the two capacitors E_2 and E_3.

(c) [5 points] Voltage sources E_1 and E_2 are removed and replaced by conductors and all the resistors have the same value $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 100$ Ohms. Voltage Souce $E_3 = 6$ V. Draw the simplest equivalent circuit. Determine and label the equivalent resistance(s) and current I_1.

(d) [5 points] Instead remove the right half of the circuit so that only the left hand loop circuit remains. The two remaining voltage sources are still $E_1 = 12$ V and $E_2 = 6$ V and that all the remaining resistors have the same value $R_1 = R_2 = R_3 = 100$ Ohms. Again draw the simplest possible equivalent circuit with labels for the components and their values including the current I_1.

(e) [5 points] Keeping again only the left half of the circuit, at time $t = 0$ replace resistor R_3 instantaneously with an inductor with $L = 10\mu$H. What is the current I_1 as a function of time?

3. [25 points] **Power to the People**

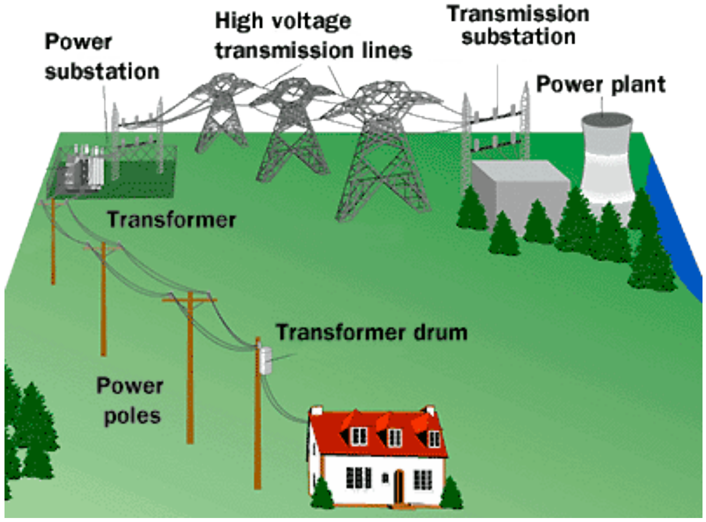

Figure 3: Power Distribution to Residence from Power Station.

(a) [4 points] The AC rms voltages of transmission are in order: (1) Power Station: $V_{PS} = 12,000$ V, (2) High Voltage Transmission: $V_{HVT} = 240,000$ V, (3) Neighbor Power Poles: $V_{pole} = 2,400$ V, and to your house or apartment residence $V = 240$ V and center tap $V = 120$ V AC. What are the ratio of the number of transformer turns at each of the various transformers?

Ratio of Primary to Secondary Number of Turns

Transformer Location	Input Voltage	Output Voltage	Ratio of Turns
Transmission Substation	12,000	240,000	
Power Substation	240,000	2,400	
Power Pole	2400	240	
Power Pole	2400	120	

(b) [6 points] A typical house uses an rms current of 50 amperes at 120 VAC rms. What is the power usage of that house? If the resistance of the long high voltage transmission line is 0.4 Ohms, what is the power cost (power lost) of transmitting that power in the high voltage transmission? What if the voltage of the transmission lines was 2,400 V instead of 240,000 V, what would the power cost be then? The average USA power transmission losses are about 7 per cent. How much would one have to lose at each transformer to have this result? Is that a reasonable amount? Why?

(c) [5 points] If a transformer on the utility pole has winding that are roughly 0.2 m in diameter and 0.4 m long with a total number of primary windings of 200, what is the inductance if it has an air core? What is the inductance, if it is wound on a ferromagnetic core with $\mu = 4000\mu_0$? What is its impedance(s) at $\omega = 2\pi\ 60\ Hz \sim 377 radians/second$? What then is the maximum current to the house?

(d) [5 points] What capacitance would you need to add to the transformer to make the phase angle between the current and voltage to be zero? Assume that the residence has a zero phase load. What would be the natural frequency of that inductor capacitor system?

(e) [5 points] What is the required rate of generator rotation to produce 60 Hz AC? If the generator's magnetic field is about a 0.3 Tesla and the size of the loops is roughly 2 m by 3 m, how many turns would be required to produce the 12,000 Volts rms quoted for the power station?

4. [25 points] **Maxwell's Equations** and the Heart of Electromagnetism.

(a) [5 points] Why is a magnetic field such as $\vec{B} = B_0\hat{r}/r^2$ not physically possible, while and electric field of the form $\vec{E} = E_0\hat{r}/r^2$ is?

(b) [5 points] What is the current distribution that leads to a magnetic field of the form $\vec{B}(r) = \hat{\phi}B_0 r/R$ for $r \leq R$ and $\vec{B}(r) = \hat{\phi}B_0 R/r$ for $r \geq R$ where $\hat{\phi}$ is the unit vector in the azimuthal direction around the axis of symmetry? (Using cylindrical coordinate system)

(c) [5 points] What configuration of magnetic fields leads to an electric field distribution of the form $\vec{E}(r) = \hat{\phi}E_0 r/R$ for $r \leq R$ and $\vec{E}(r) = \hat{\phi}E_0 R/r$ for $r \geq R$ where $\hat{\phi}$ is the unit vector in the azimuthal direction around the axis of symmetry? (Using cylindrical coordinate system) Is it possible to obtain this electric field distribution from a distribution of static charges?

(d) [5 points] Why must a capacitor have a fringe field? That is why can't the field be uniform to the edge of the capacitor plates and abruptly drop to zero everywhere else?

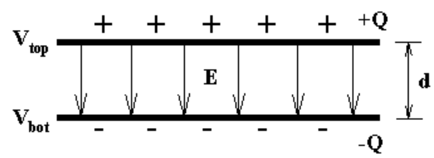

Figure 4: Capacitor shown with ideal field. Draw in necessary fringe field and explain.

(e) [5 points] If there are uniform, static electric ($E_0\hat{x}$) and magnetic ($B_0\hat{y}$) fields at right angles to each other, what velocity (speed and direction) does a particle of charge q need to have to pass through that region undeflected?

6. [25 points] **Microwave Oven**

(a) [3 points] The operating wavelength of a microwave oven is $\lambda = 12.24$ cm. What is its operating frequency?

(b) [5 points] If the power of the microwave is about 1000 watts and the microwaves are generated in a device called a klystron and transmitted to the microwave cooking area through a wave guide which is one half wavelength wide and one quarter wavelength high. What is the average Poynting vector $< |S| >$ or Intensity I, and rms Electric E_{RMS} and Magnetic B_{RMS} field values in the waveguide, when it is operating?

(c) [5 points] The volume of the microwave cooking area is 1.4 cubic feet ($3.9644 \times 10^{-2} m^3$), what is the energy density, and rms Electric and Magnetic field values in the microwave cooking area when it is operating? Assume that the Quality factor of the circuit - microwave cavity (an LC) and food (R) is such that there is about 10 times the energy stored than is being drawn out per cycle.

(d) [2 points] Why is the microwave oven enclosed in a metal container? Note that even the window in the door is covered by a metal screen and the rest of the inside is fully enclosed by metal.

(e) [3 points] Why is it a bad idea to put a cup with a gold rim (e.g. 10 cm diameter circle) in the microwave? How about food wrapped in tin foil? What about other metal utensils or Chinese food boxes with wire handles? How much power could be coupled into the circuit produced by such metal?

(f) [5 points] One warning buried in the microwave manual is that a clean smooth cup full of water can be superheated so that it has sufficient internal energy to boil but not yet started boiling because nothing has yet triggered the formation of steam bubbles. When picked up or disturbed, it will then errupt in a geyser of hot boiling water. For how long would one have to heat a container containing 100 grams of water starting at 20 C so that it was overheated and ready to errupt into boiling? Assume that the water's internal energy is distributed uniformly.

7. [25 points] A coaxial cable has an inner conductor with radius $r_1 = 1$ mm and an outer conductor with inside radius $r_2 = 2$ mm and they are concentric. They are separated by an insulating dielectric with $\kappa = 5$.

Figure 5: sketch of coaxial cable with dimensions labeled for problem 5. Treat as two concentric cylinders.

(a) [5 points] What is the capacitance per unit length of the coaxial cable?

(b) [5 points] What is the energy stored per unit length of coaxial cable, when it contains a charge q??

(c) [5 points] What is the inductance per unit length of the coaxial cable?

(d) [5 points] What is the energy stored per unit length of coaxial cable, when it carries a current I?

(e) [5 points] What is the impedance of the coaxial cable per unit length for angular frequency ω? Evaluate for $\omega = 10^8$ Hz. What is the average stored energy per unit length of coaxial cable, if it carries an RMS current I?

8. [35 points] **Tethered Satellite** for electrodynamic power generated from orbit consists of a major satellite and a secondary satellite lowered on a conducting cable. This concept was tested on a Shuttle flight in a near equitorial orbit around the Earth. The smaller satellite was lowered 20 km below the Shuttle.

(a) [5 points] If the orbital speed is 7 km/s and the mean Earth's magnetic field is 0.5×10^{-4} T, what is the voltage difference generated between the Shuttle and secondary satellite?

(b) [5 points] What charge must be on the secondary (lower) satellite for this voltage. Approximate the satellite as a conducting sphere of radius $r = 2$ m with the Shuttle very (nearly infinitely) far away? First calculate its capacitance. Then calculate the charge.

(c) [5 points] Approximate both the Shuttle and secondary satellite as two conducting spheres of radii 3 m and 2 m respectively connected by an electrical cable 20 km long and supporting the voltage difference you found in (a). Sketch the field lines for this configuration and label. Explain what is different than one would have naively thought for the electrostatic case.

(d) [5 points] Power was taken from the system by letting a current flow and power experiments on the Shuttle and tethered spacecraft. If the current is I and the voltage is V, what is the electrical power provided? How much force is exerted on the cable? Evaluate for a current of 1 ampere. Does it speed up or slow down the Shuttle-cable-satellite system? (Hint: Does it change the altitude? Which way is the force?) What makes the circuit or closed path? The basically stationary Earth's upper atmosphere, the ionosphere, is conductive and completes the circuit.

(e) [5 points] Could you run the experiment backwards? That is could the Space Station use solar cells or fuel cells to produce electrical power and run current down a tether cable to a secondary satellite and produce a force to match or overcome the residual atmospheric drag at its altitude and keep its orbit stable? How much electrical power would be required to keep the Space Station of mass 183,283 kg in its orbit, if its altitude is 353 km and loses about 2 km per year. If the tether cable is 5 km long, what is the required current and voltage?

(f) [5 points] At the Space Shuttle altitude of 400 km the typical electron density in the ionosphere is around $10^5 cm^{-3}$ or $10^{11} m^3$. The typical temperature (thermal energy) is around 1200 K. Near the second satellite (2-m radius moving 7000 m/s) what is the necessary drift velocity of the ionized electrons away from the secondary satellite to produce a 1 Ampere current? How does that compare to their thermal velocity? How much higher would the electron density need to be around the secondary satellite to make the drift velocity reasonable and comparable to their thermal velocity?

(g) [5 points] The deployment was almost complete when the unexpected happened: the tether suddenly broke and its end whipped away into space in great wavy wiggles. The failure was not caused by excessive tension, but rather that an electric current had melted the tether. Unwinding of the reel uncovered pinholes in the insulation and escaping air converted to plasma. The instruments aboard the tether satelite showed that this plasma diverted through the pinhole about 1 ampere at 3500 volts to the metal of the shuttle and from there to the ionospheric return circuit. That current was enough to melt the cable. Why is the current higher at the pinhole point where residual air plasma is escaping rather than at the surface of the Shuttle? Is the electrical power enough to melt the copper braid which is the conducting (not strength) portion of the cable? The heat capacity of copper is about 0.09 Cal/gm/K and its melting point is 1084.62 C. How large a piece of copper would be melted, if the heat were concentrated?