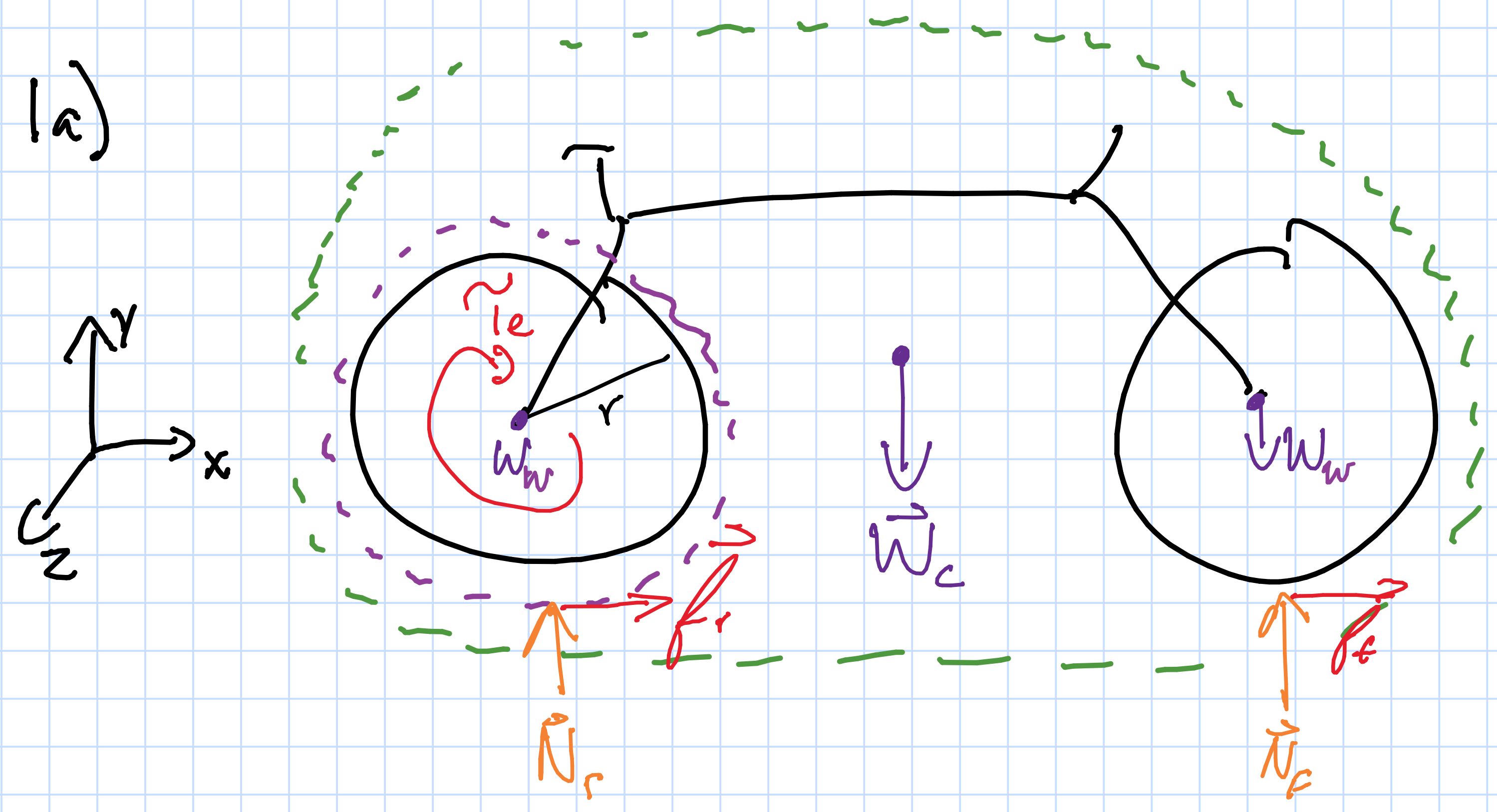
1a)



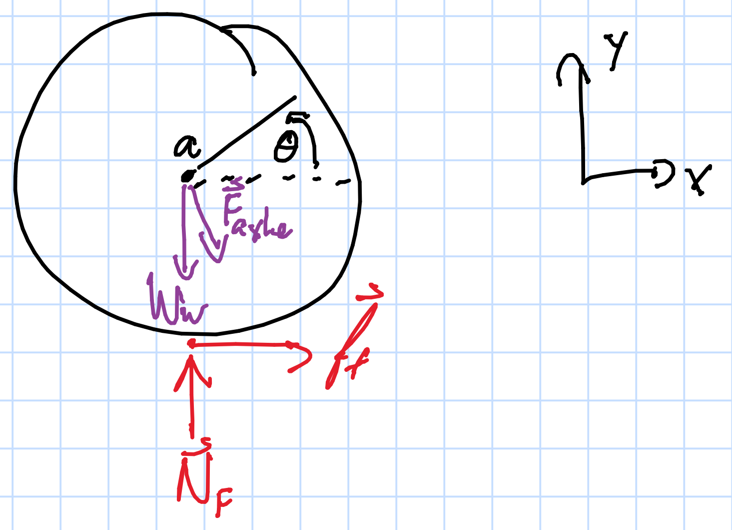
FBD: entire motorcycle

$$\Sigma \vec{F} = m\ddot{\vec{x}}$$

$\hat{\imath}: f_f + f_r = (M_c + 2M_w)\ddot{x}_c$ since $M_w \ll M_c$

$$\boxed{f_f + f_r \sim M_c \ddot{x}_c}$$

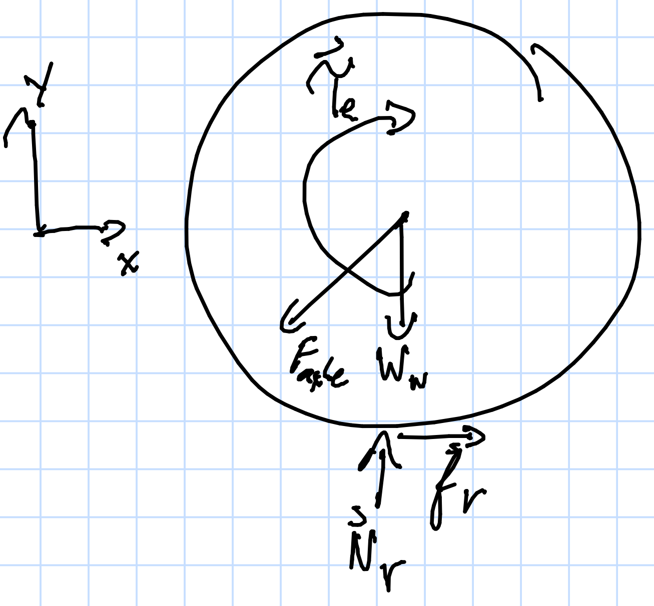
FBD front wheel



$$\Sigma \vec{\tau}_a = \dot{\vec{L}}$$

$\hat{k}: r_w f_f = m_w r_w^2 \ddot{\theta}_z$ if $m_w \ll m_c$ $f_f \sim 0$

FBD rear wheel



No slip

$\vec{V}_{eg} = \vec{V}_{ea} + \vec{V}_{ag}$

$0 = r\dot{\theta}_z \hat{\imath} + \dot{x}_c \hat{\imath}$

$\dot{\theta}_z = -\frac{\dot{x}_c}{r}$

$\ddot{\theta}_z = -\frac{\ddot{x}_c}{r}$

$$\Sigma \vec{\tau}_a = \dot{\vec{L}}$$

$\hat{k}: -\tau_e + r f \sim M_w r^2 \ddot{\theta}_z$ $M_w \sim 0$ $\therefore$

$$\tau_e = r f_r = r m \ddot{x}$$

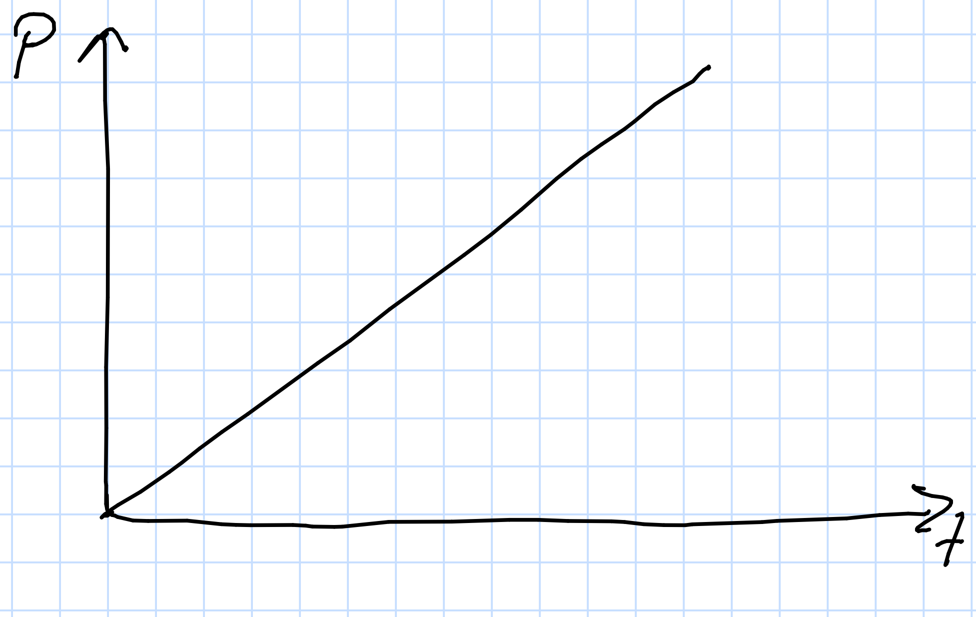
$$\boxed{\vec{\tau}_e = -r m \ddot{x} \hat{k}}$$

$$\dot{\theta}_z = \int \ddot{\theta}_z \, dt = -\frac{\ddot{x}_c}{r} t + c$$

$\dot{\theta}_z(t=0) = 0 \therefore c = 0$

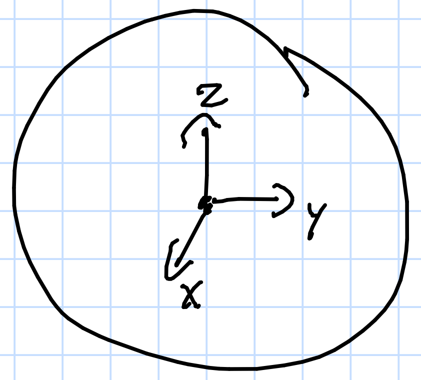
$$\dot{\theta}_z(t) = -\frac{\ddot{x}_c t}{r}$$

$$P(t) = \vec{\tau}_e \cdot \frac{d\vec{\theta}}{dt}$$
$$= (-\cancel{r} m \ddot{x} \hat{k}) \cdot \left(-\frac{\ddot{x}_c t}{\cancel{r}} \hat{k}\right)$$
$$= m \ddot{x}_c^2 t = f v_{xc}(t)$$



1b) No

1c)



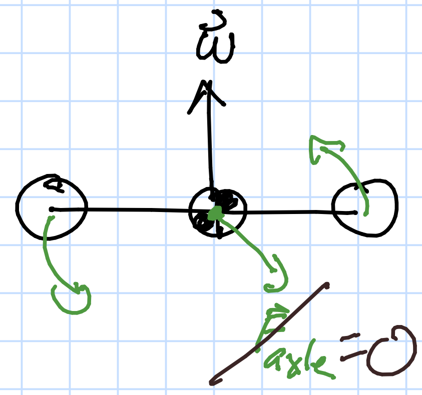
sphere is completely symmetric ∴

$$I_{xx} = I_{yy} = I_{zz} = \frac{2mr^2}{5} \qquad I_{xy} = I_{xz} = I_{yz} = 0$$

1d)

1) $\Sigma \vec{F} = m\ddot{\vec{x}}_c = 0$

2) $\Sigma \vec{\tau} = \dot{\vec{L}}$



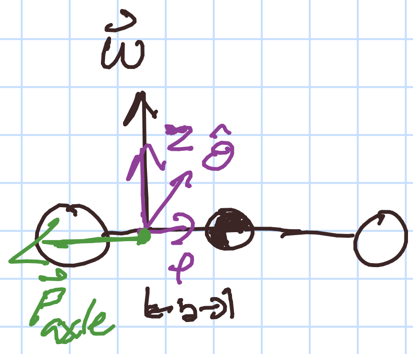
by symmetry

$$\vec{L} = I_{zz_c}\omega_z\hat{k}$$

$$\dot{\vec{L}} = I_{zz_c}\dot{\omega}_z\hat{k} = 0$$

$$\Sigma\vec{\tau} = 0$$

3) No



4) $\Sigma \vec{F}_{ext} = m\ddot{\vec{x}}_c$

$\hat{\rho}$: $F_{axle\,\rho} = m(\cancel{\ddot{r}_c} - \omega^2 b)$

$$\boxed{\vec{F}_{axle} = -m\omega^2 b\,\hat{\rho}}$$

5)

$$\vec{L}_o = M\vec{r}_c \times \dot{\vec{r}}_c + I_{zz}\omega_z\hat{k}$$

$$= (Mb^2 + I_{zz_c})\omega_z\hat{k}$$

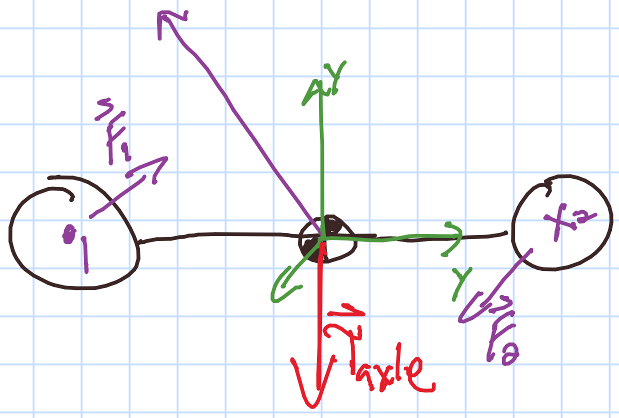
$$\dot{\vec{L}} = \underbrace{(Mb^2 + I_{zz_c})}_{I_{zz_o}}\dot{\omega}_z \boxed{= 0 = \tau_\phi}$$

6) $I_{zzo} = Mb^2 + I_{zzc}$

$$\boxed{I_{zzo} > I_{zzc}}$$

7) $\Sigma\vec{F}_{ext} = m\ddot{\vec{x}}_c = 0 \rightarrow$ NO

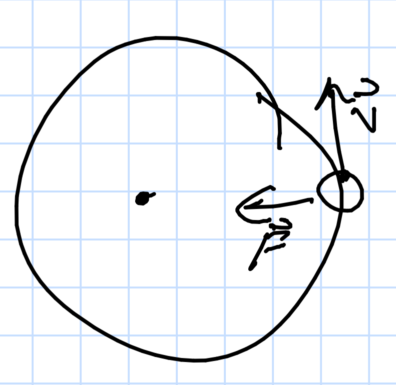
8)



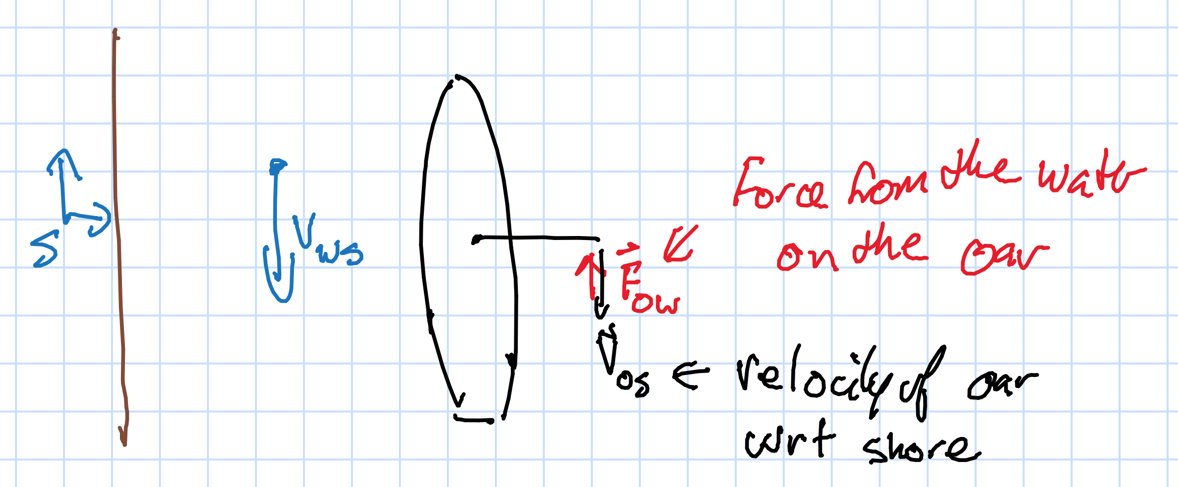
$\Sigma\vec{\tau} \neq 0$

1e) No

$$P = \vec{F} \cdot \vec{v}$$
$$= F\hat{r} \cdot v\hat{\theta} = 0$$



1f)



$$P = \vec{F}_o \cdot \vec{V} = -F_{ow} V_{os} \neq 0$$

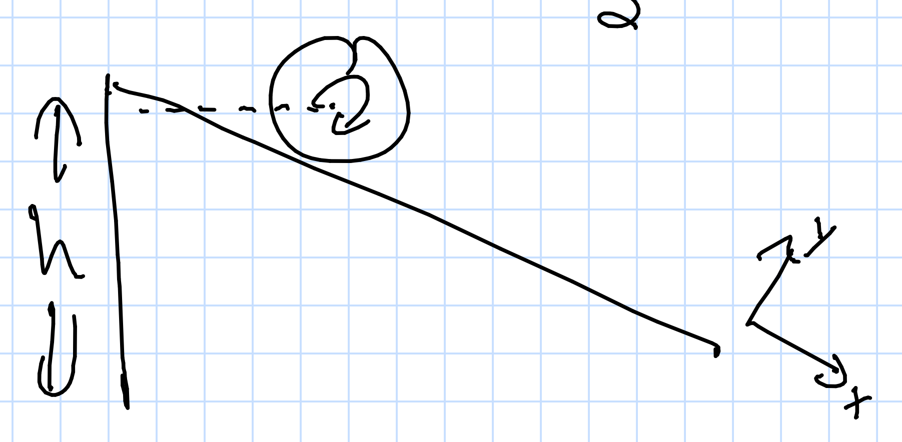
if he stops rowing $\vec{F}_{ow} = 0$ ∴

$$P = \vec{F}_o \cdot \vec{V}_{os} = 0$$

No work is being done

1g) $I_{zz_{Hollow}} = mr^2$

$$I_{zz_{solid}} = \frac{mr^2}{2} < I_{zz\,hollow}$$



@ $t=0$ $\quad E = K + U$
$$= 0 + mgh$$

@ $t=$end $\quad E = \frac{1}{2} I_{zz}\omega_z^2 + \frac{1}{2} m v_c^2$

since Energy is conserved

$$mgh = \frac{1}{2} I\omega_z^2 + \frac{1}{2} m v_c^2$$

from no slip $r\omega_z = v_c$ ∴

$$mgh = \frac{1}{2} I \left(\frac{v_c}{r}\right)^2 + \frac{1}{2} m v_c^2$$

$$2gh = \left[\frac{I}{mr^2} + 1\right] v_c^2$$

$$\sqrt{\frac{2gh}{\frac{I_{zz}}{mr^2} + 1}} = v_c$$

Since $I_{zz\,hollow} > I_{zz_{solid}}$

$$\boxed{V_{c\,Hollow} < V_{c\,solid}}$$

)

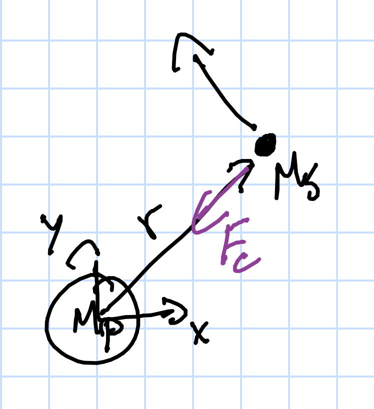
$$\vec{L} = m\vec{r} \times \vec{v}$$

$$\frac{d}{dt}\vec{L} = \frac{d}{dt}\left[m\vec{r} \times \vec{v}\right]$$
$$= m\,\cancel{\dot{\vec{r}} \times \vec{v}} + m\vec{r} \times \dot{\vec{v}}$$
$$= 0 + \vec{r} \times m\dot{\vec{v}}$$
$$= \vec{r} \times \vec{F}$$

$$\boxed{\frac{d}{dt}\vec{L} = \vec{\tau}}$$

removed

1h)



escape velocity

$E = \text{constant} = U + K$

$$E(0) = -\frac{GM_1M_s}{r(0)} + \frac{1}{2} m_s v^2(0)$$

as $t \to \infty$ $\quad r \to \infty$ for v_{escape}

$$E(\infty) = -\frac{GM_1M_s}{r(\infty)} + \frac{1}{2} m_s v^2(\infty)$$
$$= 0 + \frac{1}{2} m_s v^2(\infty)$$
$$E(\infty) = E(0) = \frac{1}{2} m_s v^2(\infty)$$

$$-\frac{GM_1M_s}{r(0)} + \frac{1}{2} M_s v^2(0) \geq 0$$

$$v^2(0) \geq \frac{2GM_1M_s}{r(0)}$$

$$\boxed{v(0) \geq \sqrt{\frac{2GM_1M_s}{r(0)}} \equiv V_{esc}}$$

$$\vec{F}_c = -\frac{GM_p\cancel{M_s}}{r^2}\hat{r} = \cancel{M_s}(\underbrace{\cancel{\ddot{r}}}_{0} - r\dot{\theta}^2)\hat{r}$$

$\hat{r}$:
$$-\frac{GM_p}{r^{\cancel{2}}} = -r\dot{\theta}^2 = \frac{v_\theta^2}{\cancel{r}}$$

$$\frac{GM_p}{r} = v_\theta^2$$

$$\sqrt{\frac{GM_p}{r}} = v_\theta$$

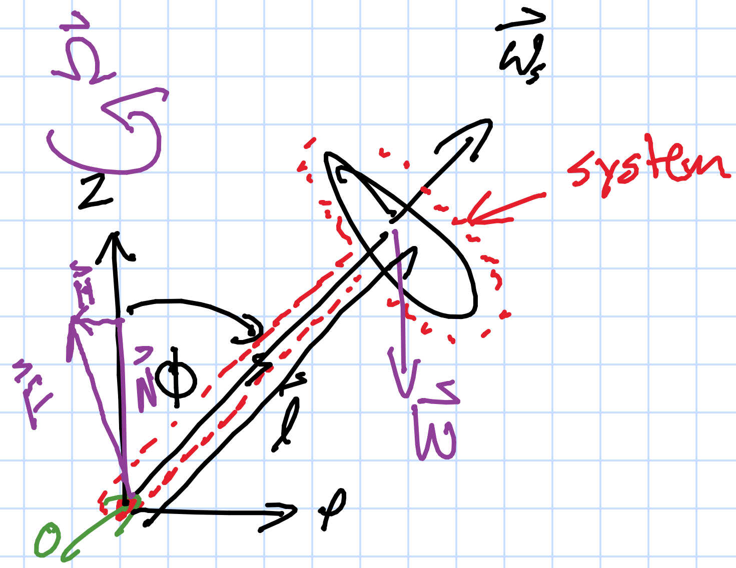
$$\boxed{V_{esc} = \sqrt{2}\, v_\theta}$$

1i)

$$\vec{F} = -\nabla U$$
$$= -\frac{\partial}{\partial r}\left[-U_0 \frac{r_0}{r} e^{-r/r_0}\right]\hat{r}$$
$$= U_0 r_0 \frac{\partial}{\partial r}\left[\frac{e^{-r/r_0}}{r}\right]\hat{r}$$
$$= \cancel{U_0 r_0}\left[\frac{+1}{r^2}\cancel{e^{-r/r_0}} + \frac{1}{r}\frac{r}{r} \cdot \frac{+1}{r_0}\cancel{e^{-r/r_0}}\right]\hat{r}$$

$$\boxed{\vec{F} = -\frac{U_0 r_0 e^{-r/r_0}}{r^2}\left(1 + \frac{r}{r_0}\right)\hat{r}}$$

2) Derive the precession frequency



Let's measure $\vec{L}$ about the fixed point

Let's calculate $\vec{L}_o$

$$\vec{L}_o = \vec{L}_c + m\vec{l}\times\vec{v}$$

$$= I_o\vec{\omega}_s + \bar{\bar{I}}\vec{\Omega} \qquad + m\vec{l}\times[\vec{\Omega}\times\vec{l}]$$

$\lim\ \bar{\Omega} << \vec{\omega}_s$

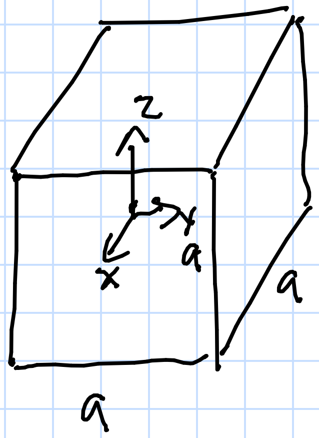
$$\vec{L}_o \sim I_o\vec{\omega}_s = I_o\omega_s[\cos(\phi)\hat{k} + \sin(\phi)\hat{\rho}]$$

$$\Sigma\vec{\tau} = lW\sin(\phi)\,\hat{\theta} = \frac{d\vec{L}_o}{dt} = I_o\omega_s\sin(\phi)\,\Omega\,\hat{\theta}$$

$$\Omega = \frac{lW\cancel{\sin(\phi)}}{I_o\omega_s\cancel{\sin(\phi)}} = \boxed{\frac{lW}{I_o\omega_s} = \Omega}$$

$\boxed{\text{as } \omega_s\downarrow \quad \Omega\uparrow}$

3-1



$$I_{zz} = \int dm\,(x^2+y^2)$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \rho\,(x^2+y^2)\,dx\,dy\,dz$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \rho\,x^2\,dx\,dy\,dz + \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \rho\,y^2\,dx\,dy\,dz$$

$$= \rho a^2 \left|\frac{1}{3}x^3\right|_{-\frac{a}{2}}^{\frac{a}{2}} + \rho a^2 \left|\frac{1}{3}y^2\right|_{-\frac{a}{2}}^{\frac{a}{2}}$$

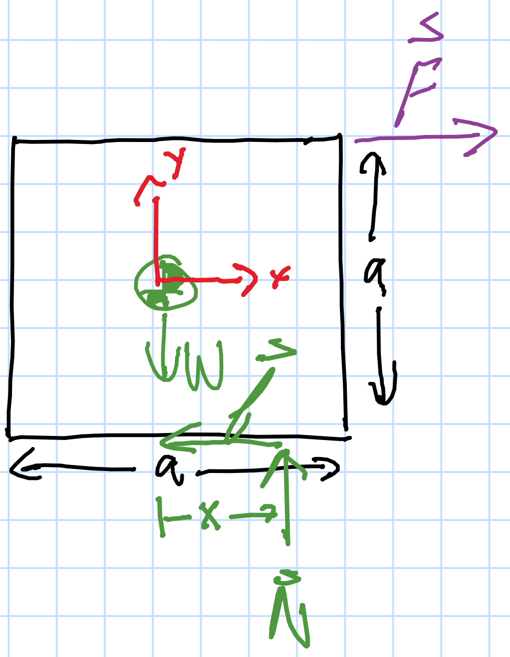
$$= 2\rho a^2\left(\frac{1}{3}\left(\frac{a}{2}\right)^3 - \frac{1}{3}\left(-\frac{a}{2}\right)^3\right)$$

$$= 4\rho a^2\,\frac{1}{3}\left(\frac{a}{2}\right)^3$$

$$= \frac{1}{6}\rho a^5 = \frac{Ma^2}{6}$$

$$\boxed{I_{zz} = \frac{Ma^2}{6}}$$

5) Slip or Tumble



the location of $\vec{N}$ $\left[\int \frac{d\vec{N}}{dx} dx\right]$ can be any where along the bottom of the cube ... i.e. x is variable, and can be anywhere from $-\frac{a}{2} < x < \frac{a}{2}$.

For equilibrium:

$$\sum \vec{F} = 0$$

$$\sum \vec{\tau}_c = 0 \quad \leftarrow \text{Let's calculate } \tau \text{ about the C of M}$$

$\underline{\sum \vec{F} = 0}$

$\hat{i}$: $F - f = 0$

$\hat{j}$: $N - W = 0$

$$\boxed{\begin{matrix} f = F \\ N = W \end{matrix}}$$

$\sum \vec{\tau} = 0$

$\hat{k}$: $-a/2\, F - \frac{a}{2} f + xN = 0$

$$-aF + xW = 0 \quad \Rightarrow \quad \boxed{x = \frac{aF}{W}}$$

At what F would the cube begin to slip?

the cube slips when $\quad F_{slip} = f_{slip} = \mu_s N = \mu_s W$

$$F_{slip} = \mu_s W$$

At what F would the cube begin to tumble?

the cube tumbles when

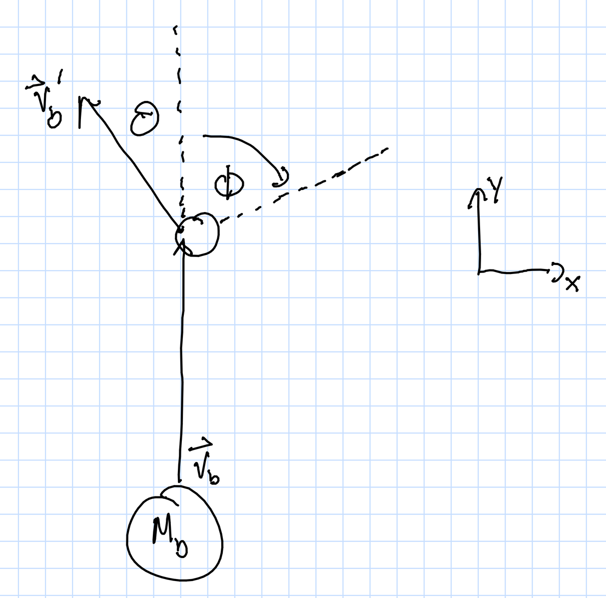
$\hat{k}$ $\quad \sum \tau \neq 0 \quad \ldots \quad x_{tumble} = \frac{a}{2}$

$$\frac{\cancel{a} F_{tumble}}{W} = \frac{\cancel{a}}{2}$$

$$F_{tumble} = \frac{W}{2}$$

$$\boxed{\begin{matrix} \text{if } \mu_s < \frac{1}{2} \text{ then } F_{slip} < F_{tumble} : \text{cube slips} \\ \text{if } \mu_s > \frac{1}{2} \text{ then } F_{tumble} < F_{slip} : \text{cube tumbles} \end{matrix}}$$

4)



a) v_b' ?

$\vec{P}$ conserved for the system
E conserved for the system

Before		After	
Momentum			
$\hat{i}$: $P_x = 0$	=	$P_x' = M_p v_p' \sin(\phi) + M_b v_b' \sin(\theta)$	1)
$\hat{j}$: $P_y = M_b v_x$	=	$P_y' = M_p v_p' \cos(\phi) + M_b v_b' \cos(\theta)$	2)
Energy			
$\frac{1}{2} M_b v_b^2$	=	$\frac{1}{2} M_p v_p'^2 + \frac{1}{2} M_b v_b'^2$	3)

3 unknowns ⇒ 3 equations: We can solve!!

from 1) $P_{bx}' = M_b v_b' \sin(\theta) = M_p v_p' \sin(\phi)$
from 2) $P_{by}' = M_b v_b' \cos(\theta) = M_b v_b - M_p v_p' \cos(\phi)$

putting into 3

$$\frac{1}{2} M_b v_b^2 = \frac{1}{2} M_p v_p'^2 + \frac{P_{bx}'^2}{2M_b} + \frac{P_{by}'^2}{2M_b}$$

$$= \frac{1}{2} M_p v_p'^2 + \frac{[M_p v_p' \sin(\phi)]^2}{2M_b} + \frac{[M_b v_b - M_p v_p' \cos(\phi)]^2}{2M_b}$$

$$\left[\frac{1}{2} M_b v_b^2 = \frac{1}{2} M_p v_p'^2 + \frac{M_p^2 v_p'^2 \sin^2(\phi)}{\cancel{2M_b}} + \frac{M_b^2 v_b^2 - 2 M_b M_p v_b v_p' \cos(\phi) + M_p^2 v_p'^2 \cos^2(\phi)}{\cancel{2M_b}}\right] 2M_b$$

$$\cancel{M_b^2 v_b^2} = M_b M_p \cancel{v_p'^2} + M_p^2 \cancel{v_p'^2} \sin^2(\phi) + \cancel{M_b^2 v_b^2} - 2 M_p M_b v_b v_p' \cos(\phi) + M_p^2 \cancel{v_p'^2} \cos^2(\phi)$$

$$0 = \left[M_b M_p + M_p^2 \cos^2(\phi) + M_p^2 \sin^2(\phi)\right] v_p'^2 - 2 M_p M_b v_b \cos(\phi) v_p'$$

$$0 = \left[M_b M_p + M_p^2\right] v_p'^2 - 2 M_p M_b v_b \cos(\phi) v_p'$$

$$v_p' = 0 \quad \text{or} \quad v_p' = \frac{2 \cancel{M_p} M_b v_b \cos(\phi)}{M_b \cancel{M_p} + M_p^{\cancel{2}}} = \frac{2 M_b v_b \cos(\phi)}{M_b + M_p}$$

4-2: speed of the ball after collision (v_b')

From 3) $\frac{1}{2} M_b v_b'^2 = \frac{1}{2} M_b v_b^2 - \frac{1}{2} M_p v_p'^2$

$$= \cancel{\tfrac{1}{2}} M_b v_b^2 - \cancel{\tfrac{1}{2}} M_p \left[\frac{2 M_b \cos(\phi)}{M_b + M_p} v_b\right]^2$$

$$\cancel{\tfrac{1}{2} M_b} v_b'^2 = \cancel{\tfrac{1}{2} M_b} v_b^2 \left[1 - \frac{4 M_b M_p \cos^2(\phi)}{(M_b + M_p)^2}\right]$$

$$v_b' = v_b \sqrt{1 - \frac{4 M_b M_p \cos^2(\phi)}{(M_b + M_p)^2}}$$

4-3: θ: from 1:

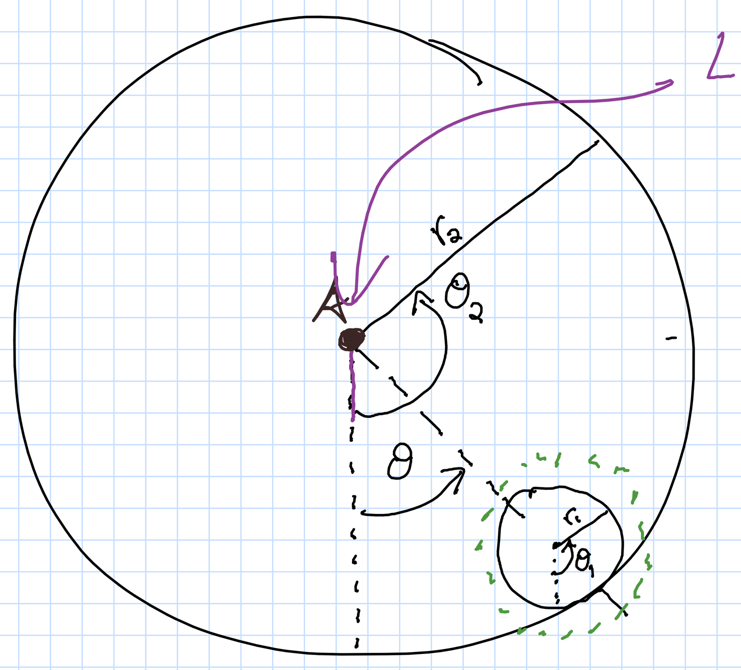
$$M_b v_b' \sin(\theta) = M_p v_p' \sin(\phi)$$

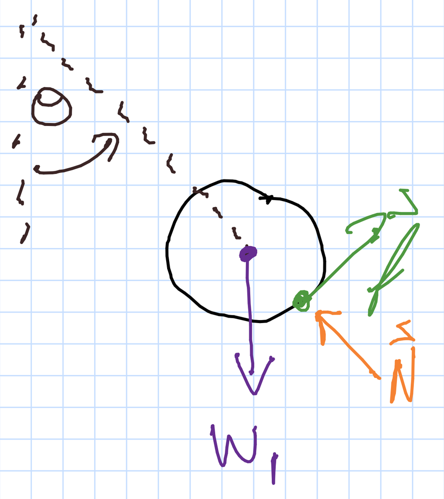
$$\sin(\theta) = \frac{M_p v_p' \sin(\phi)}{M_b v_b'}$$

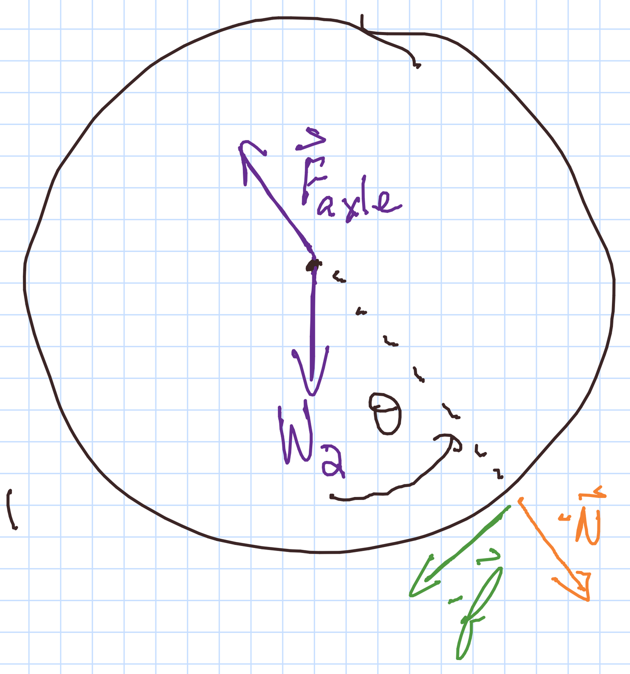
$$\sin(\theta) = \frac{M_p \left[\frac{2 \cancel{M_b} \cancel{v_b} \cos(\phi)}{M_b + M_p}\right]}{\cancel{M_b} \cancel{v_b} \sqrt{1 - \frac{4 M_b M_p \cos^2(\phi)}{(M_b + M_p)^2}}}$$

$$\boxed{\sin(\theta) = \frac{2 M_p \cos(\phi)}{\sqrt{(M_b + M_p)^2 - 4 M_b M_p \cos^2(\phi)}}}$$

Let's Calculate I about axle







$\sum \vec{\tau}_{1A} = \frac{d\vec{L}_{1A}}{dt}$

Let's calculate $\vec{L}_{1a}$ [the angular momentum of 1 about the axle]

$\vec{L}_{1a} = \vec{L}_{1C} + m_1 \vec{r}_{ca} \times \vec{v}_{ca}$

$= m_1 r_1^2 \dot{\theta}_1 \hat{k} + m_1 (r_2 - r_1)^2 \dot{\theta}$

$$\hat{k}: \boxed{-(r_2-r_1) W_1 \sin(\theta) + r_2 f = m_1 r_1^2 \ddot{\theta}_1 + m_1 (r_2-r_1)^2 \ddot{\theta}} \quad \text{eq 1}$$

$\sum \vec{\tau}_{2A} = \frac{d\vec{L}_{2A}}{dt}$

What is $\vec{L}_{2A}$?

$\vec{L}_{2A} = M_2 r_2^2 \dot{\theta}_2 \hat{k}$

$$\hat{k} \cdot \sum \vec{\tau}_{2A} = \boxed{-r_2 f = M_2 r_2^2 \ddot{\theta}_2} \quad \text{eq 2}$$

4 unknowns $[f \; \ddot{\theta} \; \ddot{\theta}_1 \; \ddot{\theta}_2]$
4 equations : we can solve for θ!

$\hat{\theta} \cdot \sum \vec{F} = m\vec{a} \cdot \hat{\theta}$ for particle 1

$\hat{\theta}: \; f - W_1 \sin(\theta) = m_1 (\cancel{2[R_2-R_1]\dot{\theta}_c} + [R_2-R_1]\ddot{\theta}_c]$

$$\boxed{f - W_1 \sin(\theta) = m_1 (R_2 - R_1) \ddot{\theta}} \quad \text{eq 3}$$

No slip condition:

$\vec{V}_{12} = \vec{0} = (R_2 - R_1)\dot{\theta} + R_1 \dot{\theta}_1 - R_2 \dot{\theta}_2$

$$\boxed{\dot{\theta} = \frac{R_2 \dot{\theta}_2 - R_1 \dot{\theta}_1}{R_2 - R_1}} \quad \text{eq 4}$$

Notice $\boxed{\dot{\theta} = 0 \text{ when } \dot{\theta}_2 = \frac{R_1}{R_2} \dot{\theta}_1}$

$[R_2 - R_1] f - [R_2 - R_1] W \sin(\theta) = m [R_2 - R_1]^2 \ddot{\theta}_c$ → eq. 3 $(R_2 - R_1)$

$R_2 f \quad - [R_2 - R_1] W \sin(\theta) = m [R_2 - R_1]^2 \ddot{\theta}_c + m r_1^2 \ddot{\theta}_1$ → eq. 1

$-R_1 f = -m_1 r_1^2 \ddot{\theta}_1$

$-R_2 f = m_2 r_2^2 \ddot{\theta}_2$ → eq. 2

solve for f → $f = m_1 r_1 \ddot{\theta}_1 = -m_2 r_2 \ddot{\theta}_2 \Rightarrow \ddot{\theta}_2 = \frac{-m_1 r_1 \ddot{\theta}_1}{m_2 r_2}$ → found $\ddot{\theta}_2(\ddot{\theta}_1)$

$-(R_2 - R_1) \sin(\theta_c) W_1 + R_2 (m_1 r_1 \ddot{\theta}_1) = m_1 r_1^2 \ddot{\theta}_1 + m_1 (r_2 - r_1)^2 \ddot{\theta}_c$

$-(R_2 - R_1) \sin(\theta_c) \cancel{W_1}^{g} = \cancel{m_1} r_1 (r_1 - R_2) \ddot{\theta}_1 + \cancel{m_1} (r_2 - r_1)^2 \ddot{\theta}_c$

$\sin(\theta_c) g = r_1 \ddot{\theta}_1 - (r_2 - r_1) \ddot{\theta}_c$ → all we need to do is rewrite $\ddot{\theta}_1$ in terms of $\ddot{\theta}$

$(R_2 - R_1) \ddot{\theta}_c = R_2 \ddot{\theta}_2 - R_1 \ddot{\theta}_1$ ⇒ No slip (4)

$= \cancel{R_2} \left[\frac{-m_1 r_1}{m_2 \cancel{r_2}} \right] \ddot{\theta}_1 - R_1 \ddot{\theta}_1$

$(R_2 - R_1) \ddot{\theta}_c = -r_1 \left[\frac{m_1}{m_2} + 1 \right] \ddot{\theta}_1$

$$\boxed{\ddot{\theta}_1 = \frac{-(R_2 - R_1) \ddot{\theta}_c}{R_1 \left(\frac{m_1}{m_2} + 1 \right)}}$$

$\sin(\theta_c) g = \cancel{r_1} \left[\frac{-(\cancel{R_2 - R_1}) \ddot{\theta}_c}{\cancel{R_1} \left(\frac{m_1}{m_2} + 1 \right)} \right] - \cancel{(r_2 - r_1)} \ddot{\theta}_c$

$$\boxed{\sin(\theta_c) g = -(R_2 - R_1) \left[\frac{1}{\left(\frac{m_1}{m_2} + 1 \right)} + 1 \right] \ddot{\theta}_c}$$