Problem 1. Simple Computation and Basic Conceptual Physics Problems (65 pts)

a) (5 pts) The resistance of a material to a sharp object penetrating it is

$$\vec{F} = -kx^4\hat{i} \tag{1}$$

Calculate the work done to force a sharp object a distance d into the material.

b) (5 pts) Why is it more difficult to do a sit-up with your hands behind your head than when your arms are folded across your chest?

c) (5 pts) Does the total work on a system depend on the choice of inertial reference frame? Explain

d) (5 pts) Mechanical Advantage: On midterm 1, you showed that with the block and tackle lifting system in Fig. 1 left you could lift a mass with a weight W, with a force of only W/6. How is this possible from an energy perspective? Does the work done by you on the mass you are lifting a distance h, depend on the number of pulleys in your block and tackle lifting system?

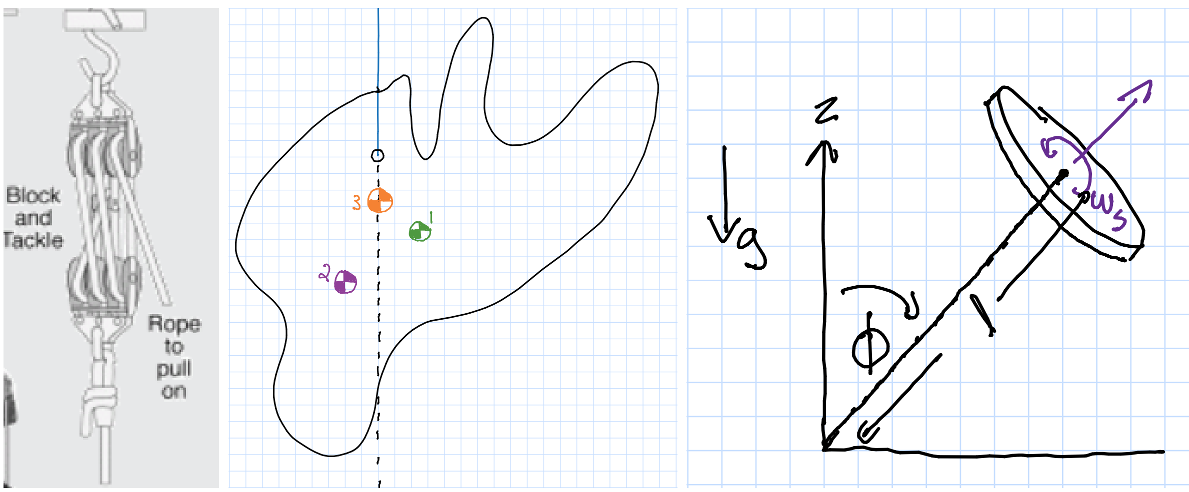

Figure 1: left: block and tackle lifting system middle: an arbitrary rigid body that is hanging from a string and can freely rotate right: A gyroscope

e) (5 pts) A rigid metal body of arbitrary 3D shape (it's thickness could vary hugely) is **statically** hanging from a thin thread (it can rotate about the point of attachment). Which of the 3 points in Fig. 1 middle is a possible center of mass? Explain (draw FBD).

f) (5 pts) Derive the precession angular velocity for a gryoscope (mass M, moment of inertia about the spin axis, I, axle length l, angle with respect to vertical,ϕ (Fig. 1 right).

g) (5 pts) An astronaut floats freely in a weightless environment. Describe how the astronaut can move her limbs so as to rotate her head and torso so that she is looking in the opposite direction? (Explicitly define the various systems and draw vectors for the various angular momentums).

h) (5 pts) When accelerating, an engine applies a torque to the axle through a set of interlocking mechanical gears (a transmission). How does this torque cause the car to accelerate forward?

Explain (draw FBD, write equations of motions and constraints). What happens if the car is on a frictionless surface?

i) (5 pts) For a rigid object to be in static equilibrium two conditions must be satisfied:

$$\sum \vec{F} = 0$$
$$\sum \vec{\tau} = 0$$

Prove that this second condition is true for torques referenced about **any** origin.

j) (5 pts) A coiled spring of mass m rests upright on a table, if you compress the spring by pressing down with your hand and then release it, can the spring leave the table? Explain, using conservation of energy?

k) (5 pts) The total energy E of an object of mass m that moves in one dimension (defined by the coordinate x) under the influence of only conservative forces can be written as

$$E = 1/2mv_x^2 + U(x) \tag{2}$$

show that $\frac{dE}{dt} = 0$ leads to Newton's 2nd law for conservative forces in 1D.

l) (5 pts) Why do tightrope walkers carry a long-narrow balancing beam?

m) (5 pts) Show that the Kinetic energy of a system of particles can be split into 2 terms, translational energy of the particles with respect to the center of mass plus the translational kinetic energy of an effective particle with the total mass of the system and the velocity of the center of mass.

Problem 2. Estimating Moment of Inertia and center of mass for weird objects (10 pts)
A **uniform** circular plate with surface density σ of radius R_0 (centered at C) has a circular hole of radius R_1 (centered a distance h away from C) cut out of it (Fig. 2 left).

a) (5 pts) Where is the center of mass of this resulting object?

b) (5 pts)What is the moment of inertia about an axis perpendicular to the page at point C?

Hint 1: think of the rigid body as a composite object of 2 pieces and use subtraction.
Hint 2: the moment of inertia for cylinder with uniform density about its center of mass is $\frac{1}{2}MR^2$

Problem 3. Playing Pool (15 pts)
Calculate the height, h, at which to hit a pool (billiards/snooker) ball such that it will roll without slipping immediately after losing contact with the cue stick no matter how big or small the coefficient of static/dynamic friction between the table and ball (Fig. 2). Assume that the cue ball is a constant density sphere (radius r, mass M and moment of inertia about the center of mass is $2/5MR^2$) that is originally at rest on a horizontal pool table before a **horizontal** impulse is given to it by the cue stick is given to it (aside: though not a part of the problem, on your way home think about the interaction between pool ball and the cue stick. Why do people put chalk on the end of their cue stick?)

Problem 4. Scattering (15 pts) Prove that in an elastic collision of 2 objects of identical mass, with one being a target initially at rest, the angle between their final velocity vectors will be 90 deg.

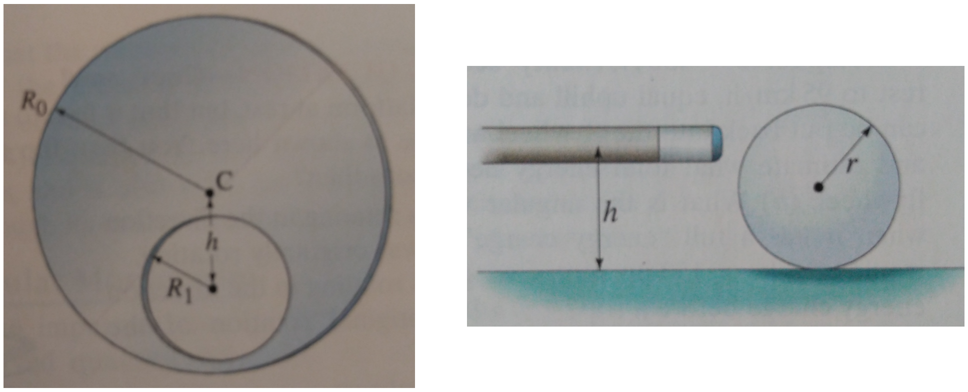

Figure 2: left: A somewhat complicated object right: a pool cue hitting a cue ball

Problem 5. Unspooling Atwood's Machine with massive pulley and friction (15 pts)
A bunch of massless string is stored on a drum with moment of inertia I_2 and radius R_2. The end of the string is connected to a mass that is hung over the side of a pulley with moment of inertia, I_1 and radius R_1 (Fig. 3). The pulley is badly oiled and has a frictional torque about it's axle of τ_f. What is the acceleration of the mass as a function of time?

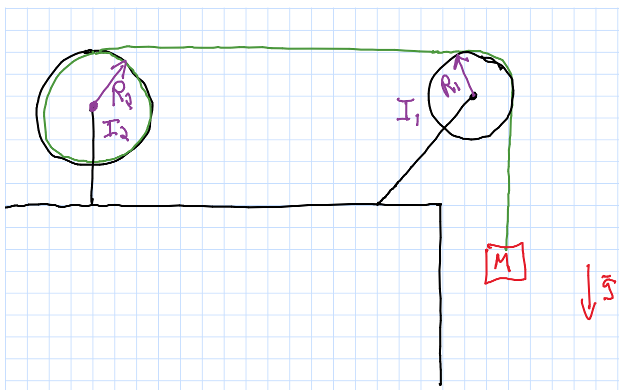

Figure 3: left: An Atwood machine with friction