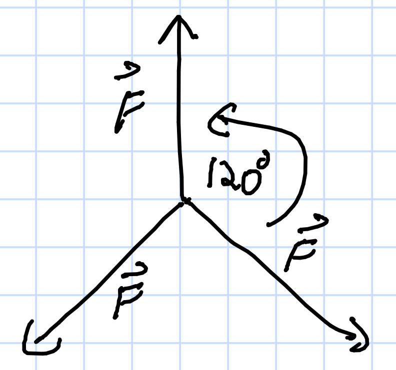
Midterm #1 solutions

Physics 5A

a)



$\Sigma \vec{F} = 0$

b) can you conclude that $\vec{a} = 0$ if $|V|$ is constant?

- NO.

$$|V| = \sqrt{\vec{v}\cdot\vec{v}} \quad \therefore$$

$$\frac{d}{dt}|V| = \frac{d}{dt}\sqrt{\vec{v}\cdot\vec{v}}$$

$$= \frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{\vec{v}\cdot\vec{v}}}\cdot\cancel{2}\frac{d\vec{v}}{dt}\cdot\vec{v}$$

$$\frac{d}{dt}|V| = \frac{d\vec{v}}{dt}\cdot\hat{v} \Rightarrow$$

if $\frac{d\vec{v}}{dt} \perp \vec{v}$ then $\frac{d}{dt}|V| = 0$

c)

Morse curve

$$U(r) = D\left(1 - e^{-a[r-r_0]}\right)^2$$

$$F = -\frac{\partial U}{\partial r} = -2D\left(1 - e^{-a[r-r_0]}\right)\left(-a\, e^{-a(r-r_0)}\right)$$

$$\boxed{F = \underbrace{2Da}_{D'}\left(1 - e^{-a(r-r_0)}\right)e^{-a(r-r_0)}}$$

What is the atomic spring constant k @ the equilibrium point

$$\frac{\partial F}{\partial r} = -\underbrace{2Da}_{D'}\left[\left(-\,-a\, e^{-a(r-r_0)}\right)e^{-a(r-r_0)} + \left(1 - e^{-a(r-r_0)}\right)-a\, e^{-a(r-r_0)}\right]$$

$$= -\underbrace{2Da}_{D'}\left[a\, e^{-2a(r-r_0)} - a\, e^{-a(r-r_0)} + a\, e^{-2a(r-r_0)}\right]$$

$$= -\underbrace{2Da}_{D'}\left[2a\, e^{-2a(r-r_0)} - a\, e^{-a(r-r_0)}\right]$$

$$\frac{\partial F}{\partial r}(r_0) = -\underbrace{2Da}_{D'}\left[2a - a\right]$$

$$= -D'a$$

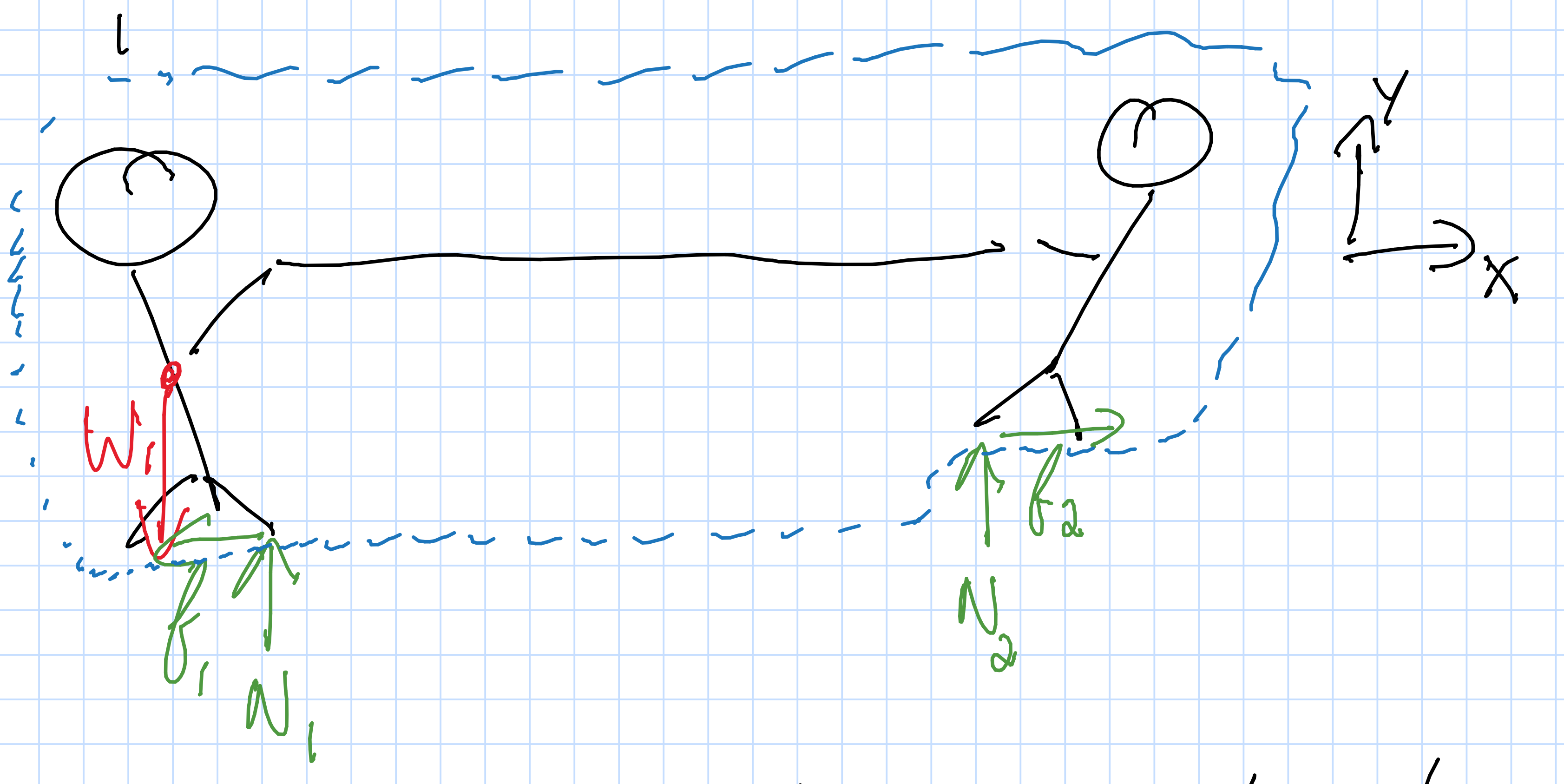
$\therefore$

$$\vec{F}(r) \sim 0 + \frac{\partial F}{\partial r}(r_0)\,(r - r_0) + \dots$$

$$\sim -\underbrace{D'a}_{k}(r - r_0)$$

$$\boxed{k = D'a}$$

e)



Let's take both person 1 and person 2 as the system

$$(M_1 + M_2)\ddot{X}_{cm} = f_2 - f_1$$

- whoever has the bigger frictional force wins!

How do you have a big frictional force?

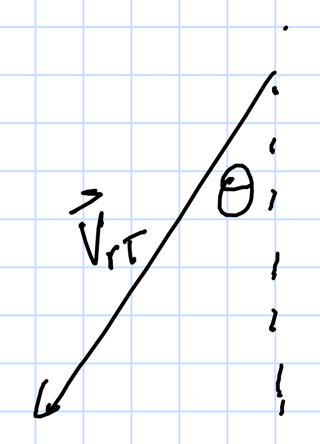
- $f = \mu_s N$

$= \mu_s W \quad \therefore$

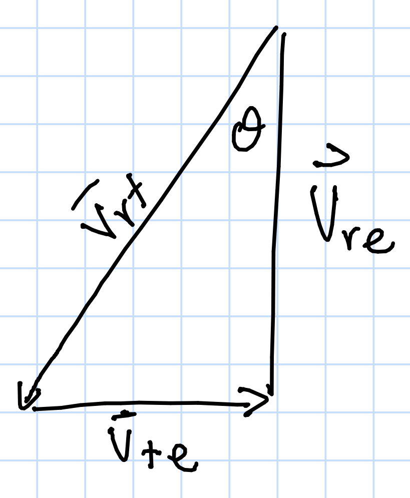
$= \mu_s M g$

- the team with greater mass usually wins
- the team with higher μ_s usually wins

f)

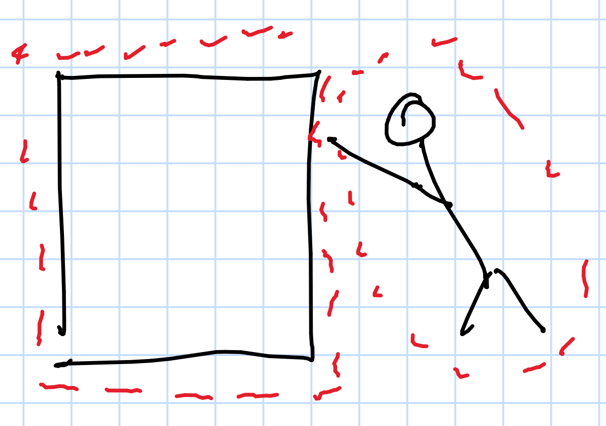


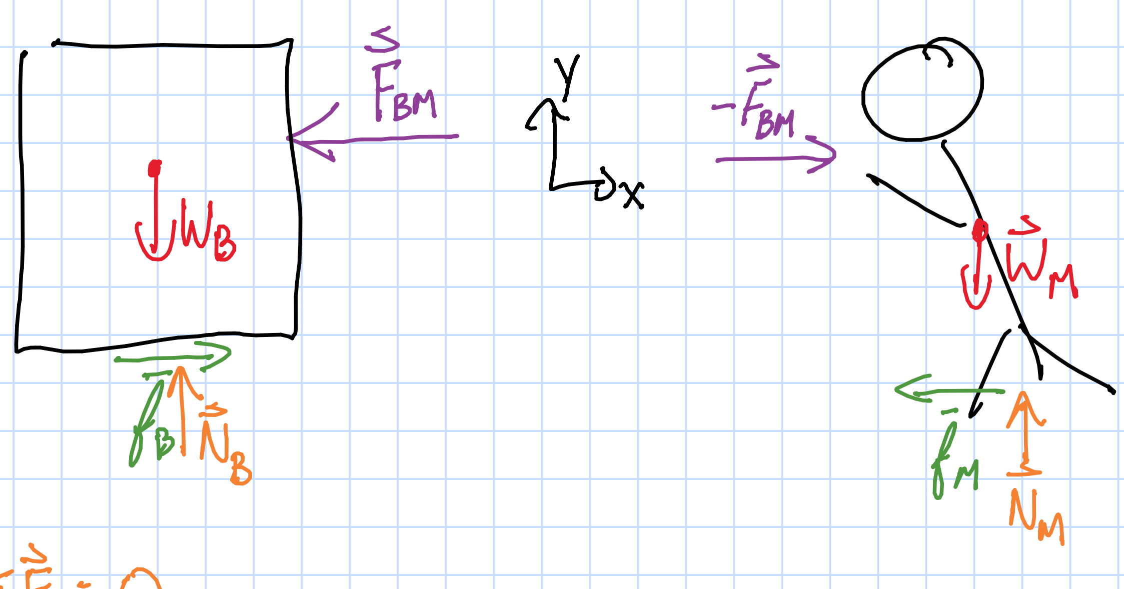
$$\vec{V}_{re} = \vec{V}_{rt} + \vec{V}_{te}$$



$$\frac{V_{te}}{V_{re}} = \tan(\theta) \Rightarrow \boxed{\frac{V_{te}}{\tan(\theta)} = V_{re}}$$

g)





for Box: $\Sigma\vec{F} = 0$

$\hat{i}$: $-F_{BM} + f_B = 0$

$\hat{j}$: $-W_B + N_B = 0 \Rightarrow N = W_B$

$f_B \leq \mu_s N_B = \mu_s W_B$

Box will move if $F_{BM} > \mu_s W_B$

For Matt:

$\Sigma\vec{F} = 0$

$\hat{i}$: $-F_{BM} + f_M = 0$ $\quad f_M = F_{BM}$

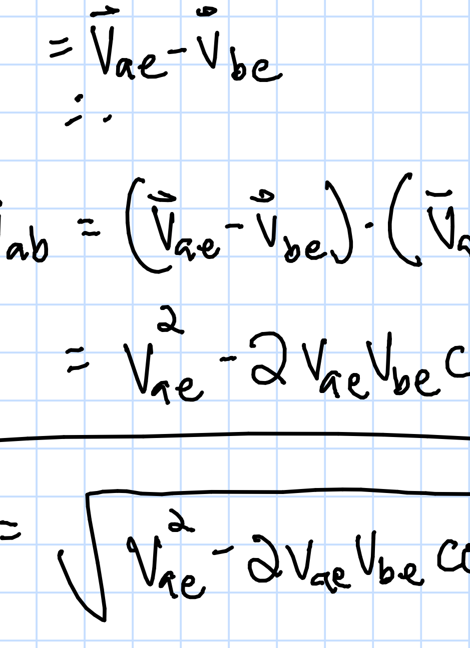
$\hat{j}$: $-W_M + N_M = 0 \Rightarrow N_M = W_M$

$F_{BM} = \mu_s W_M$

since $F_{BM} = \mu_s W_M \leq \mu_s W_B$

$\boxed{\text{the box will never move}}$

h)



$$\vec{V}_{ae} = \vec{V}_{ab} + \vec{V}_{be}$$

$$\vec{V}_{ae} - \vec{V}_{be} = \vec{V}_{ab}$$

$$\vec{V}_{ab} = \vec{V}_{ae} - \vec{V}_{be}$$

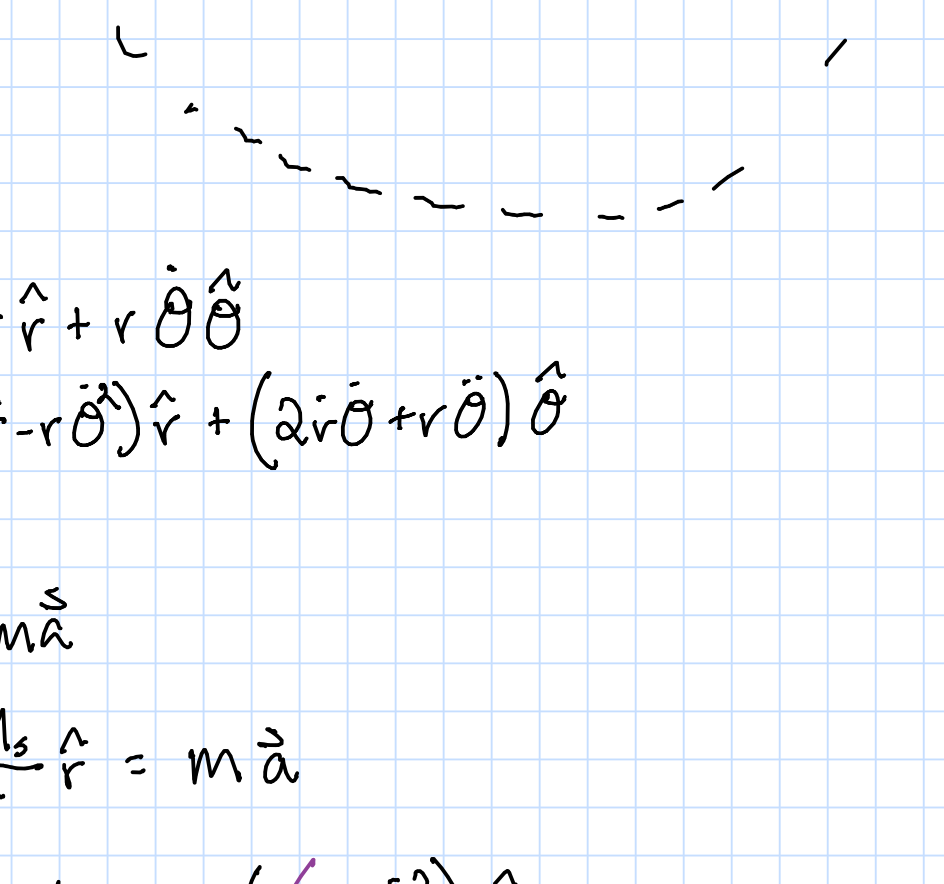
$\therefore$

$$\vec{V}_{ab} \cdot \vec{V}_{ab} = (\vec{V}_{ae} - \vec{V}_{be}) \cdot (\vec{V}_{ae} - \vec{V}_{be})$$

$$V_{ab}^2 = V_{ae}^2 - 2V_{ae}V_{be}\cos(\phi - \theta) + V_{be}^2$$

$$\boxed{V_{ab} = \sqrt{V_{ae}^2 - 2V_{ae}V_{be}\cos(\phi-\theta) + V_{be}^2}}$$

i)



$$\vec{V} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$\vec{F} = m\vec{a}$$

$$-G\frac{mM_s}{r^2}\hat{r} = m\vec{a}$$

$\hat{r}$: $-\frac{GmM_s}{r^2} = m(\cancel{\ddot{r}} - r\dot{\theta}^2)\hat{r}$

$\hat{\theta}$: $0 = m(\cancel{2\dot{r}\dot{\theta}} + \cancel{r\ddot{\theta}})$

for simple circular motion : $r = \text{constant}$ $\therefore$

$\dot{r} = 0$

$\ddot{r} = 0$

$\dot{\theta} = \text{constant}$

$\ddot{\theta} = 0$

$$-\frac{G\cancel{m}M_s}{r^2} = -\cancel{m}r\dot{\theta}^2$$

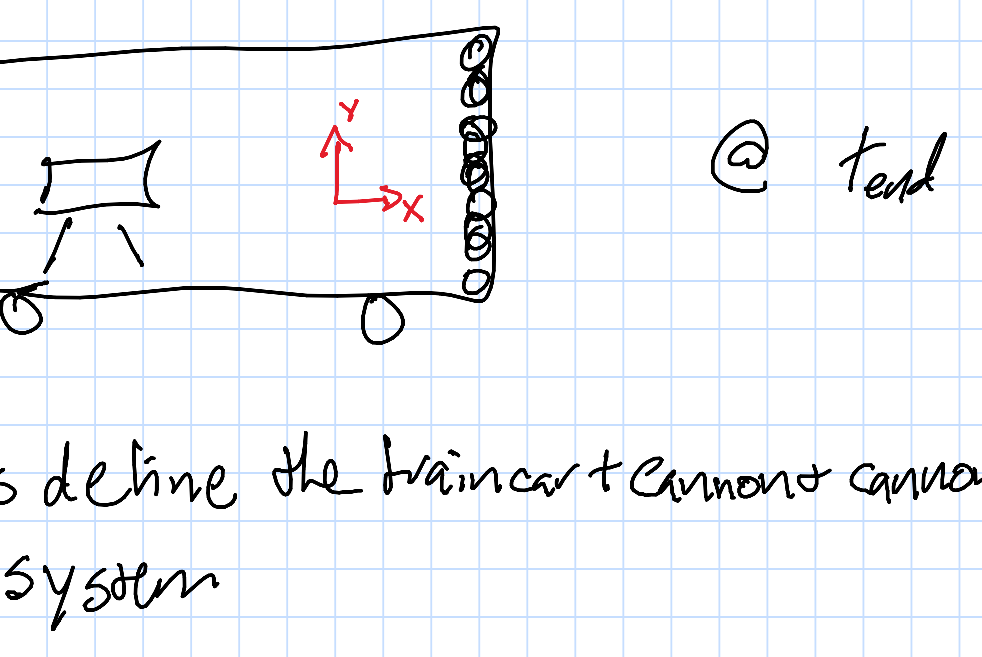
$$-GM_s = r^3\dot{\theta}^2$$

$$\dot{\theta} = \frac{2\pi}{T} \quad \therefore$$

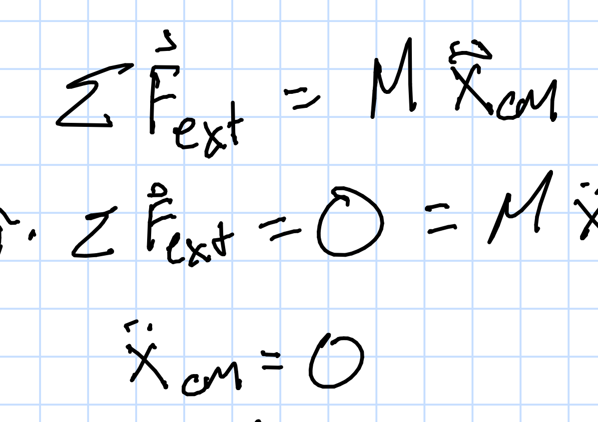
$$-GM_s = r^3\left(\frac{2\pi}{T}\right)^2$$

$$\boxed{\frac{-GM_s}{(2\pi)^2} = \frac{r^3}{T^2}}$$

j)



@ $t=0$



@ t_{end}

Let's define the traincar + cannon + cannonballs as my system

$$\Sigma\vec{F}_{ext} = M\ddot{\vec{X}}_{cm}$$

$\hat{i}$. $\Sigma\vec{F}_{ext} = 0 = M\ddot{x}_{cm}$ $\therefore$

$$\ddot{x}_{cm} = 0$$

$\therefore$

$$\int_0^{t_{end}} \ddot{x}_{cm}\,dt = \dot{x}_{cm}(t_{end}) - \dot{x}_{cm}(0)$$

$$\boxed{0 = \dot{x}_{cm}(t_{end})} - \cancel{\dot{x}_{cm}(0)} \quad = 0$$

$$\int_0^{t_{end}} \dot{x}_{cm}\,dt = x_{cm}(t_{end}) - x_{cm}(0)$$

$$0 = x_{cm}(t_{end}) - x_{cm}(0)$$

$$\Delta X_{cm} = 0$$

$$x_{cm}(0) = \frac{M_{ct}\cdot 0 + nM_B\frac{-l}{2}}{M_{ct} + nM_B} = \frac{-nM_B\, l/2}{(M_{ct} + nM_B)}$$

$$x_{cm}(t_{end}) = \frac{M_{ct}x_{ct} + nm_B(x_{ct} + l/2)}{M_{ct} + nm_B}$$

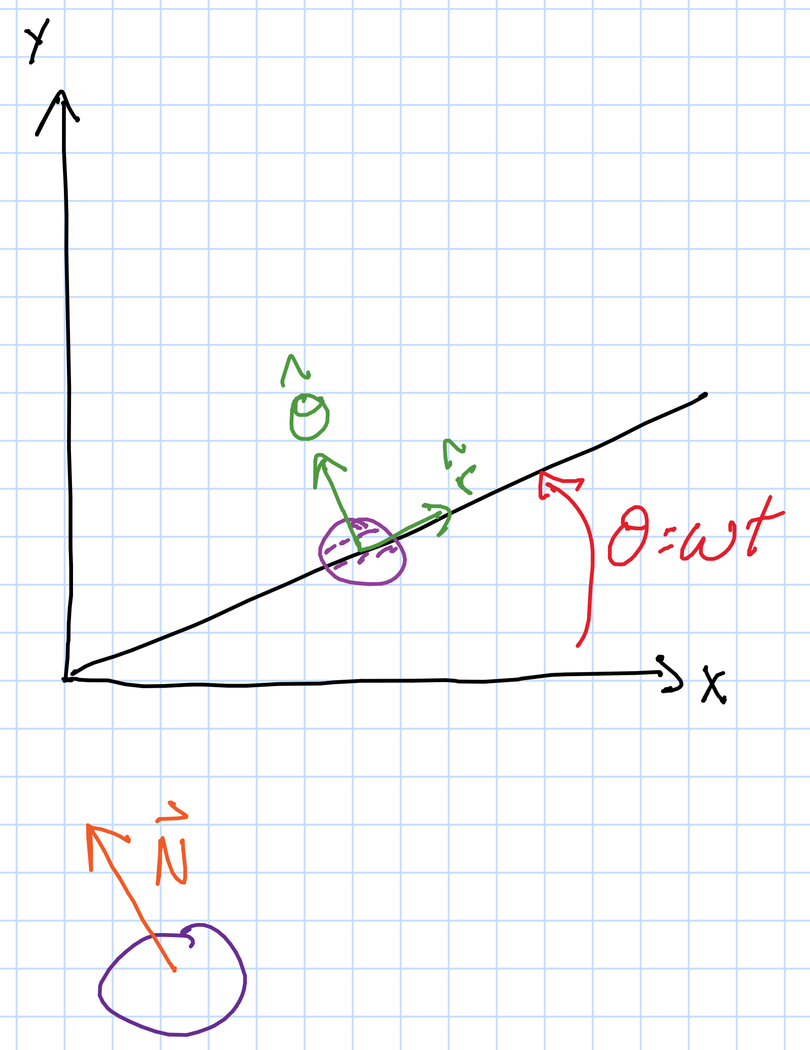
$$= x_{ct} + \frac{nm_B}{M_{ct} + nm_B}\frac{l}{2}$$

$$x_{cm}(t_{end}) = x_{cm}(0)$$

$$x_{ct}(t_{end}) + \frac{nm_B}{M_{ct} + nm_B}\frac{l}{2} = \frac{-nm_B\, l/2}{M_{ct} + nm_B}$$

$$\boxed{x_{ct}(t_{end}) = \frac{nm_B\, l/2}{M_{ct} + nm_B}}$$

2)



the only force on the bead in the horizontal plane is the Normal force from the rod

$$\vec{F} = m\vec{a}$$

$$\hat{r}: \; 0 = m[\ddot{r} - r\dot{\theta}^2]$$

$$\hat{\theta}: \; N = m[2\dot{r}\dot{\theta} + \cancel{r\ddot{\theta}}] \Rightarrow \boxed{N = 2m\dot{r}\dot{\theta}}$$

cancels

$\hat{\theta}$ equation defines the value of the normal force

$$\theta(t) = \omega t$$

$$\dot{\theta}(t) = \omega$$

$$\ddot{\theta}(t) = 0$$

$$\ddot{r} - r\omega^2 = 0$$

this is just a ODE

let's assume $r = a e^{\lambda t}$ then

$$\lambda^2 \cancel{a e^{\lambda t}} - \cancel{a e^{\lambda t}} \omega^2 = 0$$

$$\lambda^2 = \omega^2$$

$$\boxed{\lambda = \pm \omega}$$

$$r(t) = a e^{\omega t} + b e^{-\omega t}$$

let's find a + b

$$r(0) = a + b = 0 \quad \therefore \; b = -a$$

$$r(t) = a[e^{\omega t} - e^{-\omega t}]$$

$$\dot{r}(t) = \omega a[e^{\omega t} + e^{-\omega t}]$$

$$\dot{r}(0) = 2\omega a = V_r(0)$$

$$a = \frac{V_r(0)}{2\omega}$$

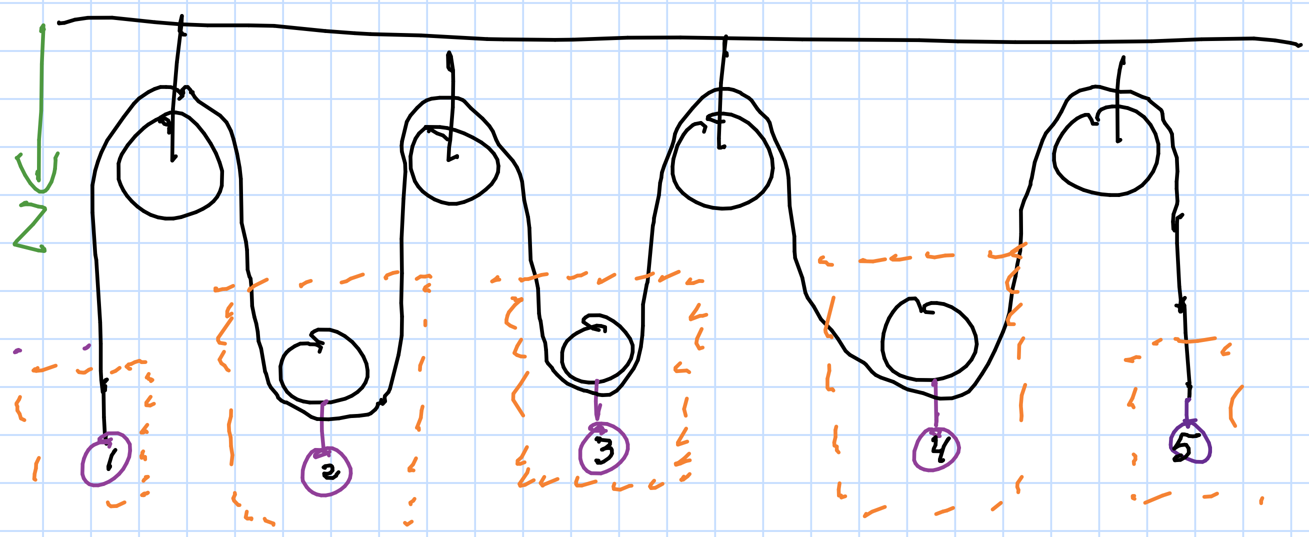
$$\boxed{r(t) = \frac{V_r(0)}{2\omega}[e^{\omega t} - e^{-\omega t}]}$$

$$\dot{r}(t) = \frac{V_r(0)}{2}[e^{\omega t} + e^{-\omega t}]$$

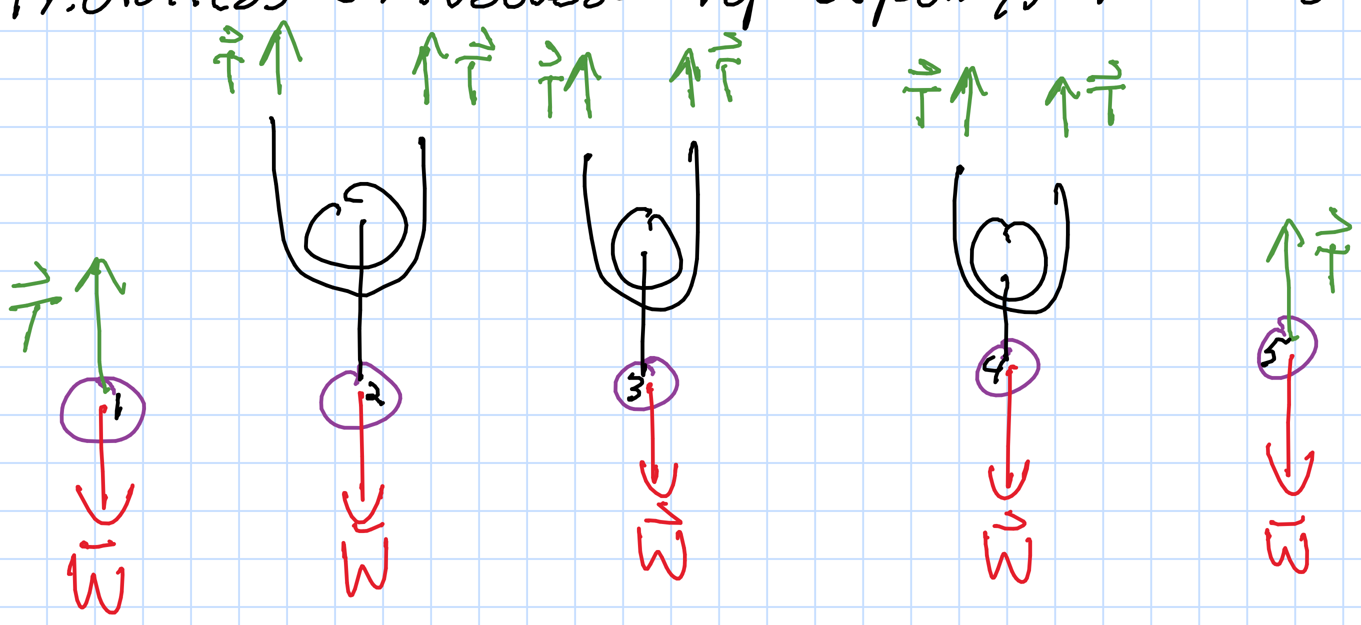
$$N = 2m\dot{r}\dot{\theta} = \frac{\cancel{2} m V_r(0)[e^{\omega t} + e^{-\omega t}]\omega}{\cancel{2}}$$

$$\boxed{\vec{N} = m\omega V_r(0)[e^{\omega t} + e^{-\omega t}]\,\hat{\theta}}$$

3)



- Frictionless & massless ropes + pulleys T = constant!



Ball 1

$\hat{k}$: $mg - T = m\ddot{z}_1$

Ball 2

$\hat{k}$: $\left[mg - 2T = m\ddot{z}_2\right] \times 2$

Ball 3

$\hat{k}$: $\left[mg - 2T = m\ddot{z}_3\right] \times 2$

Ball 4

$\hat{k}$: $\left[mg - 2T = m\ddot{z}_4\right] \times 2$

Ball 5

$\hat{k}$: $mg - T = m\ddot{z}_5$

5 eq's
6 unknowns ~

constraint: $l_{rope} = z_1 + 2z_2 + 2z_3 + 2z_4 + z_5 + 7 \cdot \pi r_{pulley}$

$$\dot{l}_{rope} = 0 = \dot{z}_1 + 2\dot{z}_2 + 2\dot{z}_3 + 2\dot{z}_4 + \dot{z}_5$$

$$\ddot{l}_{rope} = 0 = \ddot{z}_1 + 2\ddot{z}_2 + 2\ddot{z}_3 + 2\ddot{z}_4 + \ddot{z}_5$$

sum all 5 eq's together to solve for T

$$8mg - 14T = m\left[\ddot{z}_1 + 2\ddot{z}_2 + 2\ddot{z}_3 + 2\ddot{z}_4 + \ddot{z}_5\right] = 0$$

$\therefore$

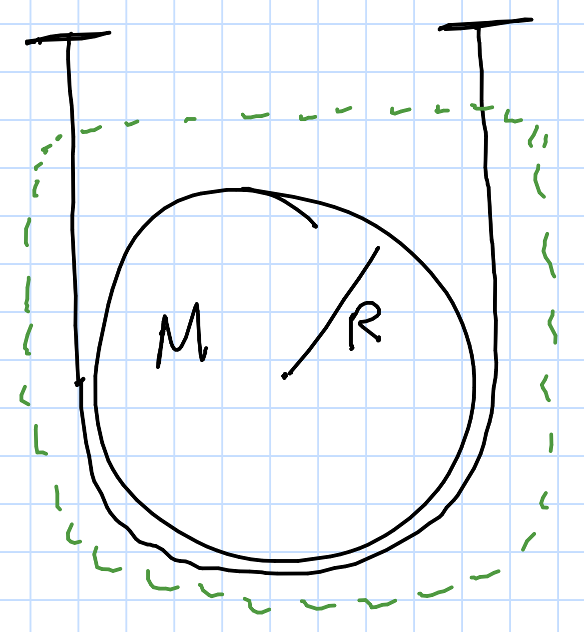
$$T = \frac{8mg}{14} = \frac{4}{7}mg$$

$$\ddot{z}_1 = \ddot{z}_5 = \frac{mg - \frac{4mg}{7}}{m} = \boxed{\frac{3}{7}g = \ddot{z}_1 = \ddot{z}_5}$$

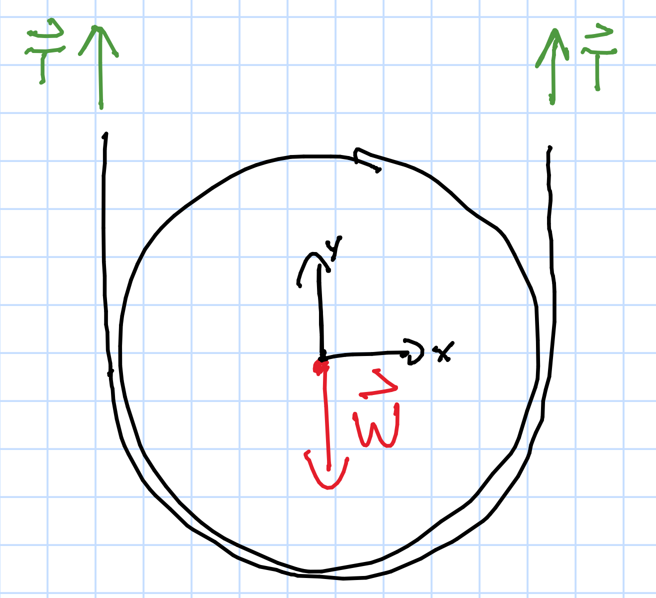
$$\ddot{z}_2 = \ddot{z}_3 = \ddot{z}_4 = \frac{mg - 2 \cdot 4mg}{7} = \boxed{-\frac{1}{7}g = \ddot{z}_2 = \ddot{z}_3 = \ddot{z}_4}$$

Problem 4

Supporting a disk



FBD:

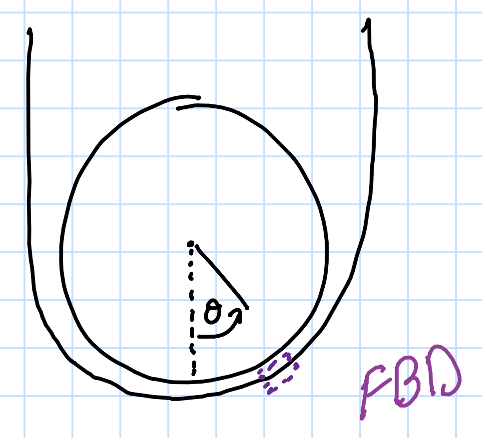


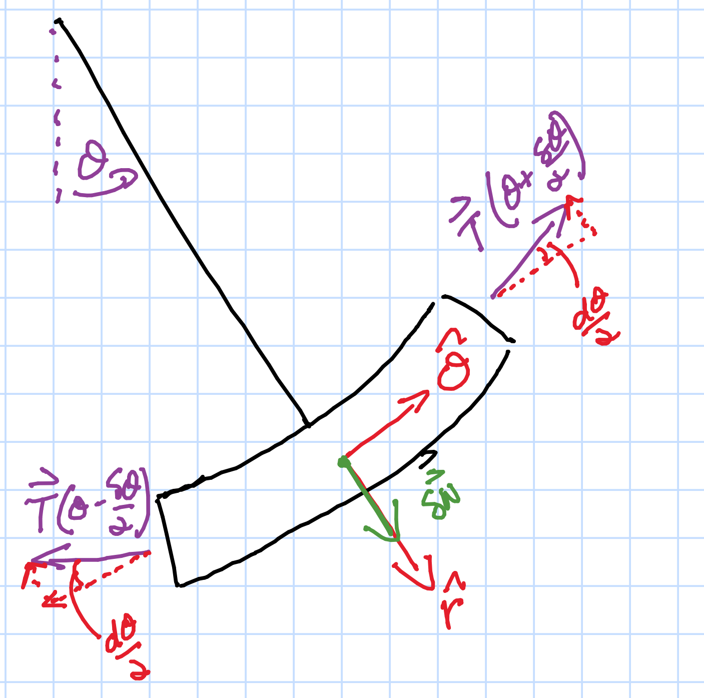
a) What is the T in the string @ the endpoints

$$\Sigma \vec{F} = m\vec{a} = 0$$

$$\hat{j}: \quad 2T - W = 0 \quad \Rightarrow \quad T = \frac{W}{2}$$

b) What is the normal force per unit length





$$\Sigma \vec{F} = 0 = \vec{T}\left(\theta - \frac{\delta\theta}{2}\right) + \vec{T}\left(\theta + \frac{\delta\theta}{2}\right) + \delta\vec{N}$$

$$\hat{r}: \quad 0 = -T\left(\theta - \frac{\delta\theta}{2}\right)\sin\left(\frac{\delta\theta}{2}\right) - T\left(\theta + \frac{\delta\theta}{2}\right)\sin\left(\frac{\delta\theta}{2}\right) + \delta N$$

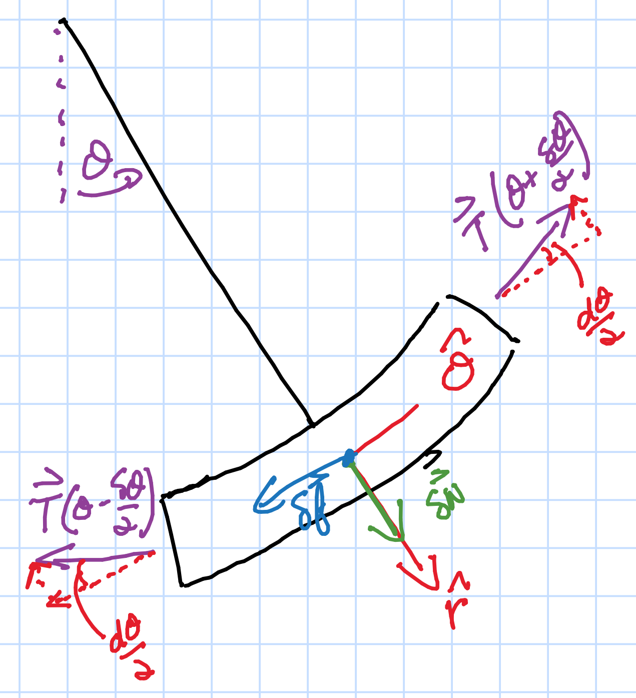
divide by $\delta\theta$ & take the lim $\delta\theta \to 0$

$$0 = -T(\theta) + \frac{dN}{d\theta} \quad \Rightarrow \quad \frac{dN}{d\theta} = T = \frac{Mg}{2}$$

or

$$\boxed{\begin{aligned}\frac{dN}{dl} &= \frac{dN}{d\theta}\cdot\frac{d\theta}{dl} \\ &= \frac{Mg}{2}\frac{1}{R}\end{aligned}}$$

b) let's add friction to the above scenario



$$\hat{\theta}: \quad 0 = T\left(\theta + \frac{\delta\theta}{2}\right)\cos\left(\frac{\delta\theta}{2}\right) - T\left(\theta - \frac{\delta\theta}{2}\right)\cos\left(\frac{\delta\theta}{2}\right) + \delta f$$

divide by $\delta\theta$ & take the lim $\delta\theta \to 0$

$$0 = \frac{\left[\cancel{T(\theta)} + \frac{\partial T}{\partial\theta}\frac{\delta\theta}{2}\right]\left[1 + \ldots\right] - \left[\cancel{T(\theta)} - \frac{\partial T}{\partial\theta}\frac{\delta\theta}{2}\right]\left[1 + \ldots\right] - \delta f}{\delta\theta}$$

$$0 = \frac{\partial T}{\partial\theta} - \frac{\partial f}{\partial\theta} \quad \Rightarrow \quad \boxed{\frac{\partial T}{\partial\theta} = \frac{\partial f}{\partial\theta}}$$

since $\delta f = \mu_s \delta N$

$$\frac{\partial T}{\partial\theta} = \mu_s \frac{\partial N}{\partial\theta} = \mu_s T$$

$$\frac{\frac{\partial T}{\partial\theta}}{T} = \mu_s$$

$$\int_0^{\pi/2} \frac{\frac{\partial T}{\partial\theta}}{T}\, d\theta = \int_0^{\pi/2} \mu_s\, d\theta$$

$$\ln(T)\Big|_0^{\pi/2} = \mu_s \pi/2$$

$$\ln\left(\frac{T(\pi/2)}{T(0)}\right) = \frac{\mu_s \pi}{2}$$

$$\frac{T(\pi/2)}{T(0)} = e^{\mu_s \pi/2}$$

$$T(0) = T(\pi/2)\, e^{-\frac{\mu_s \pi}{2}}$$

$$\boxed{T(0) = \frac{Mg}{2} e^{-\mu_s \pi/2}}$$