Physics 5A Fall 2016: Midterm 1 (100 pts)

Problem 1. Simple Computation and Basic Conceptual Physics Problems (45 pts)
The following questions are short computation questions. Please answer these questions in less than a few sentences or with just a few lines of computation. If you are doing more than 4 lines of algebra, you are doing something wrong.

a) (5 pts) Taylor Expansions: Expand $(1+x)^{1/4}$ about x=0 to 2nd order

b) (5 pts) Understanding Polar Coordinates

- (1 pts) What is $\frac{\partial}{\partial r}\hat{r}$?
- (2 pts) What is $\frac{\partial}{\partial \theta}\hat{r}$?
- (2 pts) What is $\frac{\partial}{\partial \theta}\hat{r} \cdot \hat{r}$? Why?

c) (5 pts) Basic Kinematics
$\vec{a}(t) = bt^2\hat{i} + c\cos(\omega t)\hat{j}$ for a particle with the initial conditions $\vec{v}(0) = d\hat{i}$ and $\vec{x}(0) = e\hat{j}$

- (2 pts) Calculate $\vec{v}(t)$
- (3 pts) Calculate $\vec{x}(t)$

d) (5 pts) Geostationary Orbit:

A satellite that circularly orbits the earth in the plane of the equator with an angular velocity equal to that of the earth's rotation about its axis, ω, is said to be in geostationary orbit. This is a very popular orbit for communication satellites because ground based communication antennas don't have to constantly move to track the satellite. Calculate the radius of the orbit, r_g, in terms of G, mass of the earth, M_e, and ω.

e) (5 pts) I measure my weight with a spring based scale at both the north pole and at the equator. Assuming that my mass and the mass of my clothes is identical for both measurements, will the scale measure the same value? If not which is bigger?

f) (5 pts) Jumping on a scale: Draw a free body diagram of a person standing on a stiff scale. Graph the normal force between the person and the scale as a function of time.

- For $t <= 0$: the person is just standing still.
- For $0 < t < t_{air}$: the person is in the act of jumping
- For $t > t_{air}$: the person is in the air

g) (5 pts) Block and Tackle Demo

Zoe is using a block and tackle with 3 pulleys on the top and 3 pulleys on the bottom (see the Fig. 1 left or the DEMO in the front of the room) to lift Matt up into the air. If the pulleys are frictionless and the rope is massless, what is the force Zoe needs to apply to lift Matt up?

h) (5 pts) Constraints: On a bike (Fig. 1 center), the front sprocket has a radius of R_f, while the sprocket on the back wheel has radius of R_r and the back wheel has a radius of R_w. If the wheel is rolling but not slipping and the person is pedaling with an angular velocity of ω_f, what is the speed of the bike?

Figure 1: left: Close up of picture of the pulleys in the block and tackle lifting system. center: Transmission on a bicycle. right: wheel

i) (5 pts) Rolling wheels:

A wheel is rolling on a surface with a non-zero coefficient of friction as shown in Fig. 1 right. if $v_x > \omega R$, what is the magnitude and direction of the frictional force?

Problem 3. Space Elevator (35 pts)

It takes a lot of money to put a kg of material into space. One idea to make space travel significantly cheaper is to use a "space elevator" to get out of the earth's gravitational well. The basic idea is to take a super strong **massive** cable (with density ρ and cross sectional area A) and fix one end at some point on the equator and stretch the other end radially out well past the geostationary orbital radius, r_g.

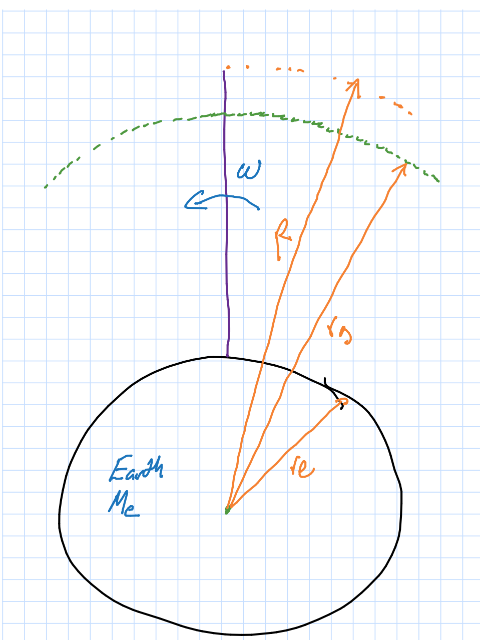

Figure 2: Space Elevator

a) (5 pts) Draw a free body diagram and write the equations of motion for a differential element of the cable

b) (2 pts) Rewrite your differential equation (specifically M_eG) in terms of the geostationary radius,r_g (which you found above), and the earth's rotation rate about it's axis, ω. (this will help you gain intuition since all terms can be easily compared)

c) (3 pts) Draw $\frac{dT}{dr}$. Where is the tension in the rope maximum?

d) (2 pts) What is the tension of the rope at its free end, $r = R$?

e) (3 pts) Though super strong, the rope is also flimsy. What happens if the tension, $T(r) < 0$ anywhere along the rope?

f) (5 pts) Let's assume that the cross sectional area of the cable,A, is constant. Calculate and graph the tension in the rope (in terms of the constants). Find the equation for the minimum allowable radius for the rope endpoint (don't try to solve it though).Note: we should really account for the weight of the car and the material that is being carried up the space elevator ... but we're going to let that slide for right now. If you account for this, then the minimum allowable length of rope is a tinge longer.

g) (3 pts) In 1975 Jerome Pearson figured out that the space elevator could be a lot lighter (and thus a lot cheaper) if the cross sectional area of the rope varied with radius, $A(r)$. Where should the cross sectional area of the rope be maximum?

h) (5 pts) Find the differential equation for A(r) assuming that the stress, $\sigma = T(r)/A(r)$ is constant at all radius and equal to the breaking stress, σ_{yield}. For this calculation, let's assume the absolute minimum allowable length.

i) (5 pts) Calculate A(r)

j) (2 pts) We want to make the rope out of materials that are strong and lightweight. What mathematical combination of ρ and σ_{yield} should we attempt to maximize in our material selection? Order the 3 materials below in suitability:

Material	density $\rho\left[\frac{g}{cm^3}\right]$	Maximum Allowable Stress $\sigma_{yield}\left[\frac{N}{m^2}\right]$
Titanium	4.51	1.3×10^9
Zylon (like Kevlar)	1.54	5.8×10^9
Carbon-Epoxy composite	1.58	1.2×10^9

Note: If you plug and chug all the numbers with a calculator you'll find that even with the best of these materials, the space elevator isn't possible. All we need is a material that is x10 stronger and the same weight as the best material shown in the table, and it becomes feasible (i.e. the maximum area is within x100 of the minimum area.). Carbon nanotubes could easily do it ...

Problem 4. Chain Heap and Block (20 pts)
A chain with mass density of λ in [kg/m] lies in a heap on the floor with one end attached to a block of mass M. The block is given a sudden kick and instantly acquires a speed v_o. Let x be the distance traveled by the block (Fig. 3). There is no friction in this problem; none with the floor, none in the chain itself.

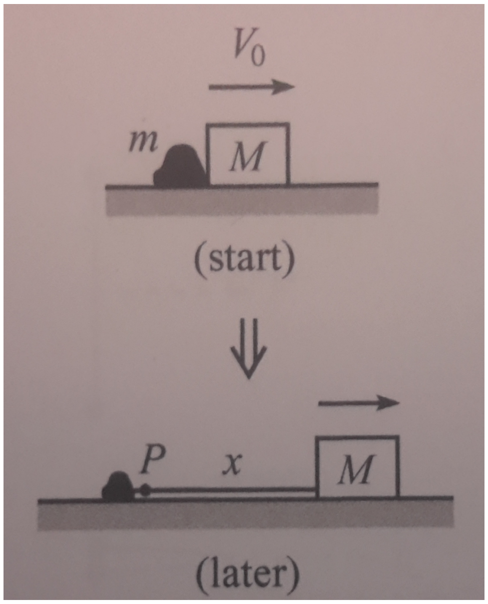

Figure 3: Chain heap and block

Choose your own adventure: Option 1: Take as your system the mass of the box and all of chain (both in the heap and stretched out):

a) (5 pts) What is the total momentum of the system immediately following the sudden kick?

b) (5 pts) What is the velocity of the block after it has travelled a distance x?

Option 2:

a) (10 pts) Using techniques developed for the rocket/fluid mechanics, what is the velocity of the M after it has travelled a distance x?

Finally,
(10 pts [hard]) What is the tension in the chain just to the right of the heap (at point P in the diagram)?
Hint: both ways get you the same answer, but from a calculation perspective option 1 is significantly easier.
Hint 2: Before frantically working, think carefully about what your system should be for part c