Midterm Exam # 1
Physics 137B, Spring 2004

PLEASE MAKE SURE YOU WRITE YOUR NAME AND STUDENT ID ON YOUR EXAM.

This exam contains 3 questions, each with multiple parts. You should answer all the questions to the best of your ability. Please show your all work. If you use paper rather than a blue book, make sure you staple all the pages together.

Calculators are not necessary. You may take ONE sheet of $8\frac{1}{2} \times 11$ inch paper with equations into this exam.

DO NOT TURN THE PAGE TO OPEN THIS EXAM UNTIL YOU ARE TOLD TO!!

1. **(30 Points)** Consider a quantum mechanical dumbbell. The Hamiltonian for the system is:

$$H_0 = \frac{L^2}{2I}$$

   (a) What are the possible energy values that the dumbbell can have and what is the degeneracy for each energy eigenvalue?

   (b) Suppose we now consider the case where the one end of the dumbbell is a charged particle with spin $\frac{1}{2}$. Let us add a new term to our Hamiltonian so that $H = H_0 + H'$ with

$$H' = \lambda \vec{L} \cdot \vec{S}$$

   (λ is a small number). Use first order perturbation theory to determine the energies and degeneracies of the $\ell = 0$ and $\ell = 1$ eigenstates.

2. **(40 Points)** Tell whether each of the following statements is True or False and give your reasons. Your answers to this question need only be a sentence or two for each part; no long calculations are required but in some cases you may need to write down an equation as part of your explanation. You will NOT be given any credit if you do not give a reason for your answer

   (a) For any wave function, the first order perturbation theory correction $\Delta E^{(1)}$ due to relativistic corrections to the kinetic energy is always positive.

   (b) Hyperfine interactions are a less important correction to the energy eigenvalues of positronium than to the energy eigenvalues of hydrogen.

   (c) A charged particle in a central potential $V(r)$ is placed in a magnetic field $\vec{B}$ that points in the x direction. The expectation value of the angular momentum in the z direction ($< L_z >$) is a conserved quantity.

   (d) A hydrogen atom is in the following state:

$$\psi(\vec{r}) = \frac{1}{5} R_{11}(r) \left(\sqrt{2} Y_{11}(\theta, \phi) + \sqrt{3} Y_{1-1}(\theta, \phi) \right)$$

   This atom is in an eigenstate of the operator $\hat{L}^2$.

3. **(30 Points)** A particle with spin=2 can have 5 possible states of S_z ($2\hbar$, $\hbar$, 0, $-\hbar$, $-2\hbar$). This means the spin wave function can be written as a 5 component column vector. Thus, the spin matrices for a spin=2 particle are (5×5) matrices. For a spin-2 particle:

   (a) What is the matrix representation of the z component of spin (S_z)?

   (b) A particle is in the state

$$\psi = \frac{1}{\sqrt{5}} \begin{pmatrix} \sqrt{3} \\ \sqrt{2} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

   What is the expectation value $< S_z >$?

   (c) What is the matrix representation of the x component of spin (S_x)?

   Note: In writing the states, define your basis so that the state with $S_z = 2$ is the first element and the state $S_z = -2$ is the last.