Physics 7B Midterm 2 - Fall 2017
Professor R. Birgeneau

Total Points: 100 (Problems)

This exam is out of 100 points. Show all your work and take particular care to explain your steps. Partial credit will be given. Use symbols defined in problems and define any new symbols you introduce. If a problem requires that you obtain a numerical result, first write a symbolic answer and then plug in numbers. Label any drawings you make. **Good luck!**

Problem 1 (20 pts.)

(a) Consider an infinitely long cylindrical conductive wire with radius "a" and linear charge density λ. Use Gauss' law to find the electric field at a distance r from the center for both $r < a$ and $r > a$.

(b) Find the potential difference between a point at a distance R from the center and $r = \infty$. Why is the result non-physical?

(c) Now suppose that there are two parallel wires, each with a radius "a", with linear charge densities $+\lambda$ and $-\lambda$. Suppose their centers are a distance d away. Find the electric field between the wires as a function of r, the distance to the center of the $+\lambda$ wire.

(d) Find the potential difference between the wires.

(e) Find the capacitance per unit length of the system.

Problem 2 (20 pts.)

(a) Consider a sphere of charge with radius "a" and charge density $\rho(r)$ that varies with radius as

$$\rho(r) = Ar^n \text{ for } r \leq a$$

where n is some number. Find the value of n such that the electric field inside the sphere is independent of r.

(b) Using your answer for n, find the electric potential as a function of r for both $r > a$ and $r < a$. Take the zero of the potential to be at $r = \infty$.

(c) Sketch the magnitude of the electric field as a function of r.

(d) Sketch the electric potential as a function of r.

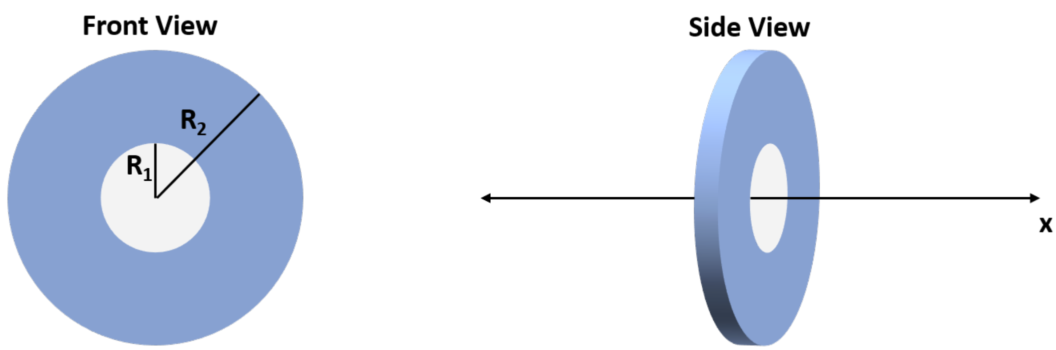
Problem 3 (15 pts.)

(a) Two wires made of the same material have identical resistances but different lengths and cross sectional areas. Assuming that one wire is twice the length of the other, and that both wires have circular cross sections, find the ratio of the diameter of the longer wire to the diameter of the shorter wire.

(b) Determine a formula for the total resistance of a spherical shell made of material whose conductivity is σ and whose inner and outer radii are r_1 and r_2, respectively. Assume the current flows radially outward.

Problem 4 (15 pts.)

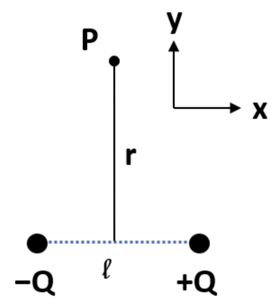
(a) An insulator in the shape of an annulus with inner radius R_1 and outer radius R_2 (see figure below) has a varying surface charge density given by $\sigma(r) = \frac{\beta}{r}$, where β is a positive constant and r is the radial distance measured from the center. Determine the magnitude of the electric field E at all points along the x axis, which is defined to be the axis of symmetry, with the annulus located at $x = 0$.



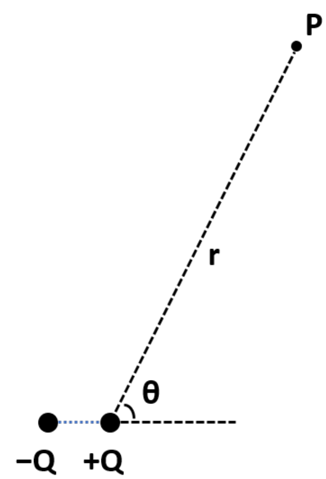
(b) What is the approximate expression for E when $x \gg R_2$?

Problem 5 (15 pts.)

(a) An electric dipole consists of point charges $+Q$ and $-Q$ separated by distance l, as shown below. Find the electric field (magnitude and direction) at the point P, located a distance r from the center of the dipole.



(b) Determine the electric potential of the dipole at the generic point P shown below, assuming $r \gg l$.

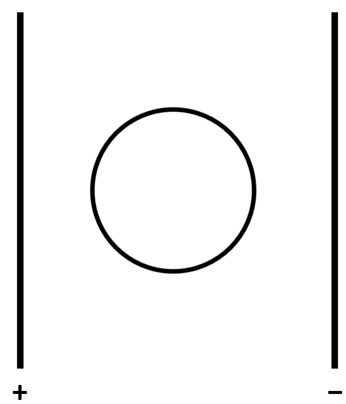


Problem 6 (15 pts.)

(a) A parallel plate capacitor of capacitance C, starting with $Q = 0$, is connected to a battery of voltage V and allowed to charge fully. Derive an expression for the energy stored in the capacitor in terms of C and V.

(b) Use the above result to determine the energy per unit volume u stored in terms of the electric field existing between the plates. Ignore any fringing effects at the edge of the plates.

(c) A conducting sphere is now placed exactly halfway between the two plates, as shown in the diagram below. Draw the electric field lines and equipotential lines in this situation. Pay particular attention to the behavior in and around the conducting sphere.



For a point charge Q:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$C = \frac{\epsilon_0 A}{d} \qquad R = \frac{\rho l}{A}$$

$$K = \frac{1}{2}mv^2$$

Volume element in spherical coordinates: $dV = r^2\sin(\theta)drd\theta d\phi$

Volume element in cylindrical coordinates: $dV = rdrdzd\phi$

Area element in polar coordinates: $dA = rdrd\theta$

$$\int \frac{1}{(a^2+x^2)^{1/2}}dx = \ln\left(x + \sqrt{a^2+x^2}\right) \qquad \int \frac{1}{(a^2+x^2)}dx = \left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{(a^2+x^2)^{3/2}}dx = \frac{x}{a^2\sqrt{a^2+x^2}} \qquad \int \frac{1}{(a^2+x^2)^2}dx = \frac{1}{2a^3}\left(\frac{ax}{a^2+x^2} + \tan^{-1}\left(\frac{x}{a}\right)\right)$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}\cdots \qquad (1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

$$\sin(x) = 1 - x + \frac{x^3}{6} + \cdots \qquad e^x = 1 + x + \frac{x^2}{2} + \cdots$$