

Department of Physics  
University of California, Berkeley

Mid-term Examination 1  
Physics 7B, Section 1

7:00 pm - 9:00 pm, 22 February 2006

Name: .....

SID No: .....

Discussion Section: .....

Name of TA: .....

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	

Score: .....

Answer all six problems. Write clearly and explain your work. Partial credit will be given for incomplete solutions provided your logic is reasonable and clear. Cross out any parts that you don't want to be graded. Enclose your answers with boxes. **Express all numerical answers in SI units.** Answers with no explanation or disconnected comments will not be credited. If you obtain an answer that is questionable, explain why you think it is wrong.

### Constants and Conversion factors

Avogadro number, $N_A$	$6.022 \times 10^{23}$
Universal gas constant, $R$	$8.315 \text{ J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1} = 1.99 \text{ cal}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$
Boltzmann constant, $k$	$1.381 \times 10^{-23} \text{ J}\cdot\text{K}^{-1}$
Stefan-Boltzmann constant, $\sigma$	$5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$
Acceleration due to gravity, $g$	$9.8 \text{ m}\cdot\text{s}^{-2}$
Specific heat of water	$1 \text{ kcal}\cdot\text{kg}^{-1}\cdot^\circ\text{C}^{-1}$
Heat of fusion of water	$80 \text{ kcal}\cdot\text{kg}^{-1}$
Heat of vaporization of water	$539 \text{ kcal}\cdot\text{kg}^{-1}$
1 atm	$1.013 \times 10^5 \text{ N}\cdot\text{m}^{-2}$
1 kcal	$4.18 \times 10^3 \text{ J}$
1 hp	$746 \text{ W}$ 1 liter $10^3 \text{ cm}^3$

### Some useful equations

$$\Delta L = \alpha L \Delta T$$

$$W = \int p dV$$

$$Q = mc\Delta T$$

$$Q = nC\Delta T, \text{ where } C = C_V \text{ or } C_P$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}}$$

$$\frac{dN_v}{dv} = 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}$$

$$PV^\gamma = \text{constant} \quad TV^{\gamma-1} = \text{constant} \quad \gamma = \frac{C_P}{C_V}$$

$$\frac{dQ}{dt} = \sigma \epsilon AT^4$$

$$\frac{dQ}{dt} = -kA \frac{dT}{dx}$$

1. [25 points] Short Questions

(a) [5 points] Circle T or F for True or False

T F (i) A microscopic description of a system includes the details of the kinetic properties of each degree of freedom, e.g. motion and relevant location of all the atoms or molecules of a gas.

T F (ii) A macroscopic description is given in terms of quantities that are an average over the microscopic details and are called state variables.

T F (iii) The temperature of a system is directly proportional to the heat that it contains.

T F (iv) Even though two bodies are in thermal equilibrium with a third, that does not mean that they are in thermal equilibrium with each other.

T F (v) The mean free path for air molecules at standard temperature and pressure (STP) is about  $10^{-7}$  m.

T F (vi) The internal energy of a system is strictly the sum of its heat and chemical energy.

T F (vii) The specific heat of a body is the amount of heat that it contains at a given temperature.

T F (viii) Diffusion is described in the kinetic theory of gases as a result of the random motion of molecules.

T F (ix) Evaporation of a substance spontaneously cools the remaining substance and can be used to turn heat into work.

T F (x) A closed system is one for which no mass enters or leaves, an isolated system does not exchange either mass or energy, and an open system allows anything in or out.

T F (xi) According to the Third Law it is not possible for the most ideal heat engine to be perfect.

(b) [5 points] Circle correct answer

(i) An inventor claims he has produced an engine for ships that takes in water, extracts thermal energy, propels the ship forward, and ejects ice out the back. He is seeking investors.

You chose not to invest because this violates

- (A) the First Law of Thermodynamics.
- (B) the Second Law of Thermodynamics.
- (C) the Third Law of Thermodynamics.
- (D) the First and Second Law of Thermodynamics.
- (E) the Second and Third Law of Thermodynamics.

(ii) Oceans as Global Thermometer: Various climate modelers predict that the Temperature of the Earth will rise between 1.5 and 5°C in the next 30 years. Water has a volume coefficient of expansion of  $2.1 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ . Assume the volume coefficient for earth is  $0.1 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$ . The mean depth of the Ocean is about 12,000 ft (3700 m). How much will the level of the Ocean rise?

- (A) no change at all
- (B) 1 to 4 feet (0.3 to 1.3 m)
- (C) 3 to 12 feet (1 to 4 m)
- (D) 0.3 to 1.2 feet (0.1 0 to 0.4 m)
- (E) 300 feet (100 m)

(ii) You are going on a trip to Paris and the weather forecast is for high of 22°C and a low of 14°C for 22 February 2006. Approximately what temperature are these in Farhenheit?

- (A) 54° F and 46° F.
- (B) -5.6° F and -10° F.
- (C) 22° F and 14° F.
- (D) 40° F and 25° F.
- (E) 71° F and 57° F.

(iii) On a cold day you want a hot cup of coffee or tea. However, the water is a cool 20°C, so you fill the microwave-safe cup with 200 cm<sup>3</sup> of water and place it in your 500 watt (delivered power) microwave. If you want to heat the water to just below boiling, what time setting should should you use? (Assume that the heat capacity of the cup is less than 10% that of the water.)

- (A) 33 seconds
- (B) 55 seconds
- (C) 1 minute and 11 seconds
- (D) 1 minute and 44 seconds
- (E) 2 minutes and 22 seconds
- (F) 3 minutes and 33 seconds.
- (G) 4 minutes and 44 seconds

(iv) The fog blows in across the Bay and quickly (no time for heat to flow) up to the room of this exam an altitude of about 200 m. If the temperature on the Bay is  $22^{\circ}\text{C}$  and the pressure is  $10^3\text{N}/\text{m}^2$  and the density is roughly  $1\text{ kg}/\text{m}^3$ . What will the difference in its pressure and temperature in the room? ( $\Delta T = T_{\text{room}} - T_{\text{bay}}$ )

- (A) Pressure is the same and temperature is same.
- (B) Pressure difference about  $2000\text{N}/\text{m}^2$  lower and temperature  $6^{\circ}\text{C}$  higher
- (C) Pressure difference about  $2000\text{N}/\text{m}^2$  lower and temperature  $6^{\circ}\text{C}$  lower
- (D) Pressure difference about  $2000\text{N}/\text{m}^2$  lower and temperature  $1.7^{\circ}\text{C}$  lower
- (E) Pressure difference about  $2000\text{N}/\text{m}^2$  lower and temperature  $0.6^{\circ}\text{C}$  lower
- (F) Pressure difference about  $200\text{N}/\text{m}^2$  lower and temperature  $0.6^{\circ}\text{C}$  lower
- (G) Pressure difference about  $200\text{N}/\text{m}^2$  lower and temperature  $1.6^{\circ}\text{C}$  lower

(v) A 1 meter radius very elastic balloon containing an ideal gas is cooled from 300 K to 200 K. How much does its volume and surface area change assuming the external pressure is constant?

- (A) decrease by 33% and 22%.
- (B) decrease by 33% and 11%.
- (C) decrease by 25% and 17%.
- (D) no change. increase by by 25% and 17%.
- (E) increase by by 25% and 17%.
- (F) increase by 33% and 22%.
- (G) increase by 33% and 11%.

(c) [5 points] Circle correct answer

(i) What is the mass of air in this room ( take  $V = 3000 \text{ m}^3$ ) at STP and how many molecules assuming a mean molar weight of 28.8 gm.

- (A) 50 kg and  $10^{27}$  molecules
- (B) 38 kg and  $8 \times 10^{26}$  molecules
- (C) 380 kg and  $8 \times 10^{27}$  molecules
- (D) 3800 kg and  $8 \times 10^{28}$  molecules
- (E) 38000 kg and  $8 \times 10^{29}$  molecules
- (F) 380000 kg and  $8 \times 10^{30}$  molecules

(ii) What is the mean speed of these air molecules at STP?

- (A) 0.5 m/s
- (B) 5 m/s.
- (C) 50 m/s
- (D) 500 m/s.
- (E) 5000 m/s.

(iii) The critical temperature of a real gas is

(A) the temperature at and above which vapor of the substance cannot be liquefied, no matter how much pressure is applied

(B) the temperature of the PV isotherm curve goes horizontal and goes through the critical point.

(C) both (A) and (B).

(D) the division between gas and vapor

(E) All of the above.

(iv) What is boiling?

(A) When water has bubbles in it.

(B) When the saturated vapor pressure equals the external pressure.

(C) It is the reverse of the dew point.

(D) It is when the vapor pressure is saturated.

(v) What is the triple point?

(A) It is the critical point.

(B) The point at which liquid can no longer exist.

(C) The point at which vapor can no longer exist

(D) The point where solid, liquid, and vapor can coexist in equilibrium

(E) The point where solid, liquid, and gas can coexist in equilibrium

(F) The point where gas, vapor, and liquid can coexist in equilibrium

(d) [5 points] Circle correct answer

(i) The van der Waals equation of state is

(A) a simple correction to the ideal gas law taking into account the finite volume of the molecules

(B) a simple correction to the ideal gas law taking into account the small attractive forces of the molecules on each other

(C)  $P \left( \frac{V}{n} - b \right) = RT$

(D)  $\left( p + \frac{a}{(V/n)^2} \right) \left( \frac{V}{n} - b \right) = RT$

(E)  $P_1 + P_2 = P_{total} = (n_1 + n_2)RT/V$ .

(ii) What is the internal energy  $U$  of a diatomic ideal gas?

(A)  $U = \frac{1}{2}Nk_B T = \frac{1}{2}nRT$

(B)  $U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$

(C)  $U = \frac{5}{2}Nk_B T = \frac{5}{2}nRT$

(D)  $U = \frac{6}{2}Nk_B T = 3nRT$

(E)  $U = \frac{7}{2}Nk_B T = \frac{7}{2}nRT$

(iii) Which of these is **not** a latent heat?

(A) heat of fusion.

(B) heat of vaporization.

(C) heat of phase change.

(D) heat of friction

(E) none of the above

(iv) What is **not** a statement of the First Law of Thermodynamics?

(A) Energy is conserved.

(B)  $dU = dQ - dW$

(C) The change in a system's internal energy is the difference between the heat added to the system and the work done by the system.

(D) Heat is a transfer of energy due to a difference in temperature.

(v) Which of these is **not** true for free expansion of a gas. I.e. The gas is initially in only one side of the container and allowed to expand without interference or resistance.

(A) No work is done in free expansion.

(B) Free expansion is adiabatic.

(C) The internal energy of the gas remains constant.

(D) Free expansion is a good example of a non-equilibrium process..

(E) The entropy of the gas is unchanged in free expansion.

(e) [5 points] Circle correct answer

(i) Which of these is **not** a possible efficiency for a heat engine working between two heat reservoirs at  $127^\circ$  and  $27^\circ\text{C}$ ?

- (A) zero.
- (B) 0.1
- (C) 0.2
- (D) 0.25
- (E) 0.3
- (F) all above are acceptable

(ii) What is the factor by which the number of accessible states changes when 1 kg of water temperature increases  $1^\circ\text{C}$  from  $27^\circ\text{C}$ ?

- (A)  $10^{24}$
- (B)  $10^{25}$
- (C)  $10^{4.3849 \times 10^{24}}$
- (D)  $10^{4.3849 \times 10^{25}}$
- (E)  $10^{4.3849 \times 10^{26}}$

(iii) The mean time between collisions of air molecules at standard temperature and pressure is roughly

- (A)  $10^{-25}$  sec
- (B)  $10^{-20}$  sec
- (C)  $10^{-15}$  sec.
- (D)  $10^{-10}$  sec.
- (E)  $10^{-5}$  sec

(iv) You wear a pair of 2-mm wool socks with thermal conductivity  $k = 0.1 \times 10^{-4} \text{ kcal}/(\text{s m K})$  while standing on floor  $10^\circ\text{C}$  cooler than your feet. Assuming your foot print is  $200 \text{ cm}^2$ , how much heat power do you lose through the bottom of your feet?

- (A) 0.4 watts
- (B) 0.8 watts
- (C) 4 watts
- (D) 8 watts.
- (E) 40 watts.
- (F) 80 watts

(v) If the entropy of a substance is proportional to its absolute temperature,  $T$ , then its heat capacity must be proportional to

- (A)  $1/T$ .
- (B) constant ( $T^0$ )
- (C)  $T$ .
- (D)  $T^2$ .
- (E)  $T^3$ .



2. [25 points] West Wall Windows, there for the view, are the equivalent of 10 windows set side by side each with an area of  $3 \text{ m}^2$  and a glass thickness of  $3 \text{ mm}$ .
- (a) [4 points] What is the heat loss through the windows in watts per second for a  $10^\circ\text{C}$  temperature difference (cold day)?  $k_{\text{glass}} = 0.84 \text{ W/m/C}$
- (b) [4 points] Now account for the fact that on a calm day natural convection effectively leaves  $1.5 \text{ mm}$  of still air ( $k_{\text{air}} = 0.023 \text{ W/m/C}$ ) on either side of the glass.
- (c) [1 point] Does it matter whether the wind is blowing or not? Why?
- (d) [4 points] What is the heat loss if the windows are covered with cellular (honeycomb) blinds which have an R factor of  $3.4 \text{ hr ft}^2\text{°F} / \text{Btu}$ ? (The glass has an R value of about 1.)
- (e) [2 points] What if I like the view, can I make up the heat loss with a gas furnace or gas fire place? Assume that on a peak month PG&E will ration me to  $80 \text{ Therms/month}$  of gas. (Therm is  $100,000 \text{ Btu}$ ;  $1 \text{ watt} = 3.4 \text{ Btu/hr}$ )
- (f) [4 points] How much heat flow can I gain from the Sun at mid-afternoon if it is clear?
- (g) [2 points] What should be the effective R value, if I replaced the single pane windows with double pane windows spaced by  $3 \text{ mm}$  of air with  $k_{\text{air}} = 0.023 \text{ W/m/C}$ ? I.e. the ratio of heat loss to single pane glass with  $k_{\text{glass}} = 0.84 \text{ W/m/C}$  to the dual pane system?
- (h) [2 points] In practice the effective R value of a dual pane window with air between the windows, is not much different than the cellular shades. Why?
- (i) [4 points] The thermal coefficient of expansion of glass is about  $9 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$  and the wood frame's coefficient of expansion is about  $1.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ . How much does the  $2\text{-m}$  height of the window want to change for the  $10^\circ\text{C}$  temperature change? How much does the wood frame want to expand? If the Young's modulus is  $E_{\text{wood}} = 10^9 \text{ N/m}^2$  for wood and  $E_{\text{glass}} = 72 \times 10^9 \text{ N/m}^2$ , what is the stress in the glass from this temperature change?

3. [18 points] Because of heavy rains and minor flooding, you decide to purchase a dehumidifier to dry out your house and prevent mold and mildew.

The dehumidifier is designed to have the water condensing coils at just above  $0^\circ\text{C}$  when the room is at  $20^\circ\text{C}$ . The dehumidifier is sized to remove 1 liter of water per hour.

(a) [4 points] How much heat is must be removed to condense water vapor at  $20^\circ$  to 1 liter of water at  $0^\circ\text{C}$ ?

(b) [2 points] Why does the humidifier cut off rather than cool the air below  $0^\circ\text{C}$ ?

(c) [2 points] Is this a heat engine or a refrigerator? What is its ideal efficiency or coefficient of performance?

(d) [2 points] What is the minimum electric power needed to condense 1 liter of water per hour?

(f) [4 points] In order to get one liter of water, how much air must be processed and how much heat must be removed to cool the air coming in with 80% relative humidity water vapor? [The Saturated Vapor Pressure of water at  $20^\circ$  is  $2,330\text{ N/m}^2$  (this is the same as 17.5 torr and standard atmospheric pressure is 760 torr.)]

Exempt the water portion calculated in earlier part of problem or state the total. What is the temperature the air blown out of the dehumidifier which takes away the waste heat, if it were the same air? (i.e. If no additional air were added to air flow stream to take away waste heat.)

4. [15 points] For a modern power plant, the pressure of the steam can be as high as 70 atmospheres at  $300^{\circ}\text{C}$  (573 K); the exhaust is at one atmosphere and perhaps  $40^{\circ}\text{C}$  (313 K).

(a) [1 points] For such a power plant, the maximum thermodynamic efficiency is?

(b) [2 points] For such an ideal power plant, suppose that the rate of waste heat out is  $1.2\text{GW} = 1.2 \times 10^9 \text{watts}$ , what is the rate of input heat? What is the rate of entropy production?

(c) [2 points] For an actual engine, which necessarily runs irreversibly, the actual efficiency,  $e = 1/3$ , is less than the maximum efficiency. If the output electric power is  $1\text{GW} = 10^9 \text{watts}$ . What are the rates of waste heat and entropy production?

The temperature difference between the outside and inside of your house is  $5^{\circ}\text{C}$  and you need 1 kilowatt of heat flow to keep your bathroom at your ideal temperature of  $22^{\circ}\text{C}$ .

(d) [3 points] If you use an ideal heat pump to get this heat flow, what electric input power do you need? What total input energy is needed at the power plant (33% efficiency) to produce this? What is the rate of total entropy production including the power plant?

(e) [2 points] If you use an electric resistance (turns electric power to heat) heater, what total input energy is needed at the power plant (33% efficiency) to produce this? What the rate of total entropy production including the power plant?

(f) [2 points] What power do you need if you burn the fuel in a space heater in the bathroom, if one third the heat goes out the vent required to prevent CO build up? What is the rate of total entropy production assuming the combustion temperature is  $300^{\circ}\text{C}$ ?

(g) [3 points] Draw and label a PV diagram and show the beginning temperature and pressure and ending temperature and pressure as circled points on the PV diagram. To be specific consider one mole of steam. Sketch the following curves starting from the initial temperature and pressure: (i) isothermal, (ii) adiabatic, (iii) estimated actual path.

5. [15 points] A binary alloy is made by mixing atoms of two distinct substances A and B. For this problem the total of the two substances sums to Avogadro's Number,
- (a) [4 points] What is the number of possible arrangements for  $n$  atoms of A and  $N - n$  of B? What is the corresponding entropy? Evaluate for a half and half mixture.  
(Stirling's approximation is  $\log(n!) \approx n \log(n)$ .)
- (b) [4 points] The entropy that is created in making the alloy is called the mixing entropy. If no work was required to mix the alloy at  $27^\circ\text{C}$ , show that implies there is an energy of mixing. What is the form and evaluate the mixing energy for the whole batch and per atom for 50% mixing? (If you do not have the formula for the mixing entropy from part (a) assume it is of the form  $S = k_B N \sin(\pi x)$  where  $x = n/N$ .)
- (c) [2 points] What is the increase in heat capacity? Evaluate for 50% mixing?
- (d) [4 points] Assume there is an actual chemical-type bonding energy  $E_{AB}$  between nearest pairs of atoms of the distinct components A and B and that there is only one such nearest neighbor per atom. Find a formula for the total binding energy as a function of  $n$  the number or  $x$  the fraction of A. Make a diagram of energies versus mixing fraction. What is the boundary binding energy between stable or unstable for the 50% mixture? (Unstable means that the mixture would tend to unmix and segregate into components.) How does the entropy and heat capacity change?

6. [11 points] Brownian Motion observed by botanist Robert Brown in 1827 and explained by Albert Einstein in 1905 confirmed the kinetic model and gave the size of atoms. Brown observed that tiny (1 micron =  $10^{-6}$ m) sized pollen particles moved about in tortuous (called random walk) paths. Assume that the water was at  $27^{\circ}\text{C}$  and the pollen has a density of  $1\text{ gm/cm}^3$ .

(a) [1 points] What is the mass of the pollen grain?

(b) [3 points] What is the rms velocity of the pollen particle according to kinetic model? What is the average velocity of the pollen particle?

(c) [1 points] How many water molecules on average coming from one side must hit a pollen grain to change its velocity by its rms amount?

(d) [3 points] How many water molecules hit the pollen grain per unit time? First calculate the typical or rms velocity of a water molecule toward the pollen grain. Also calculate the mean spacing between water molecules.

(e) [2 points] Although the existence of atoms and molecules was still open to objection in 1905, Einstein predicted that the random motions of molecules in a liquid impacting on larger suspended particles would result in irregular, random motions of the particles, which could be directly observed under a microscope. The predicted motion corresponded precisely with the puzzling Brownian motion! From this motion Einstein accurately determined the dimensions of the hypothetical molecules and allowed a precise determination of Avogadro's number. How many molecules, given that the water molecules are hitting from random directions and with random momenta, does it take to randomize the direction of motion? [Hint the mean square scatter for a Gaussian is  $\sigma^2$  and for water molecule velocity or momentum is of order  $kT$ . The mean square scatter for the sum of  $n$  gaussians is just the sum of the mean square scatters  $\sigma_{average\ of\ n}^2 = \sum_{i=1}^n \sigma_{for\ one}^2 = n\sigma_{for\ one}^2$ ] What is the mean time  $\delta t$  between randomly changing direction? (i.e.  $\langle \vec{v}_{pollen}(t + \delta t) \cdot \vec{v}_{pollen}(t) \rangle \approx 0$ )

(f) [2 points] Einstein realized that the diffusion equation described the relevant process and eventually the pollen particle motion. The diffusion coefficient  $D$  depends upon the properties of the diffusee (pollen grains) and the diffuser (water molecules). What is the diffusion equation and why is the solution a probability distribution of the distance from the starting point of the pollen particle a Gaussian (Normal Probability Curve) with a variance that increases linearly with time?

$$P(d) = \frac{1}{\sqrt{2\pi 2Dt}} e^{-\frac{d^2}{4Dt}}$$