Solutions to C110 Final

Prepared by Dong Wei

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Note: Grading is based on the point allocation in blue. There is no further partial credits. That is, you either get the whole points in blue, or 0.

Question 1 (10 pts)

Note that we can write player i's payoff function as

$$u_i(s_i, s_{-i}) = (1-c)s_i + \sum_{j \neq i} s_j$$

When $c < 1$, the unique NE, which is symmetric, is $s_i = 5$ for all i (2 pts); this NE is Pareto efficient, because for any other strategy profile s' s.t. $s'_i < 5$ for some i, $s = (5, 5, ..., 5)$ Pareto dominates s' (1 pt).

When $c > 1$, the unique NE, which is symmetric, is $s_i = 0$ for all i (2 pts); this NE is Pareto efficient if and only if $c \leq n$. This is because when $c > n$, $\sum_{i=1}^n u_i(s) = (1-c)\sum_{i=1}^n s_i \leq 0$ for all s, with strict inequality if $s \neq 0$. Therefore, for any $s \neq (0, ..., 0)$, the payoff of some i must be negative, while for $s = (0, ..., 0)$, the payoff of all i is 0. Therefore, $(0, ..., 0)$ is Pareto efficient. On the other hand, when $c \leq n$, $(0, ..., 0)$ is Pareto dominated by $(5, ..., 5)$, thus $(0, ..., 0)$ is not Pareto efficient (2 pts).

When $c = 1$, the set of NEs is $\{(s_1, ..., s_n) : s_i \in [0, 5] \text{ for all } i\}$. Among all NEs, the symmetric equilibria are $(s, ..., s)$ for $s \in [0, 5]$ (2 pts). Only $(5, ..., 5)$ is Pareto efficient, because all other equilibria are dominated by $(5, ..., 5)$, while $(5, ..., 5)$ is not Pareto dominated by any strategy profile, as can be shown (1 pt).

Question 2 (10 pts)

Game 1

(i) Players: $\{1, 2\}$; histories: $\{\emptyset, A, B, BC, BD, BDE, BDF\}$; terminal histories: $\{A, BC, BDE, BDF\}$; player function: $p(\emptyset) = 1, p(B) = 2, p(BD) = 1$; preferences: $BC \succ_1 A \succ_1 BDF \succ_1 BDE$, $BDF \succ_2 BC \succ_2 A \sim_2 BDE$. (1 pt)

(ii) The strategic form of Game 1 is as follows. (1 pt)

		Player 2	
		C	D
Player 1	AE	$2,0$	$2,0$
	AF	$2,0$	$2,0$
	BE	$3,1$	$0,0$
	BF	$3,1$	$1,2$

(iii) The set of pure strategy NEs is $\{(AE, D), (AF, D), (BE, C)\}$. (2 pts)

(iv) The unique SPNE is (AF, D). (1 pt)

Game 2

(i) Players: $\{1, 2\}$; histories: $\{\emptyset, L, M, R, LA, LB, MC, MD, RE, RF\}$; terminal histories: $\{LA, LB, MC, MD, RE, RF\}$; player function: $p(\emptyset) = 1, p(L) = 2, p(M) = 2, p(R) = 1$; preferences: $RE \sim_1 MC \succ_1 LA \sim_1 RF \succ_1 LB \sim_1 MD$, $LB \sim_1 MD \succ_1 LA \sim_1 RF \succ_1 RE \sim_1 MC$. (1 pt)

(ii) The strategic form of Game 2 is as follows. (1 pt)

			Player 2		
		AC	AD	BC	BD
Player 1	LE	$2,2$	$2,2$	$1,3$	$1,3$
	LF	$2,2$	$2,2$	$1,3$	$1,3$
	ME	$3,1$	$1,3$	$3,1$	$1,3$
	MF	$3,1$	$1,3$	$3,1$	$1,3$
	RE	$3,1$	$3,1$	$3,1$	$3,1$
	RF	$2,2$	$2,2$	$2,2$	$2,2$

(iii) The set of pure strategy NEs is $\{(RE, AC), (RE, AD), (RE, BC), (RE, BD)\}$. (2 pts)

(iv) The unique SPNE is (RE, BD). (1 pt)

Question 3 (10 pts)

(i) We define grim trigger strategy as follows: For each player, start by playing C; at any history s.t. all player has been playing C, play C; at any history s.t. some player played D in at least one period prior to the current period, play D.

To check whether such a strategy profile is a Nash equilibrium, we need to compute the *best* deviation of each player on the equilibrium path. By symmetry, it is sufficient to do it for player 1. Notice that after player 1 deviates to playing D in some period, player 2 will keep playing D from the next period on, no matter what player 1 plays later on. Therefore, the best payoff sequence that player 1 can achieve by deviating to D is $(y, 0), (1, 1), (1, 1), \ldots$ So grim trigger is an NE, iff

$$\begin{aligned} \frac{x}{1-\delta} &\geq y + \frac{\delta}{1-\delta} \\ \iff \quad \delta &\geq \frac{y-x}{y-1} \text{ (5 pts)} \end{aligned}$$

(ii) We define tit-for-tat strategy as follows: For each player, start by playing C; at any period after the first period, play what the other player played in the last period.

To check whether such a strategy profile is a Nash equilibrium, we need to compute the *best* deviation of each player on the equilibrium path. By symmetry, it is sufficient to do it for player 1. Suppose tit-for-tat is an NE. The best deviation results in one of the following action sequences, whichever gives player 1 a higher payoff:

$$\begin{aligned} &(D, C), (D, D), (D, D), (D, D), \ldots \\ &(D, C), (C, D), (D, C), (C, D), \ldots \end{aligned}$$

Therefore, tit-for-tat is an NE, iff

$$\begin{aligned} \frac{x}{1-\delta} &\geq \max\{y + \frac{\delta}{1-\delta}, \frac{y}{1-\delta^2}\} \\ \iff \quad \delta &\geq \max\{\frac{y-x}{y-1}, \frac{y-x}{x}\} \text{ (5 pts)} \end{aligned}$$