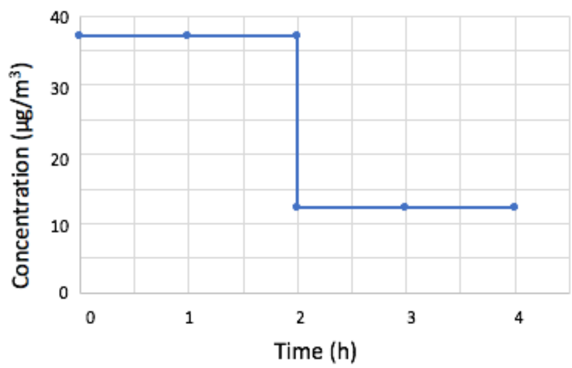
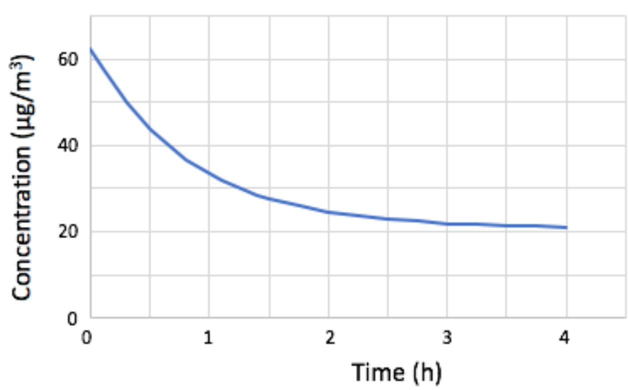
ANSWERS

1. Transient response of reactor models

(a) For a PFR with first-order decay, the outlet concentration, C, in steady state is $C_{in}\exp(-k\theta)$, where $\theta = V/Q$. In this problem, $\theta = 2$ h and $k\theta = 1.4$. So, the outlet concentration is $0.247 \times C_{in}$. When $C_{in} = 150\ \mu g/m^3$, then $C = 37.0\ \mu g/m^3$. Similarly, when $C_{in} = 50\ \mu g/m^3$, then $C = 12.3\ \mu g/m^3$. The step change from high to low happens at the inlet at time t = 0. At the outlet, there is a corresponding step change that occurs at time $\theta = 2$ h later. Sketch is presented below.



(b) For a CMFR with first-order decay, the steady state concentration at the outlet, C, is $C_{in}/(1+k\theta)$. For the given conditions, $1/(1+k\theta) = 0.417$. The initial condition that applies for $(t \leq 0)$ is the steady-state solution when $C_{in} = 150\ \mu g/m^3$; that is $62.5\ \mu g/m^3$. For $t > 0$, the ultimate steady-state concentration applies when $C_{in} = 50\ \mu g/m^3$; that is $20.8\ \mu g/m^3$. The time pattern of response follows an exponential decay from the initial to the final steady state value with a characteristic response time $\tau \sim \theta/(1+k\theta) = 0.83$ h. The governing equation is $d(CV)/dt = C_{in}Q - (Q + kV)\,C$ or $dC/dt = (Q/V)\,C_{in} - (Q/V + k)\,C$. This is of the form $dC/dt = S - LC$ and the characteristic response time is $1/L$. The concentration sketch versus time appears below.



2. Sedimentation for particle control in drinking water treatment

(a) The overflow rate is Q/A_s. Here, $Q = 6\text{ m} \times 3\text{ m} \times 600\text{ m/d}$ and $A_s = 6\text{ m} \times 25\text{ m}$. So, $Q/A_s = 1800/25 = \boxed{72\text{ m/d}}$.

(b) This is just a unit conversion, since the critical settling velocity equals the overflow rate for a sedimentation basin. Since 72 m = 7200 cm and 1 d = 86400 s, the critical settling velocity is 7200/86400 = 0.083 cm/s.

(c) A particle will be captured with 75% efficiency if its settling velocity is equal 75% of the critical settling velocity, i.e. 0.0625 cm/s. Let's guess that Stokes law holds. From equation in handout, and ignoring the slip correction factor, we have $d_p^2 = (18\ \mu\ v_s)/[g\ (\rho-\rho_f)]$. Look up parameter values and use cgs system consistently: $\mu = 0.01$, $v_s = 0.0625$, $g = 980$, $\rho-\rho_f = 1.5$. Substitute and solve for $d_p = 0.0028$ cm = 28 µm. (Check $Re_p = 0.0028 \times 0.0625 \times 1/0.01 = 0.02 < 0.3$, so Stokes law is okay!)

3. Disinfection performance in drinking water treatment

(a) At 0.2 mg/L, we can read from the plot that 99% inactivation occurs with a contact time of three minutes. We can recognize that each log removal requires the same amount of time. The 99% inactivation corresponds to 2-log removal; the target is 3-log removal. Each 1-log removal requires 3 min/2 = 1.5 minutes, so, to realize the design goal, we would need a hydraulic detention time of 3 × 1.5 = 4.5 min. (Can also solve the problem by computing the rate constant, k, for the given concentration: $N/N_o = 0.01 = \exp(-kt)$, so at 0.2 mg/L, $k = \ln(100)/3$ min = 1.53 per minute. To achieve 99.9% inactivation, we need a contact time that satisfies $t = \ln(1000)/1.53 = 4.5$ min.)

(b) For the CMFR, must solve for k. The figure shows that with 3 mg/L concentration, the time needed in a batch reactor to achieve 99% inactivation would be 0.1 min. The corresponding rate constant is ln(100)/0.1 = 46 per minute. The steady-state performance of a CMFR yields N/No = 1/(1+k theta). With given k and theta = 5 min, we have k theta = 230 and so N/No = 1/231 = 0.004. Consequently, these conditions yield 99.6% inactivation. The corresponding n value is obtained from - log(0.004) = 2.4 log inactivation.

3. Characteristic response time for a lake following contaminant spill

Perhaps the strongest way to approach this problem is to write a time-dependent material balance equation for the contaminant in the lake: $d(CV)/dt = QC - k_{gl}\ A\ C - k\ V\ C$. Let's treat V as constant so it can be taken outside the derivative and we can divide both sides by V. The result is of the form $dC/dt = S - LC$, where $S = 0$. The characteristic response time is $1/L$, where $L = Q/V + k_{gl}\ A/V + k$. That is:

$$\tau \sim \frac{1}{Q/_V + k_{gl}\, A/_V + k}$$