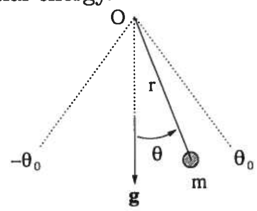
	Average	St. Dev
P1	86.98	16.83
P2	61.51	17.00
Total	148.49	29.08

University of California, Berkeley
Mechanical Engineering
ME104 Engineering Mechanics
Test 1 F17 *Prof S. Morris*

NAME ______________________

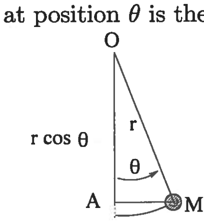
1. (100) The pendulum swings about the pivot O; it is released from rest at angle θ_0. The rod has constant length r and negligible mass. By using conservation of energy, derive the relation giving $\dot{\theta}^2$ as function of θ, the parameters g, r and amplitude θ_0. As part of your solution, you must identify the reference level being used for the potential energy.



Solution

(a) Let the reference level be the height of the mass at $\theta = 0$. When the mass is at position θ, its height above the reference level is $r - r\cos\theta$, the second term following from the geometry of the right triangle OAM. The potential energy of the mass at position θ is therefore $mgh = mgr(1 - \cos\theta)$.

(25 pts)



(25 pts)

Because energy is conserved in this problem, the sum of the kinetic and potential energy is independent of θ:

$$\frac{1}{2}m(r\dot{\theta})^2 + mgr\{1 - \cos\theta\} = mgr\{1 - \cos\theta_0\}, \qquad (1.1)$$

(35 pts)

because when $\theta = \theta_0$, the mass is instantaneously at rest.

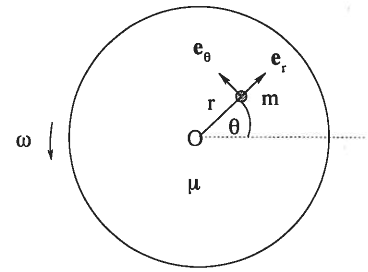
Hence

$$\dot{\theta}^2 + 2\frac{g}{r}\{\cos\theta_0 - \cos\theta\} = 0. \qquad (1.2)$$

(15 pts)

PRINT YOUR NAME ON THIS PAGE

2. (100) The turntable spins about O so that a point on the turntable at distance r from O has velocity $\mathbf{v} = \omega r \mathbf{e}_\theta$ (constant ω). Gravity acts into the page.



(a) A box of mass m is placed on the turntable at distance r_0 from O. Assuming the box does not slide, draw the free–body diagram showing the three forces acting on the box. Hence determine the largest value of r_0 for which there is no sliding (coefficient of static friction, μ_s). If the box is placed at a smaller value of r_0, what is the magnitude of the friction force? **Given :** $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$.

(b) Suppose now that the box is placed so that it slides. Draw its free–body diagram (coefficient of kinetic friction μ_k) showing the single (friction) force acting in the plane of the turntable. To receive credit, your figure must show the magnitude and the *direction* of this force correctly: remember that the friction force acts antiparallel to the velocity of the box *relative* to the turntable.

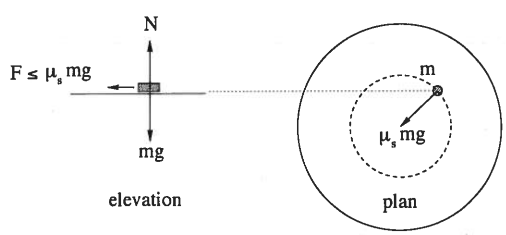
(c) Hence determine the radial and circumferential components of the friction force. **Given :** $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$, and $\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\,\mathbf{e}_\theta$. (This problem can be solved either by geometry, or by first determining the unit vector parallel to the relative velocity of the box.)

Solution

(a) Free–body diagram: because the box does not slide, it follows the circular path (dotted in the plan view). The friction force exerted by the turntable on the box must act in the direction of the centripetal acceleration of the box: its magnitude is $mr_0\omega^2$. Because the friction force can not exceed $\mu_s mg$, the box slides if

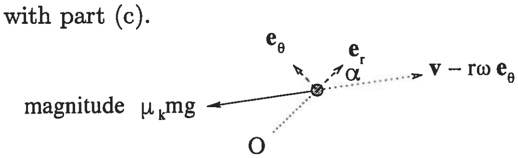
$$r_0 > \frac{\mu_s g}{\omega^2} \tag{2.1}$$

(40 pts)



If the box is placed at a value of r smaller than that given by (2.1), the friction force per unit mass balances the centripetal acceleration: $F = mr_0\omega^2$.

(b) Free–body diagram: for clarity, only the force acting in the plane on the turntable is shown. This friction force has magnitude $\mu_k mg$ and acts antiparallel to the velocity of the box *relative* to the turntable: let $\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\,\mathbf{e}_\theta$ be the absolute velocity of the box; then its velocity relative to the turntable is $\mathbf{v} - r\omega\mathbf{e}_\theta$, by using the formula given with part (c).



(40 pts)

(c) From the figure,

$$\mathbf{F} = \mu_k mg\, \mathbf{e}_F\,; \tag{2.2}$$

the unit vector $\mathbf{e}_F$ is antiparallel to the relative velocity.

To determine $\mathbf{e}_F$, we need the components of the relative velocity. By using the formula given,

$$\mathbf{v} - r\omega\, \mathbf{e}_\theta = \dot{r}\, \mathbf{e}_r + r(\dot{\theta} - \omega)\, \mathbf{e}_\theta, = |\mathbf{v} - r\omega\, \mathbf{e}_\theta| \left\{ \frac{\dot{r}}{|\mathbf{v} - r\omega\, \mathbf{e}_\theta|}\mathbf{e}_r + \frac{r(\dot{\theta} - \omega)}{|\mathbf{v} - r\omega\, \mathbf{e}_\theta|}\mathbf{e}_\theta \right\}. \tag{2.3a, b}$$

In (2.3b), the term in braces represents the unit vector *parallel* to the relative velocity. Hence

$$\mathbf{e}_F = -\frac{\dot{r}}{|\mathbf{v} - r\omega\, \mathbf{e}_\theta|}\mathbf{e}_r - \frac{r(\dot{\theta} - \omega)}{|\mathbf{v} - r\omega\, \mathbf{e}_\theta|}\mathbf{e}_\theta. \tag{2.4}$$

(20 pts)

It follows that

$$\mathbf{F} = \mu_k mg \left\{ -\frac{\dot{r}}{|\mathbf{v} - r\omega\, \mathbf{e}_\theta|}\, \mathbf{e}_r + \frac{r(\omega - \dot{\theta})}{|\mathbf{v} - r\omega\, \mathbf{e}_\theta|}\, \mathbf{e}_\theta \right\}. \tag{2.5}$$

Because we expect $\dot{\theta} < \omega$, in going from (2.4) to (2.5), we have combined minus signs to make it clear that $F_\theta > 0$ as shown in the free–body diagram.