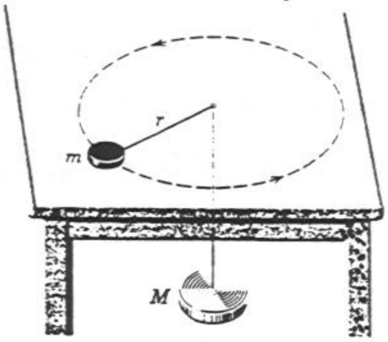
Physics 7A (Sec. 2) Midterm Exam #1 Oct. 1, 2002

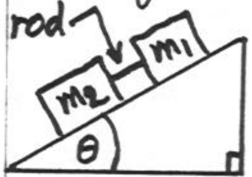
You may use one (1) card, 3" × 5", as a memory aid. Exam = 200 points

(30) (1) A cannon is located at the top of a vertical cliff of height h; the barrel of the cannon makes an angle θ with the horizontal. The cannon fires a projectile with an initial velocity of magnitude v_0, and the projectile lands at a horizontal distance R from the foot of the cliff. Calculate h (in terms of R, v_0, θ, and g).

(30) (2) A mass m on a frictionless table is attached to a hanging mass M by a massless cord passing through a hole in the table, as shown in the drawing. Calculate the value of the tangential velocity v_T (magnitude only) of mass m, moving in a circle of radius r, which will result in mass M remaining at rest.



(30) (3) Two masses, m_1 and m_2, are connected by an inextensible rod of negligible mass. The masses slide down a non-frictionless incline which makes an angle θ with the horizontal. The coefficients of kinetic friction of the two masses with the incline are, respectively, μ_1 and μ_2.



(a) Calculate the acceleration of each mass; (b) If the incline were frictionless, show that your answer to (a) reduces to the proper value. [(a) = 25 points, (b) = 5 points]

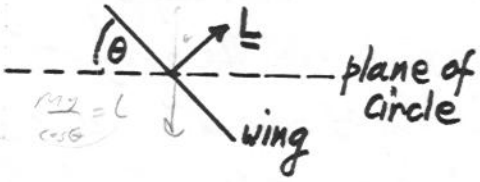
(continued →)

(25)(4) An airplane flies in a horizontal circle of radius R. The wings of the airplane make an angle θ with the horizontal. The "aerodynamic lift" $\underline{L}$ on the wing is normal to the wing, and the tangential speed of the airplane is v. Calculate the value of the angle θ, expressed in terms of v, R, and constants.

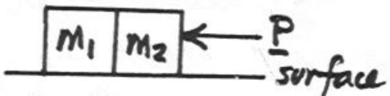
$L\cos\theta = mg$

$L\sin\theta = \frac{mv^2}{r}$

$\frac{mg}{\cos\theta} = L$



(30)(5) As shown, a constant force $\underline{P} = P(-\hat{x})$ is applied to two (non-compressible) blocks which are in contact with each other and are sitting on a frictionless horizontal surface. The masses of the blocks are m_1 and m_2. Calculate the force $\underline{F}_1$ exerted on m_1 by m_2. There is no vertical motion of the blocks.

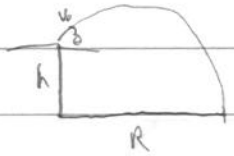


$\frac{m_1 P}{m_1+m_2}$

(30) (6) Given a particle, moving in a circle of radius B, whose angular coordinate $\theta(t)$ varies with time as $\theta(t) = at^2$, where a is constant. (a) Calculate the position vector $\underline{r}(t)$ of the particle; (b) Calculate the velocity vector $\underline{v}(t)$ of the particle; (c) Calculate the speed $|\underline{v}|$; (d) Calculate the acceleration vector $\underline{a}(t)$ of the particle; (e) Calculate $\underline{v}$ and $\underline{a}$ of the particle when $t = 1$ sec.; (f) Make a sketch showing the vector $\underline{a}$ obtained in Part (e) above. [Each part = 5 points]

(25)(7) A sphere of mass m falls vertically downward (from rest) in a liquid which exerts a vertically upward frictional force f (proportional to the sphere's velocity) on the sphere. Describe a possible experiment which measures the maximum value of the frictional force f.

1,



$x = v_{0x} t$

$y = -\frac{1}{2}gt^2 + v_{0y}t + h$

// in time t, x distance traveled is R, use this to find h in y-equati

$R = v_{0x} t$

$\frac{R}{v_{0x}} = t$

$y = \frac{-gR^2}{2v_{0x}^2} + \frac{v_{0y} R}{v_{0x}} + h$

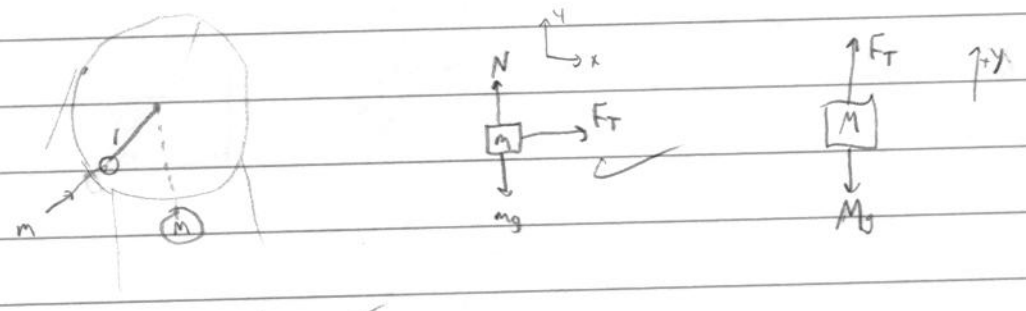
$\frac{gR^2}{2v_{0x}^2} - \frac{v_0 \sin\theta R}{v_0 \cos\theta} = h$

$\boxed{\frac{gR^2}{2v_0^2\cos^2\theta} - R\tan\theta = h}$

30/30

30/30

2.



$\sum F_x = F_T = \frac{m v_T^2}{r}$ ✓

$\sum F_y = M\cancel{a}^{\to 0}$ because we want M to be at rest.

so $F_T - Mg = 0$

$F_T = Mg$ ✓

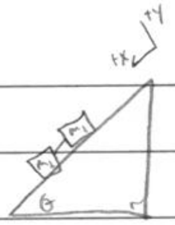
Tension will provide centripetal force:

$Mg = \frac{m v_T^2}{r}$ ✓

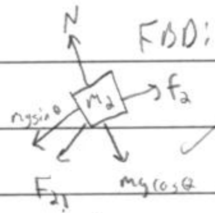
$\boxed{\sqrt{\frac{rMg}{m}} = v_T}$ ✓ So m must move at this v_T to keep M at rest.

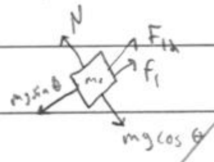
30 | 30

3.



FBD:





F_{21} = F exerted on 2 by 1

F_{12} = F exerted on 1 by 2

ΣF_{x2} $\quad mg\sin\theta + F_{21} - f_2 = m_2 a_2$

ΣF_{y2} $\quad N - mg = 0$ // not moving in y-direction

$N = mg$

$\boxed{a_2 = a_1 = a}$ // system has uniform acceleration

$+\ \Sigma F_{x1}$ $\quad m_1 g\sin\theta - F_{21} - f_1 = m_1 a_1$ $\qquad f = \mu N = \mu m g\cos\theta$

$m_1 g\sin\theta + m_2 g\sin\theta - f_2 - f_1 = (m_1 + m_2)a$

$$= \frac{m_1 g\sin\theta + m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - \mu_1 m_1 g\cos\theta}{m_1 + m_2} = a = \frac{(m_1 + m_2)g\sin\theta - g\cos\theta(\mu_2 m_2 + \mu_1 m_1)}{m_1 + m_2}$$

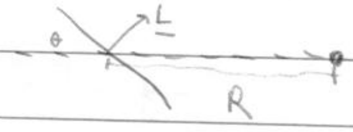
$$= \boxed{g\sin\theta - \frac{g\cos\theta(\mu_2 m_2 + \mu_1 m_1)}{m_1 + m_2}}$$

b. if there is no friction, $\mu_1 = \mu_2 = 0$

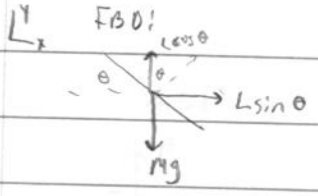
$$a = \frac{g\sin\theta\cancel{(m_1 + m_2)}}{\cancel{(m_1 + m_2)}} = g\sin\theta$$

expected acceleration if only gravity were acting on the system.

4.



FBD:



ΣF_y: $L\cos\theta - mg = 0$; ΣF_x: $L\sin\theta = \frac{mv^2}{R}$

$L = \frac{mg}{\cos\theta}$

$\cancel{m}g\tan\theta = \frac{\cancel{m}v^2}{R}$

$\tan\theta = \frac{v^2}{gR}$

$$\boxed{\theta = \tan^{-1}\left[\frac{v^2}{gR}\right]}$$

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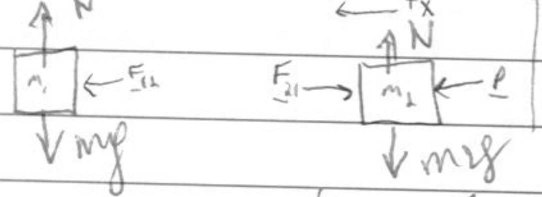
5.

FBD:

Y-components cancel out as stated in problem

F_{21} = force exerted on m_2 by m_1

F_{12} = force exerted on m_1 by m_2



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$$P - F_{21} = m_2 a_2$$

$$\boxed{a_1 = a_2 = a}$$

$$\boxed{F_{21}} = \boxed{F_{12}}$$

$$F_{12} = m_1 a_1$$

$$P = (m_1 + m_2)a$$

$$\frac{P}{m_1+m_2} = a$$

$$\text{so } \boxed{F_{12} = F_{21} = m_1 a = \frac{m_1 P}{m_1+m_2}}$$

$a(m_1+m_2) - m_2 a_2$

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6. a. radius = B $\theta(t) = at^2$ $(\theta = \theta(t))$

$$r(t) = B\hat{U}_R = B[\hat{x}\cos\theta(t) + \hat{y}\sin\theta(t)] = B[\hat{x}\cos(at^2) + \hat{y}\sin(at^2)]$$ +5

b. $$\bar{v}(t) = \frac{d\theta(t)}{dt} = \frac{d}{dt}\left[B\hat{U}_R\right] = B\left[\hat{x}(-\sin\theta)\frac{d\theta}{dt} + \hat{y}\cos\theta\frac{d\theta}{dt}\right] = B\frac{d\theta}{dt}\left[(-\sin at^2)\hat{x} + (\cos at^2)\hat{y}\right]$$

$$= B\cdot\frac{d\theta}{dt}\hat{U}_\theta = B[2at]\hat{U}_\theta$$ +5

c. $$|\bar{v}| = (\bar{v}\cdot\bar{v})^{1/2} = \left[B^2(2at)^2\cdot(\hat{U}_\theta\cdot\hat{U}_\theta)\right)^{1/2} = \left[B^2(2at)^2\right]^{1/2} = \boxed{B\cdot 2at}$$ +5

d. $$\bar{a}(t) = \frac{d\bar{v}(t)}{dt} = B\frac{d}{dt}(2at)\hat{U}_\theta$$

$$= B\left[2at\cdot\frac{d}{dt}\hat{U}_\theta + \hat{U}_\theta\cdot 2a\right]$$

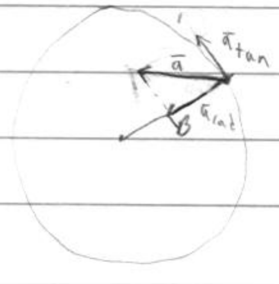
$$= B\left[2at\cdot\frac{d\theta}{dt}\left[-\hat{U}_R\right] + 2a\hat{U}_\theta\right]$$

$$= B\left[4a^2t^2[-\hat{U}_R] + 2a\hat{U}_\theta\right]$$ +5

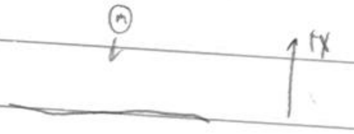
e. $\bar{v}(1) = B[2a]\hat{U}_\theta$

$$\bar{a}(1) = B\left[4a[-\hat{U}_R] + 2a\hat{U}_\theta\right]$$ +5

f.



+5

7. 

$f = \alpha v$

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if we drop the sphere and measure its displacement d and its initial velocity v_0, we can find the acceleration a by:

$$\overset{0}{\cancel{v_f^2}} = v_0^2 + 2(-a)d \qquad v_0 = \text{velocity just before it hits the water}$$

$$v_0^2 = 2ad \qquad v_f = \text{final velocity} = 0$$

$$a = \frac{v_0^2}{2d}$$

then we can use:

$$f - mg = ma$$

$$f = mg + ma$$

$$= m\left(g + \frac{v_0^2}{2d}\right)$$

will tell us the magnitude of f.

If we repeat this experiment many times with different velocities we can extrapolate α from the data - we will know f + v.

α