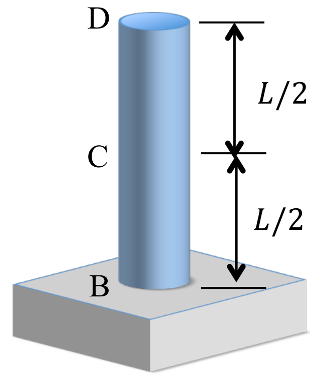
Problem 1. (40 points)



Consider a uniform rod with cross sectional area A, length L, Young's modulus E, linear coefficient of thermal expansion α, and mass density (mass per unit volume) ρ. The rod is oriented vertically and rests on end B. There are no external forces applied, but the rod deforms due to the effects of gravity g and of temperature change ΔT from a reference temperature T_0.

Determine the deflection relative to the fixed end and the average axial stress at points B, C and D. Express your answers in terms of the parameters given: A, L, E, α, ρ, g and ΔT.

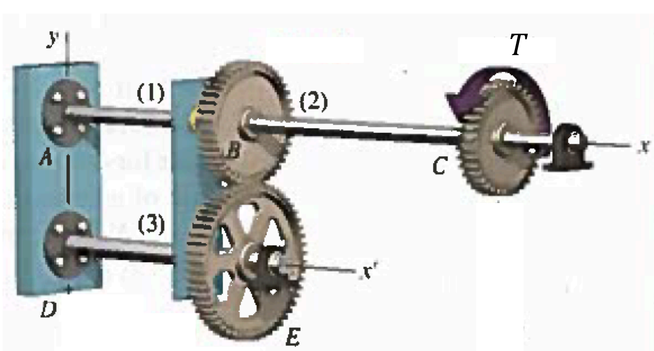
Problem 2. (60 points)

Torque T is applied to gear C as shown. The shafts are fixed at points A and D. Both of the shafts have the same diameter, and so the same polar moment of inertia J, and are made of the same material with shear modulus G. The gears at E, B and C have radii of r_E, r_B and r_C, respectively, and all gears are solidly attached to the shafts. Shaft ABC is continuous, and the shaft lengths from A to B, B to C and D to E are all L.

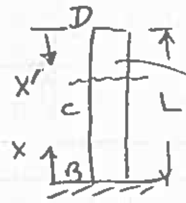
Determine:

(a) the reaction torque at point A,
(b) the rotation of gear B, and
(c) the rotation of gear C.

Express your results in terms of the parameters given: r_E, r_B, r_C, L, J and G.



1.



$x' = L - x$

FBD FROM TOP TO x' BELOW TOP:

$\rho g V = \rho g A x'$ = WEIGHT OF THIS SECTION

$N(x')$ VOLUME

$$\Sigma F_x = -\rho g A x' - N(x') \Rightarrow N = -\rho g A x' = -\rho g A(L-x)$$

AVERAGE STRESS DUE TO WEIGHT: $\sigma_g(x) = \dfrac{N}{A} = -\rho g (L-x)$

POINT B: $x = 0$, $\sigma_g = -\rho g L$

POINT C: $x = L/2$, $\sigma_g = -\rho g L/2$ (COMPRESSIVE)

POINT D: $x = L$, $\sigma_g = 0$

DEFLECTION DUE TO WEIGHT: $\delta = \int_0^x \varepsilon(x)\,dx = \int_0^x \dfrac{\sigma(x)}{E}\,dx$

$$\delta_g = -\int_0^x \frac{\rho g}{E}(L-x)\,dx = -\frac{\rho g}{E}\left(Lx - \frac{x^2}{2}\right)$$

POINT B: $x = 0$, $\delta_g = 0$

POINT C: $x = L/2$, $\delta_g = -\dfrac{3}{8}\dfrac{\rho g L^2}{E}$

POINT D: $x = L$, $\delta_g = -\dfrac{1}{2}\dfrac{\rho g L^2}{E}$ (75% OF THE DEFLECTION DUE TO THE BAR'S WEIGHT OCCURS IN THE BOTTOM HALF)

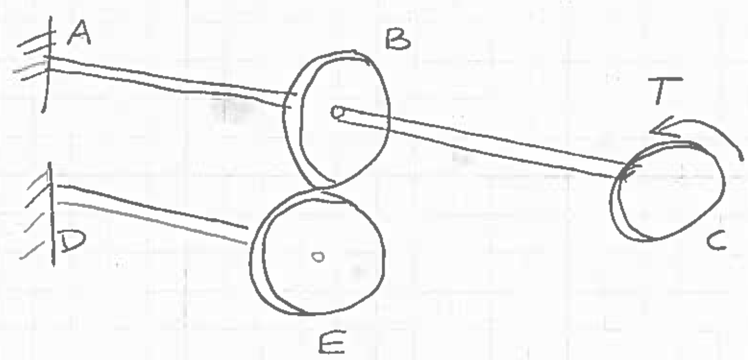
THERMAL EXPANSION: THE BAR IS NOT RESTRAINED AGAINST EXPANSION, SO THERE IS NO THERMAL STRESS. ∴ THE AVERAGE AXIAL STRESS IS THAT DUE TO THE BAR'S WEIGHT.

THE DEFLECTION DUE TO ΔT IS $\delta_T(x) = \alpha x \Delta T$

$$\delta = \delta_T + \delta_g = \alpha x \Delta T - \frac{\rho g}{E}\left(Lx - \frac{x^2}{2}\right)$$

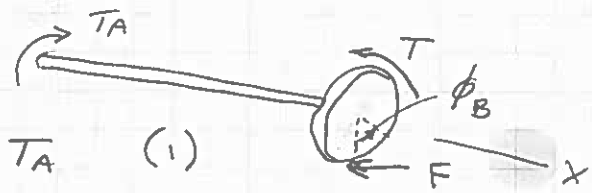
$$\boxed{B:\ \delta = 0, \quad C:\ \delta = \frac{\alpha \Delta T L}{2} - \frac{3}{8}\frac{\rho g L^2}{E}, \quad D:\ \delta = \alpha \Delta T L - \frac{1}{2}\frac{\rho g L^2}{E}}$$

2.



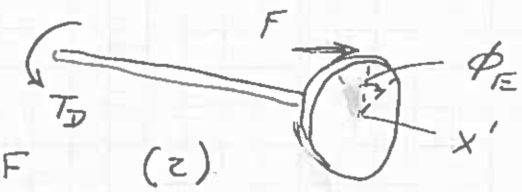
THE TORQUE T APPLIED AT C ACTS DIRECTLY ON GEAR B.

FBD OF AB:



$$\Sigma M_x = 0 = T - r_B F - T_A \qquad (1)$$

FBD OF DE:



$$\Sigma M_{x'} = 0 = T_D - r_E F \qquad (2)$$

$$F = \frac{T_D}{r_E} \Rightarrow T = T_A + \left(\frac{r_B}{r_E}\right) T_D \qquad (3)$$

<u>EQUILIBRIUM</u> $\Rightarrow$ STATICALLY INDETERMINATE PROBLEM.

<u>COMPATIBILITY</u>: $r_B \phi_B = r_E \phi_E \qquad (4)$

<u>TORQUE-TWIST</u>: $\phi_B = \dfrac{T_A L}{GJ}, \quad \phi_E = \dfrac{T_D L}{GJ} \qquad (5)$

USING (5) IN (4) $\Rightarrow r_B \dfrac{T_A L}{GJ} = r_E \dfrac{T_D L}{GJ} \Rightarrow T_D = \left(\dfrac{r_B}{r_E}\right) T_A \qquad (6)$

(a)

PLUG (6) INTO (3) $\Rightarrow T = T_A + \left(\dfrac{r_B}{r_E}\right)^2 T_A \Rightarrow \boxed{T_A = \dfrac{T}{1 + \left(\frac{r_B}{r_E}\right)^2}}$

(b)

$$\boxed{\phi_B = \frac{T_A L}{GJ} = \frac{T L}{GJ\left(1 + \left(\frac{r_B}{r_E}\right)^2\right)}}$$

(c) $\phi_C = \phi_B + \phi_{C/B} = \dfrac{TL}{GJ\left(1 + \left(\frac{r_B}{r_E}\right)^2\right)} + \dfrac{TL}{GJ}$ $\boxed{= \dfrac{TL}{GJ}\left[1 + \dfrac{1}{1 + (r_B/r_E)^2}\right] = \dfrac{TL}{GJ}\left[\dfrac{2 + (r_B/r_E)^2}{1 + (r_B/r_E)^2}\right]}$