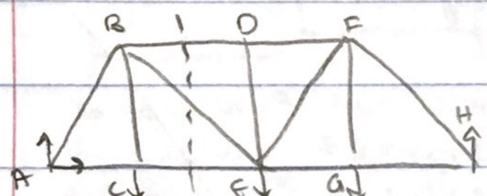
Problem 1

$\sigma = 21$ ksi $A = ?$



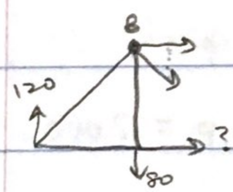
Entire Structure

$\Sigma F_x = 0 \rightarrow A_x = 0$

$\Sigma F_y = 0 = -80 - 80 - 80 + A_y + H_y$

$\Sigma M_A = 0 = (-9 \times 80) + (-18 \times 80) + (-27 \times 80) + (36 \times H_y)$

$H_y = 120 \quad A_y = 120 = 80 + 80 + 80 - 120$



Cut section

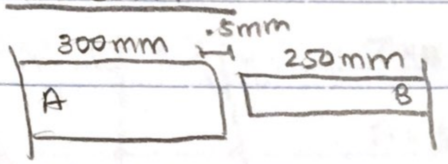
$\Sigma M_B = 0 = (120 \times 9) + (12 \times CE)$

$CE = 90$ kips

$\sigma = \frac{P}{A} = \frac{CE}{A}$

$A = \frac{CE}{\sigma} = \frac{90 \text{ kips}}{21 \text{ ksi}} = \boxed{4.29 \text{ in}^2}$

Problem 2



$T_0 = 20°C$

$T_f = 120°C$

$\Delta T = 100°C$

$A_a = 3000\,mm^2$ $A_s = 1000\,mm^2$

$E_a = 100\,GPa$ $E_s = 200\,GPa$

$\alpha_a = 20\times10^{-6}/°C$ $\alpha_s = 15\times10^{-6}/°C$

$A_a = 3000\,mm^2 \rightarrow .003\,m^2$

$A_s = 1000\,mm^2 \rightarrow .001\,m^2$

a) $\sigma = \dfrac{P}{A_a}$

$\delta = \delta_T + \delta_M$ $\delta = .0005\,m$

$.0005 = \Delta T(\alpha_a L_a + \alpha_s L_s) + P\left(\dfrac{L_a}{E_a A_a} + \dfrac{L_s}{E_s A_s}\right)$

$.0005 = 100(20\times10^{-6}\cdot .3m + 15\times10^{-6}\cdot .25m) +$
$P\left(\dfrac{.3}{100\times10^{9}\cdot .003} + \dfrac{.25}{200\times10^{9}\cdot .001}\right)$

$.0005 = 9.75\times10^{-4} + P(2.25\times10^{-9})$

$P = -211{,}111\,N \rightarrow$

$\sigma = \dfrac{-211{,}111}{.003} = \boxed{-70.4\,MPa}$

$-70370370\,Pa$

b) $\delta_a = \delta_{Ta} + \delta_{Ma} = \alpha\Delta T L + \dfrac{PL}{EA}$

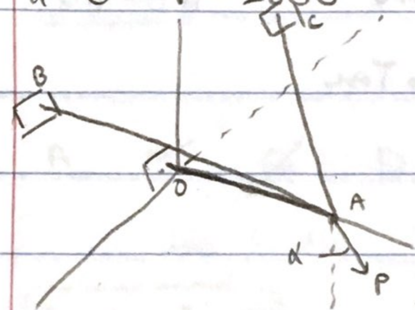
$= 100(20\times10^{-6}\cdot .3) + \dfrac{-211{,}111(.3)}{100\times10^{9}\cdot .003}$

$= 6\times10^{-4} - 2.11\times10^{-4}$

$= \boxed{3.89\times10^{-4}\,m \rightarrow}$

Problem 3

$\alpha = 0 \quad P = 2000 \quad T_{AB} = ? \quad T_{AC} = ?$



$A = (.6, 0, 0) \quad B = (0, .36, .27)$

$C = (0, .32, -.51)$

$\vec{r}_{OA} = .6\vec{i}$

$\vec{r}_{AB} = (-.6, .36, .27) \quad \|\vec{r}_{AB}\| = .75$

$\vec{r}_{AC} = (-.6, .32, -.51) \quad \|\vec{r}_{AC}\| = .85$

$\lambda_{AB} = \left(-\frac{.6}{.75}, \frac{.36}{.75}, \frac{.27}{.75}\right) \quad \lambda_{AC} = \left(\frac{-.6}{.85}, \frac{.32}{.85}, -\frac{.51}{.85}\right)$

$\vec{T}_{AB} = T_{AB}\left(-\frac{.6}{.75}\vec{i} + \frac{.36}{.75}\vec{j} + \frac{.27}{.75}\vec{k}\right)$

$\vec{T}_{AC} = T_{AC}\left(-\frac{.6}{.85} + \frac{.32}{.85}\vec{j} - \frac{.51}{.85}\vec{k}\right)$

$\Sigma M_O = 0 = \Sigma(r \times F) = (\vec{r} \times \vec{P}) + (\vec{r} \times \vec{T}_{AB}) + (\vec{r} \times \vec{T}_{AC})$

$$= \begin{vmatrix} i & j & k \\ .6 & 0 & 0 \\ 0 & -2000 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ .6 & 0 & 0 \\ T_{AB}\left(-\frac{.6}{.75}\right) & T_{AB}\left(\frac{.36}{.75}\right) & T_{AB}\left(\frac{.27}{.75}\right) \end{vmatrix} +$$

$$\begin{vmatrix} i & j & k \\ .6 & 0 & 0 \\ T_{AC}\left(\frac{-.6}{.85}\right) & T_{AC}\left(\frac{.32}{.85}\right) & T_{AC}\left(\frac{-.51}{.85}\right) \end{vmatrix}$$

$\Sigma M_{Ox} = 0$

$\Sigma M_{Oy} = 0 = -\left(.6 \cdot T_{AB}\left(\frac{.27}{.75}\right)\right) + +\left(.6 \cdot T_{AC}\left(+\frac{.51}{.85}\right)\right)$

$.36 T_{AB} = .6 T_{AC} \rightarrow T_{AB} = 1.67 T_{AC}$

$\Sigma M_{Oz} = 0 = (.6 \times -2000) + \left(.6 \cdot T_{AB}\left(\frac{.36}{.75}\right)\right) + \left(.6 \cdot T_{AC}\left(\frac{.32}{.85}\right)\right)$

NEXT PAGE $\longrightarrow$

$2000 = .48T_{AB} + .376T_{AC} \longrightarrow T_{AB} = 1.67T_{AC}$

$2000 = .48(1.67T_{AC}) + .376T_{AC}$

$2000 = 1.176T_{AC}$

$T_{AC} = 1700\ N$

$T_{AB} = 2833.3\ N$

$$\boxed{\begin{aligned} T_{AC} &= 1700\ N \\ T_{AB} &= 2830\ N \end{aligned}}$$

Problem 4

A. $\boxed{B, D}$

B. A ~~B~~ ~~C~~ D E F

$\mu_k < \mu_s$

$\boxed{A, D, E, F}$

C. $J_o = I_x + I_y = \frac{1}{8}\pi r^4 + \frac{1}{8}\pi r^4 = \frac{1}{4}\pi r^4 \to$ about AA'

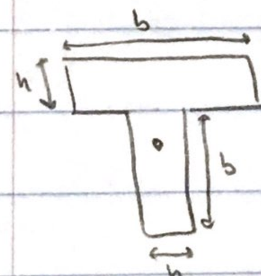
About centroidal axis

$J_{AA'} = J_{x'} + Ad^2$

$\frac{1}{4}\pi r^4 = J_o + Aa^2$

$J_o = \frac{\pi r^4}{4} - a^2 A$

$\boxed{B}$

D. 

A	$\bar{y}$	$A\bar{y}$
hb	$b/2$	$hb(b/2)$
hb	$b+h/2$	$(b+\frac{h}{2})hb$

$\bar{Y} = \frac{\Sigma \bar{y} A}{A} = \frac{\cancel{hb}(\frac{b}{2}+b+\frac{h}{2})}{2\cancel{hb}} = \frac{3b}{4} + \frac{h}{4} = \frac{1}{4}(3b+h)$

$\boxed{B}$

Next Page $\longrightarrow$

E. $I = I_1 + I_2$

$I_1 = I_c + Ad^2$

$= \frac{1}{12} bh^3 + bh\left(b + \frac{h}{2} - \frac{3}{4}b - \frac{1}{4}h\right)^2$

$I_1 = \frac{1}{12} bh^3 + bh\left(\frac{1}{4}(b+h)\right)^2$

$I_2 = \frac{1}{12} hb^3 + bh\left(\frac{3}{4}b + \frac{1}{4}h - \frac{1}{2}b\right)^2$

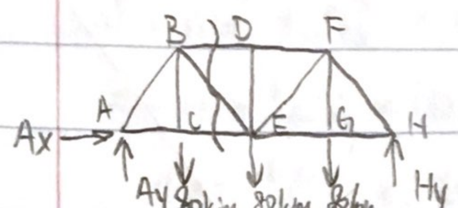
$= \frac{1}{12} hb^3 + bh\left(\frac{1}{4}(b+h)\right)^2$

$I = \frac{bh^3}{12} + \frac{hb^3}{12} + 2bh\left(\frac{1}{4}\right)^2 (b+h)^2$

$\frac{bh^3}{12} + \frac{hb^3}{12} + \frac{1}{8} bh(b+h)^2$

$\boxed{C}$

P1.



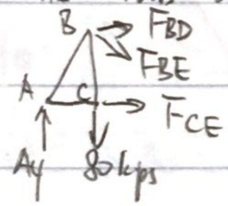
$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0 \Rightarrow A_y + H_y - 3 \times 80\,\text{kips} = 0$

By symmetry, $A_y = H_y$

$\therefore A_y = H_y = 120$ kips.

Cut the truss as shown above:



$\Sigma M_B = 0 \Rightarrow -120\text{kips} \times 9\,\text{ft} + F_{CE} \times 12\,\text{ft} = 0$

$\Rightarrow \boxed{F_{CE} = 90\ \text{kips}}$

$\sigma_{all} = \frac{F_{CE}}{A_{CE}} = 21$ ksi

$\Rightarrow \boxed{A_{CE} = \frac{90}{21}\,\text{in}^2 = 4.29\,\text{in}^2}$

P2.

$\Delta^T = \Delta_a^T + \Delta_s^T = \alpha_a \Delta T L_a + \alpha_s \Delta T L_s$

$= 20\times10^{-6}/°C \times (120°C - 20°C) \times 0.3m +$

$15\times10^{-6}/°C \times (120°C - 20°C) \times 0.25m$

$= 6\times10^{-4}m + 3.75\times10^{-4}m = 9.75\times10^{-4}m$

$\Delta^P = \Delta_a^P + \Delta_s^P = f_a P + f_s P = (f_a + f_s) P$

$f_a = \frac{L_a}{E_A A_a} = \frac{0.3m}{100\times10^9 Pa \times 3000\times10^{-6} m^2} = 1\times10^{-9} m/N$

$f_s = \frac{L_s}{E_s A_s} = \frac{0.25m}{200\times10^9 Pa \times 1000\times10^{-6} m^2} = 1.25\times10^{-9} m/N$

$\therefore f_a + f_s = 2.25\times10^{-9} m/N$

$\Delta = \Delta^T + \Delta^P = \Delta Gap = 0.5\times10^{-3} m$

$\Rightarrow 9.75\times10^{-4}m + 2.25\times10^{-9} \times P \, m = 0.5\times10^{-3}m$

$\Rightarrow P = -211.11\times10^3 N$

a) $\boxed{\sigma_a = \frac{P}{A_a} = -70.37\times10^6 Pa}$

b) $\Delta_a = \Delta_a^T + \Delta_a^P = 6\times10^{-4}m + f_a P$

$= 6\times10^{-4}m + 1\times10^{-9} \times (-211.11\times10^3) m$

$= \boxed{3.889\times10^{-4} m}$

P3.

$A(0.6, 0, 0)$ $B(0, 0.36, 0.27)$

$C(0, 0.32, -0.51)$

$\vec{r}_{AB} = -0.6\hat{i} + 0.36\hat{j} + 0.27\hat{k}$ $|\vec{r}_{AB}| = 0.75$

$\vec{\lambda}_{AB} = \dfrac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \dfrac{1}{0.75}(-0.6\hat{i} + 0.36\hat{j} + 0.27\hat{k})$

$\vec{F}_{AB} = T_{AB} \cdot \vec{\lambda}_{AB} = \dfrac{T_{AB}}{0.75}(-0.6\hat{i} + 0.36\hat{j} + 0.27\hat{k})$

$\vec{r}_{AC} = -0.6\hat{i} + 0.32\hat{j} - 0.51\hat{k}$ $|\vec{r}_{AC}| = 0.85$

$\vec{F}_{AC} = T_{AC} \cdot \vec{\lambda}_{AC} = \dfrac{T_{AC}}{0.85}(-0.6\hat{i} + 0.32\hat{j} - 0.51\hat{k})$

$\Sigma M_O = 0 \Rightarrow \vec{r}_{OA} \times (\vec{F}_{AB} + \vec{F}_{AC} + \vec{P}) = 0$

$$\Rightarrow \frac{T_{AB}}{0.75}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.6 & 0 & 0 \\ -0.6 & 0.36 & 0.27 \end{vmatrix} + \frac{T_{AC}}{0.85}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.6 & 0 & 0 \\ -0.6 & 0.32 & -0.51 \end{vmatrix}$$

$$+ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.6 & 0 & 0 \\ 0 & -2000 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \boxed{\begin{array}{l} T_{AB} = 2833.33\,N \\ T_{AC} = 1699.98\,N \end{array}}$$

P4.

A. b) d)

B. a) d) e) f)

C. b)

D. b)

E. c)

1 — Global eq: $\uparrow A_y$ $\uparrow H_y$

$A_x \rightarrow$

$\Sigma M_A = 0$

$-(9)(80) - (18)80 - 27(80) + H_y 36 = 0$

$H_y = 120$ Kips

$\Sigma F_y = 0$

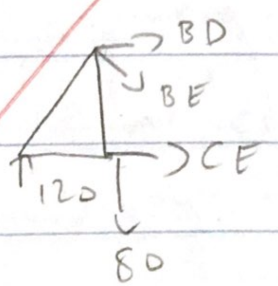
$120 - 3(80) + A_y = 0$

$A_y = 120$ kips

$\Sigma F_x = 0$

$A_x = 0$

section through BD, BE, CE



$\Sigma M_B = 0 = 12CE - 120(9)$

$CE = 90$ kips

$$\sigma = \frac{F}{A}$$

$$A = \frac{90}{21} = \boxed{4.29\,in^2}$$

2. $A_a = 3 \times 10^{-3}\ m^2$ $\quad$ $A_s = 1 \times 10^{-3}\ m^2$

$E_a = 100 \times 10^{9}\ Pa$ $\quad$ $E_s = 200 \times 10^{9}\ Pa$

$\alpha_a = 20 \times 10^{-6}\ /°C$ $\quad$ $\alpha_s = 15 \times 10^{-6}\ /°C$

① unrestrained thermal expansion

$$(\Delta L)_T = (\alpha_a L_a + \alpha_s L_s)\Delta T$$

$$= \left[(20 \times 10^{-6})(0.3) + (15 \times 10^{-6})(0.25)\right]100$$

$$= 0.975 \times 10^{-3}\ m$$

② shortening due to constraint

$$0.975\ mm - 0.5\ mm = 0.475\ mm$$

$$\Delta L_P = 0.475\ mm$$

$$(0.475\times10^{-3}) = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P$$

$$(0.475\times10^{-3}) = P\left[\frac{(0.3)}{(100\times10^{9})(3\times10^{-3})} + \frac{0.25}{(200\times10^{9})(1\times10^{-3})}\right]$$

$$P = 211.11 \text{ kN}$$

$$\sigma_a = \frac{-P}{A_a} = \boxed{-70.37 \text{ MPa}}$$

b) $$\Delta L_A = (\Delta L_T)_A - (\Delta L_P)_A$$

$$= \alpha_a L_A \Delta T - \frac{PL_a}{E_aA_a}$$

$$= (20\times10^{-6})(0.3)(100) - \frac{(211.11\times10^{3})\,0.3}{(100\times10^{9})(3\times10^{-3})}$$

$$= 600\times10^{-6} - 211.11\times10^{-6}$$

$$\boxed{= 388.89\times10^{-6} \text{ m}}$$

3_ $A = (0.6, 0, 0)$

$B = (0, 0.36, 0.27)$

$C = (0, 0.32, -0.51)$

$\vec{r}_{AB} = B - A = (-0.6, 0.36, 0.27)$

$\vec{r}_{AC} = C - A = (-0.6, 0.32, -0.51)$

$$\vec{\lambda}_{AB} = \frac{1}{0.75}(-0.6, 0.36, 0.27)$$

$$\vec{\lambda}_{AC} = \frac{1}{0.85}(-0.6, 0.32, -0.51)$$

$$\vec{F}_{AB} = \frac{T_{AB}}{0.75}(-0.6, 0.36, 0.27)$$

$$\vec{F}_{AC} = \frac{T_{AC}}{0.85}(-0.6, 0.32, -0.51)$$

$$\Sigma M_o = 0 = \vec{r}_{OA} \times (\vec{F}_{AB} + \vec{F}_{AC} + \vec{P})$$

$$= T_{AB}\begin{vmatrix} i & j & k \\ 0.6 & 0 & 0 \\ -0.8 & 0.48 & 0.36 \end{vmatrix} + \frac{T_{AC}}{0.85}\begin{vmatrix} i & j & k \\ 0.6 & 0 & 0 \\ -0.6 & 0.32 & -0.51 \end{vmatrix}$$

$$+ \begin{vmatrix} i & j & k \\ 0.6 & 0 & 0 \\ 0 & -2000 & 0 \end{vmatrix} = 0$$

$$= T_{AB}\left(0i - j(0.216) + k(0.288)\right)$$
$$+ \frac{T_{AC}}{0.85}\left(0i - j(-0.306) + k(0.192)\right)$$

$$+ \; -1200k$$

$$-0.216T_{AB} + 0.36T_{AC} = 0$$
$$T_{AB} = 1.667T_{AC}$$

$$0.288T_{AB} + 0.2259T_{AC} - 1200 = 0$$
$$0.48T_{AC} + 0.2259T_{AC} = 1200$$

$$\boxed{\begin{aligned} T_{AC} &= 1700\,N \\ T_{AB} &= 12833.8\,N \end{aligned}}$$

4A b, d

B. a, d, e, f

C. $\bar{I}_{x'} = I_{AA'} - d^2 A$

$= \frac{\pi r^4}{8} - a^2 A$

$I_x = \frac{\pi r^4}{8} - a^2 A + b^2 A$

$J_0 = \bar{I}_{x'} + I_y \Rightarrow$ b

D $\frac{\Sigma \bar{y} A}{\Sigma A} = \frac{\frac{bh}{2}(3h + b)}{2bh}$

$= \frac{3h + b}{4}$

b

E - $I_{x1} = \frac{hb^3}{12} + \frac{(h+b)^2 bh}{16}$

$I_{x2} = \frac{bh^3}{12} + \frac{(h+b)^2 bh}{16}$.

$I_x = I_{x1} + I_{x2} = \frac{bh^3 + hb^3}{12} + \frac{(h+b)^2 bh}{8}$

C_1